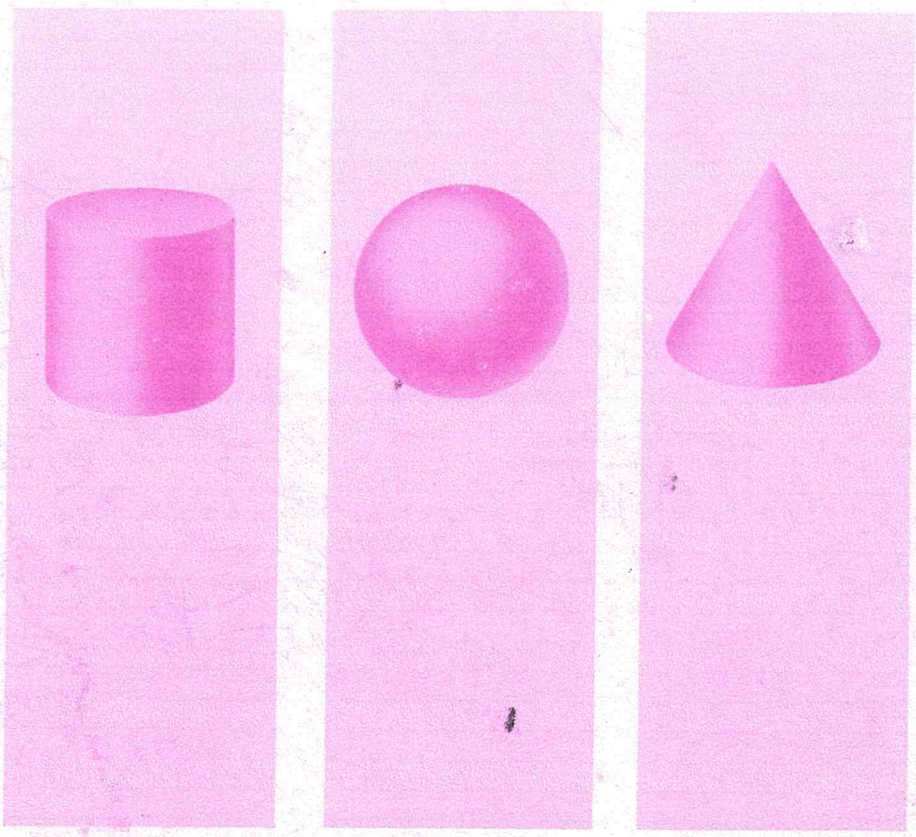
Modern School Mathematics

Structure and Use



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Illustrations by Tom Cooke

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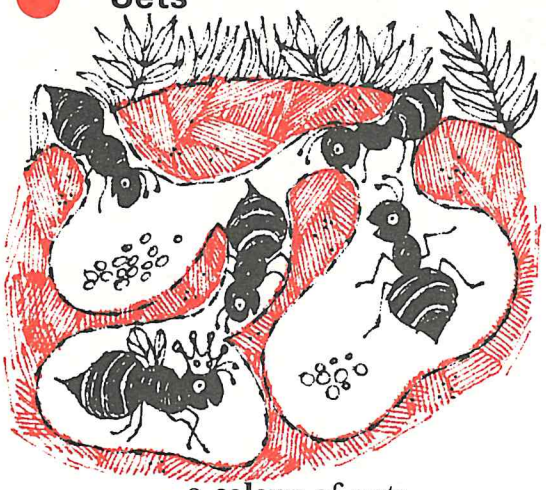
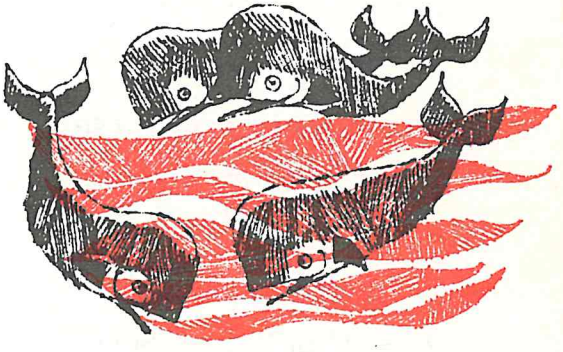
Sets



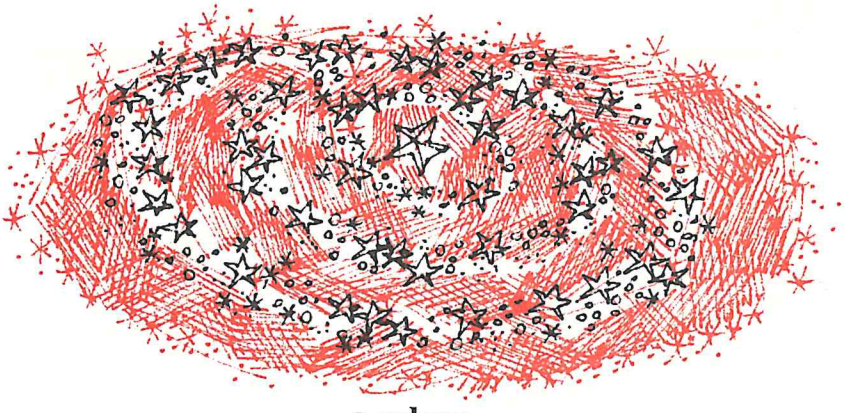
a **colony** of ants

a **covey** of quail

a **company** of firemen



a **gam** of whales



a **galaxy**

These are all examples of **sets.** Can you think of some others?

Sets

At the beginning of the year, Maxine listed as a **set** the subjects she would study in school.

{English, Mathematics, Spelling, Reading, Art, History, Music, Science}

We often use the symbols { }, called braces, to show a set.

EXERCISES

List the objects in the sets.

1. {the first 6 months of the year}
2. {the days of the week whose names begin with T}
3. {the odd numbers between 10 and 20}
4. {the 5 senses}
5. {the even numbers between 3 and 15}
6. {the 2 brothers who invented the first airplane}

Describe the objects in the sets. The first one is done for you.

$A = \{0, 1, 2, 3, 4, 5, 6\}$ $A = \{$The first seven whole numbers$\}$

7. $D = \{$North, East, South, West$\}$
8. $B = \{0, 2, 4, 6, 8, 10\}$
9. $C = \{$Santa Maria, Niña, Pinta$\}$
10. $E = \{$ounce, pound, ton$\}$

Two sets are equal only if they contain the same members. Name the pairs of equal sets.

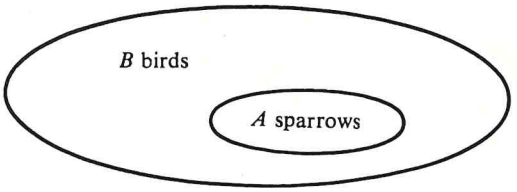
11. {○ □ △ ◇}
12. {□ ◇ ○ △}
13. {the liquid measures}
14. {pint, quart, pound, gallon}
15. {sphere, plane, cone, prism}
16. {plane, cylinder, prism, circle}
17. {the first 10 counting numbers}
18. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Complete the sentence.

19. The set with no members, { }, is called the ____?____ set.

A = {● ●}
B = {● ● △ △ ☆}

All the members of A are in B.
We say "A is a **subset** of B" and write $A \subset B$, or we say "B is a **superset** of A" and write $B \supset A$.



The diagram shows:
All sparrows are birds.
Set A is a subset of set B, $A \subset B$.

EXERCISES

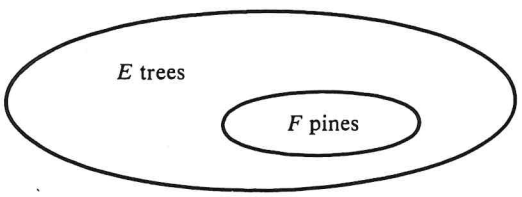
Write 1 to 4. Next to each numeral write *T* or *F* to show whether the statement is true or false.

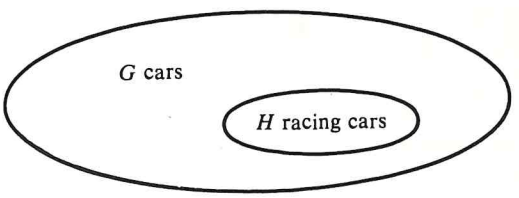
R = {● ● ● ▲ ▲ ▲ ★ ★ ★}
S = {● ▲ ★}

L = {● ● ▲ ▲ ★ ★ ■ ■ ◆ ◆}
M = {● ▲ ▲ ◆}

1. S is a subset of R.
2. { } is a subset of R and S.
3. M is a subset of L.
4. { } is a subset of L and M.

Use the diagrams to help you complete the sentences.



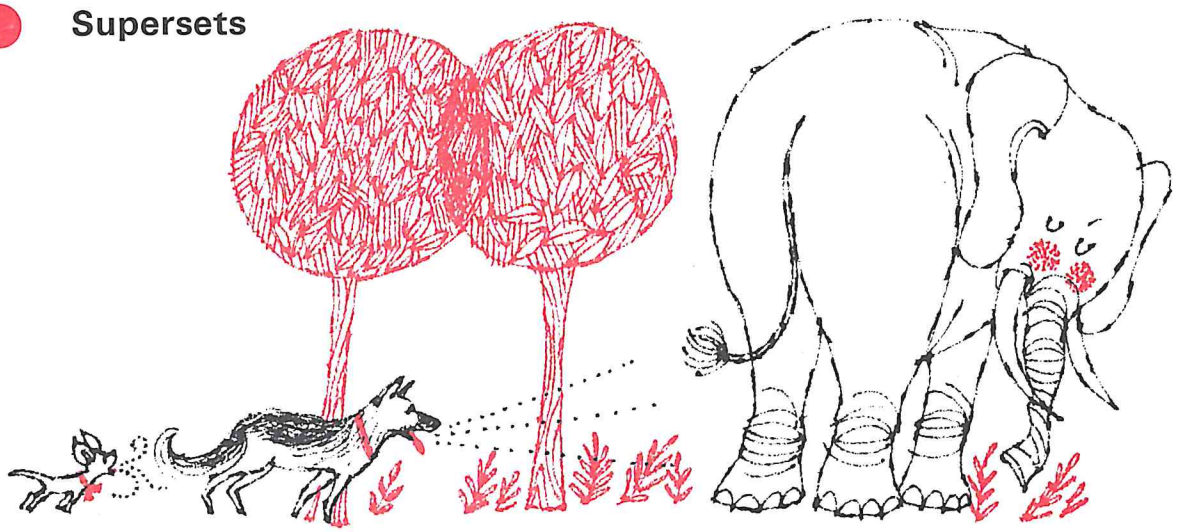


5. ___?___ pines are trees.
6. There are ___?___ trees that are not pines.
7. Set ___?___ is a subset of ___?___.
8. ___?___ racing cars are cars.
9. There are ___?___ cars that are not racing cars.
10. Set ___?___ is a subset of ___?___.

Complete the sentence.

11. The empty set is a subset of ___?___ set.

Supersets



A German shepherd is *big* if we are thinking of dogs, but *not big* if we are thinking of all kinds of animals.

It is important to know *the superset you are thinking of.* We shall call the superset we are thinking of our **universal set** or **universe.**

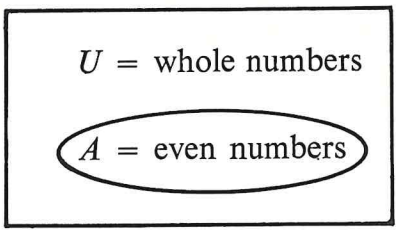
EXERCISES

Name a set that could be the universe for each of the following sets.

1. M = {horse, sheep, cat, dog}
2. R = {Arithmetic, Reading, History}
3. U = {0, 2, 6, 12, 22, 40, 68}
4. K = {2, 3, 5, 7, 11, 13, 17}
5. J = {carrot, cabbage, pea, celery, lettuce}
6. A = {counting numbers between 3 and 25}
7. E = {the first 3 days in the week}
8. L = {fractions which have 4 as their numerator}
9. S = {the colors of the American flag}
10. N = {New Zealand, France, Spain, Italy}
11. T = {Nile, Mississippi, Amazon, Rhine, Volga}
12. V = {3, 6, 9, 12, 15, 18, 21}
13. P = {a, e, o, u}
14. W = {fir, maple, oak, elm}
15. B = {Ford and Chevrolet cars}
16. F = {the Great Lakes in the United States}
17. P = {the 3 highest mountains in the world}
18. T = {animals that have 2 legs}

The diagram shows:

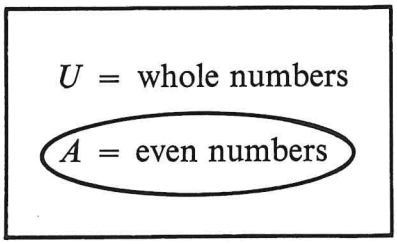
The universe (U) is the set of whole numbers.
Set A is a subset of U, $A \subset U$.



EXERCISES

Use the diagrams to help you name the universal set and its subset.

1.



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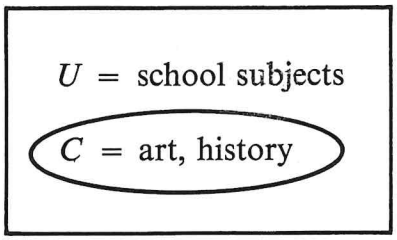

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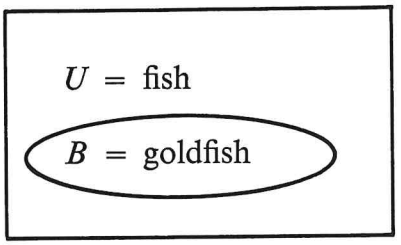

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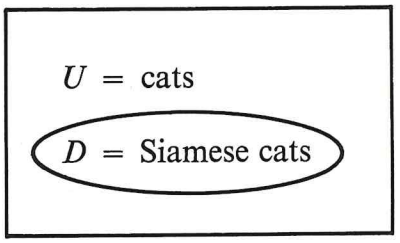


List the members in each of the following subsets of U.

$$U = \{1, 3, 4, 7, 8, 11, 15, 22\}$$

5. $A = \{\text{odd numbers}\}$

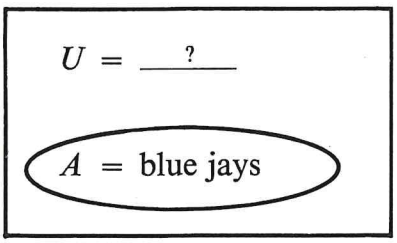
6. $B = \{\text{even numbers}\}$

7. $C = \{\text{fractional numbers}\}$

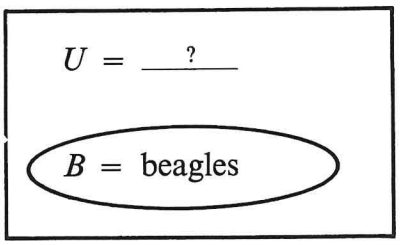
8. $D = \{\text{numbers less than } 15\}$

Name a set that could be the universe for the subsets in the diagrams.

9.



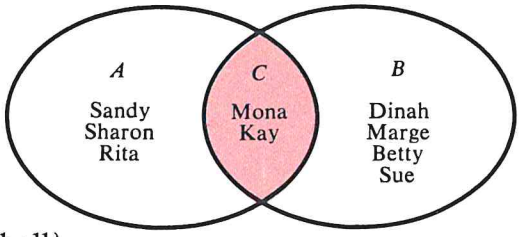
10.



The intersection of two sets

The diagram shows:

A = {girls who play basketball}
B = {girls who play softball}



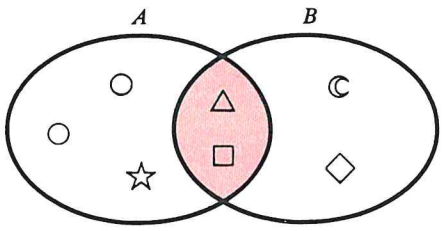
The shaded part shows:

C = {girls who play both basketball *and* softball}

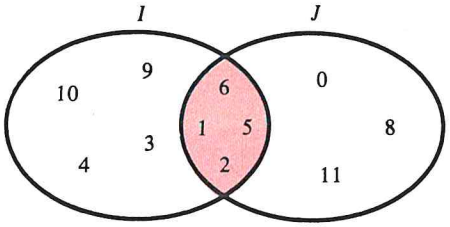
Set C is the **intersection** of sets A *and* B. The intersection of A and B, written $A \cap B$, is {Kay, Mona}.

EXERCISES

Use the diagrams to help you name the intersections of the sets.



1. $A \cap B$



2. $I \cap J$

Name the intersection of each pair of sets.

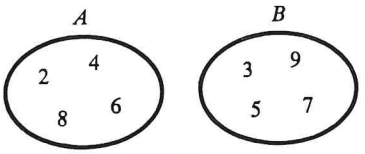
3. {a, b, c, d, e, f, g, h} and {a, e, i, o, u}

4. {China, Chile, England, Brazil} and {Denmark, India, Chile, Finland, England}

5. {cotton, wool, silk} and {nylon, wool, rayon, cotton, silk}

6. {0, 2, 4, 6, 8} and {0, 1, 2, 3, 4, 5, 6}

If 2 sets have no common members they are said to be *disjoint*. The intersection is the empty set. $A \cap B = \{\ \}$.



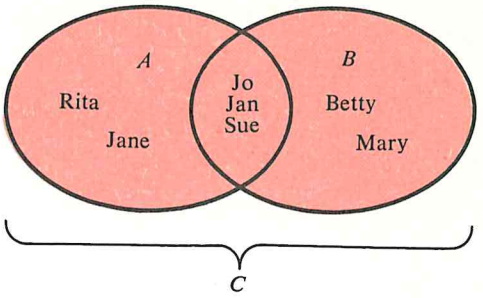
Name the intersection of each pair of sets.

7. A = {Sue, Frank, Jim, Mike} and B = {Louise, Joan, Linda}

8. E = {0, 3, 10, 16, 17} and F = {2, 4, 6, 8, 12}

The diagram shows:

A = {girls who play basketball}
B = {girls who play softball}



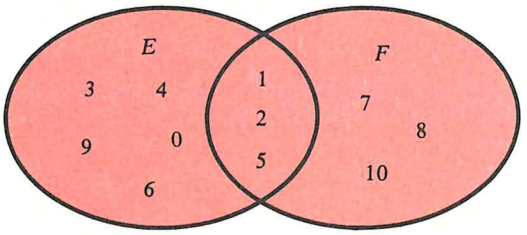
The shaded part shows:

C = {girls who play basketball *or* softball}

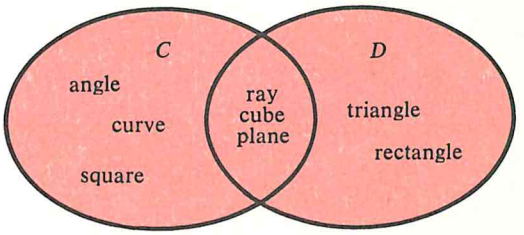
Set C is the **union** of A and B. The union of A and B, written $A \cup B$, is {Rita, Sue, Jo, Jan, Mary, Jane, Betty}.

EXERCISES

Use the diagrams to help you name the union of the sets.



1. $E \cup F$



2. $C \cup D$

Name the union of each pair of sets.

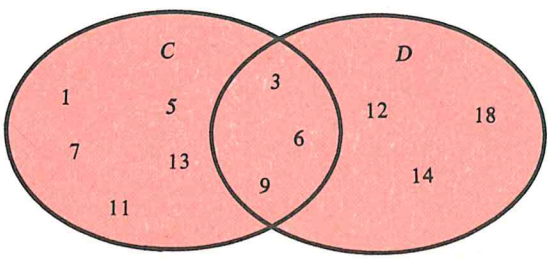
3. {Bob, Nick, Pete, Joe} and {Tom, Jack}

4. {Asia, North America, South America} and {Africa, Europe, Antarctica, Australia}

5. {butter, milk, eggs} and {toast, juice, bacon, jam, milk}

6. {○ △ ☆} and {◇ ℂ □}

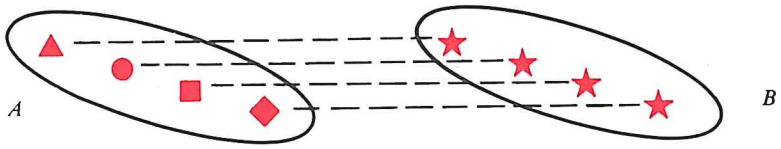
The intersection of C and D is a *subset* of both C and D. The union of C and D is a *superset* of both C and D.



*Name the sets.

7. $C \cup D$ **8.** $C \cap D$

Cardinal number



The **cardinal number** of a set tells *how many* objects the set contains. The cardinal number of A, written $n(A)$, is 4.

Sets A and B have the same cardinal number. They are **equivalent.** For each object in A there is only one object in B and for each object in B there is only one object in A.

EXERCISES

Pair the equivalent sets.

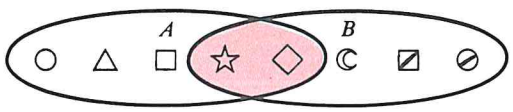
1. $A = \{\triangle\ \square\ \bigcirc\ \star\ \mathbb{C}\ \diamond\ \square\}$
2. $C = \{a, e, i, o, u\}$
3. $E = \{0, 2, 4, 6, 8, 10, 12, 14, 16\}$
4. $B = \{4 \text{ boys and } 5 \text{ girls}\}$
5. $D = \{1, 2, 3, 4, 5, 6, 7\}$
6. $F = \{\text{the odd numbers less than } 10\}$

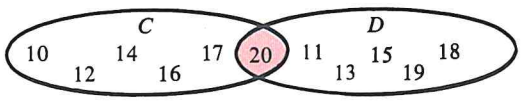
7–12. Use the n( ) symbol to name the cardinal number of each set above. For example, write $n(A) = 7$.

Complete the sentences.

13. If $n(A) = 5$ and $n(B) = 5$, then set A is ___?___ to set B.
14. If $n(C) = n(D)$, then set C is ___?___ to set D.
15. The cardinal number of the empty set is ___?___.

Study these sets, then name the cardinal numbers.





16. $n(A) = a$
17. $n(B) = b$
18. $n(A \cup B) = c$
19. $n(A \cap B) = d$
20. $n(C) = e$
21. $n(D) = f$
22. $n(C \cup D) = g$
23. $n(C \cap D) = h$

If we can list *all* the members of a set and the set has a cardinal number, it is a **finite set.** If the set is not finite, it is **infinite.**

A finite set: {Mona, Fay, Lucy}
An infinite set: {1, 2, 3, 4, 5, 6, . . .}
The 3 dots mean "and so on in the same way."

EXERCISES

Can you list the members of the following sets? Tell whether each set is finite or infinite.

1. {all names for the number 5}
2. {all counting numbers less than 14}
3. {the digits we use in writing numerals}
4. {all even numbers}
5. {all odd numbers less than 20}

Are these sets finite? Name the cardinal number of each set.

6. {all children in your classroom}
7. {all the months in a year}
8. {all the states in the United States}
9. {the seasons in the year}

Tell whether each set is finite or infinite.

10. A = {1, 2, 3, 4, . . .}
11. C = {all the trees in the nearest woods}
12. E = {0, 2, 4, 6, 8, . . .}
13. B = {all odd numbers}
14. D = {all the students who go to school}
15. F = {cars in the world}

Answer these questions.

16. If you were to place together all the grains of rice in the world at any moment to form a curve which is not closed, the curve would go around the earth many times. Would the curve have a beginning and an end? Is the set of grains of rice in the world at any moment finite or infinite?
17. In your lifetime you could not write compact numerals for all the whole numbers from 1 to 1 billion, even if you were to write one digit every second every hour every day. Is {whole numbers from 1 to 1 billion} finite or infinite?

Numbers — numerals

One symbol or name for the cardinal number of $\{\bigcirc\ \triangle\ \square\ \star\ \mathbb{C}\}$ is 5. The name for a number is called a **numeral.** Every number has many numerals. Other numerals for the cardinal number of $\{\bigcirc\ \triangle\ \square\ \star\ \mathbb{C}\}$ are five, $1 + 1 + 1 + 1 + 1$, $4 + 1$, $3 + 2$, $2 + 3$, $7 - 2$, and $25 \div 5$.

EXERCISES

Match the numerals on the right with the sets on the left.

1.	E = {all girls over 10 feet tall}	$13 - 9$
2.	B = {football, basketball, baseball, tennis ball}	$8 \div 4$
3.	J = {ring, bracelet, pin}	$6 - 1$
4.	S = {s, t, u, v, w}	0
5.	T = {3, 6, 9, 12, 15, 18, 21, 24}	$1 + 1 + 1$
6.	N = {107, 108, 109, 110, 111, 112}	$4 + 2$
7.	G = {Gretchen, Alison}	$32 \div 4$

Write the numeral on the right that does *not* name the cardinal number of the set on the left.

8.	B = {days of the week}	$5 + 2$, $12 - 5$, $4 + 4$, $28 \div 4$, 7
9.	C = {days in April}	$15 + 15$, 30, $33 - 3$, $26 + 4$, $24 + 7$
10.	D = {cookies in a dozen}	14, $20 - 8$, $5 + 7$, $24 \div 2$, $3 + 5 + 4$

Study the sets. From X, choose the name for each cardinal number.

$A = \{\bullet\ \mathbb{C}\ \blacktriangle\ \blacksquare\}$ $\quad B = \{\blacksquare\ \blacklozenge\ \mathbb{C}\ \bullet\ \bigstar\ \blacklozenge\ \blacktriangle\}$

$C = \{a, b, c, d, e\}$ $\quad D = \{f, b, g, d, h, e, i, j, k, l\}$

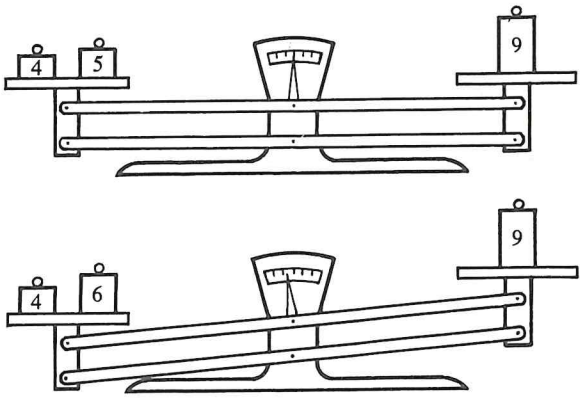
$X = \{2 + 1, 10 - 2, 12 \div 3, 6 + 5, 1 + 1 + 1 + 2, 13 - 6, 9 + 1, 6 \times 2\}$

11. $n(A)$ **12.** $n(B)$ **13.** $n(C)$ **14.** $n(D)$ **15.** $n(A \cap B)$

16. $n(A \cup B)$ **17.** $n(C \cap D)$ **18.** $n(C \cup D)$

$4 + 5 = 9$
This is an **equation.** It states that $4 + 5$ and 9 are two numerals for the same number.

$4 + 6 \neq 9$
The $\neq$ sign means "is not equal to." This is an **inequality.** If $4 + 6$ is not equal to 9, it must be less than 9 or greater than 9: $4 + 6 < 9$ *or* $4 + 6 > 9$.



EXERCISES

These number sentences must be true or false. Write *T* or *F* to show whether the number sentence is true or false.

1. $3 + 8 = 11$
2. $7 - 3 \neq 4$
3. $4 \times 8 = 31$
4. $8 + 9 \neq 5$
5. $28 \div 7 = 4$
6. $9 > 2 + 5$
7. $3 \times 7 < 13$
8. $4 + 7 < 2 \times 6$
9. $36 \div 9 > 2 \times 2$
10. $16 - 7 \neq 3 + 5$
11. $14 = 4 + 10$
12. $36 > 3 + 6$

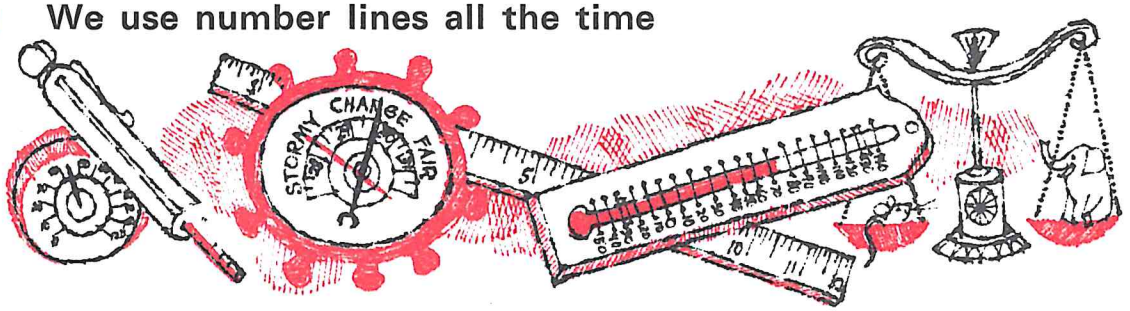
Write a numeral which will make each of these sentences true.

13. $6 + 7 = a$
14. $28 - 10 = b$
15. $2 + 3 > c$
16. $11 < 5 + d$
17. $54 \div e > 7$
18. $4 \times 6 > 23 + f$
19. $15 - g < 6$
20. $2 \times 9 = 3 \times h$
21. $6 + i \neq 14$

In each row there are 3 sentences using the same numbers. Name the 2 true sentences in each row.

22.	$3 \neq 7$	$3 < 7$	$3 > 7$
23.	$6 + 2 \neq 11$	$6 + 2 < 11$	$6 + 2 > 11$
24.	$3 \times 4 \neq 5 \times 2$	$3 \times 4 < 5 \times 2$	$3 \times 4 > 5 \times 2$
25.	$42 \div 6 \neq 54 \div 9$	$42 \div 6 < 54 \div 9$	$42 \div 6 > 54 \div 9$
26.	$5 \times 4 = 3 \times 6$	$5 \times 4 > 3 \times 6$	$5 \times 4 \neq 3 \times 6$
27.	$63 \div 7 \neq 4 \times 3$	$63 \div 7 < 4 \times 3$	$63 \div 7 > 4 \times 3$
28.	$13 - 8 \neq 2 + 1$	$13 - 8 > 2 + 1$	$13 - 8 < 2 + 1$
29.	$6 \times 8 \neq 9 \times 5$	$6 \times 8 < 9 \times 5$	$6 \times 8 > 9 \times 5$

We use number lines all the time



Number lines are used to show numbers in order. On our number line the greater numbers are on the right.

EXERCISES

Complete the sentences. Write $<$, $>$, or $=$ to replace each $\bigcirc$.

1. On the number line, 4 is to the right of 3, so $4 \bigcirc 3$.
2. On the number line, 7 is to the left of 9, so $7 \bigcirc 9$.
3. On the number line, $3 + 5$ and 8 name the same point, so $3 + 5 \bigcirc 8$.

Our number line continues on and on in either direction. Think of the number line extended to the right and name the two whole numbers just to the right of

4. 13 **5.** 739 **6.** 4768 **7.** 29,374 **8.** 401,576

Name the two whole numbers on the number line just to the left of

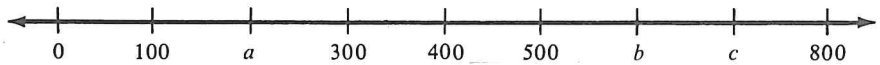
9. 17 **10.** 102 **11.** 7680 **12.** 83,924 **13.** 2799

Name the whole numbers on the number line between

14. 67 and 70 **15.** 184 and 191 **16.** 3996 and 4003

Copy and complete the number lines.

17.



18.

In our system of numerals we use the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to name any number however great or small. We do this by giving three values to each digit.

The **face value** of the 6 in 645 is *6*.
The **place value** of the 6 in 645 is *hundreds*.
The **total value** of the 6 in 645 is *600*.

EXERCISES

Name the face value of the digit in the tens' place of each numeral.

1. 73 **2.** 684 **3.** 92,024 **4.** 4691 **5.** 236,587

Name the place value and the total value of the 7 in each numeral.

6. 79 **7.** 207 **8.** 7009 **9.** 375 **10.** 467,398

The compact numeral 645 may be written as a sum of the *total values* of its digits: $645 = 600 + 40 + 5$. This is called an *expanded numeral*.

Write an expanded numeral for each compact numeral.

11. 67 **12.** 283 **13.** 4007 **14.** 706 **15.** 289

Write a compact numeral for each expanded numeral.

16. $70 + 3$
17. $40 + 9$
18. $300 + 20 + 3$
19. $800 + 50 + 6$
20. $900 + 10 + 1$
21. $500 + 60 + 7$
22. $700 + 10 + 8$
23. $2000 + 900 + 20 + 1$
24. $7000 + 300 + 60 + 7$
25. $4000 + 500 + 40 + 8$
26. $50{,}000 + 7000 + 300 + 30 + 5$
27. $90{,}000 + 9000 + 900 + 90 + 9$

Complete the sentences.

28. If the face value of a digit is 7 and its place value is hundreds, its total value is ____?____.

29. If the face value of a digit is 5 and its place value is thousands, its total value is ____?____.

Properties

To work with numerals for numbers greater than 9 we need to remember the properties of addition and multiplication.

Study this chart.

addition	multiplication
The Commutative Property The order of the addends does not affect the sum. $6 + 2 = 2 + 6$	The Commutative Property The order of the factors does not affect the product. $6 \times 2 = 2 \times 6$
The Associative Property The way addends are grouped does not affect the sum. $(3 + 4) + 5 = 3 + (4 + 5)$	The Associative Property The way factors are grouped does not affect the product. $(3 \times 4) \times 5 = 3 \times (4 \times 5)$
Zero as an Addend When 0 is one addend the sum and other addend are the same. $4 + 0 = 4, \quad 4 - 0 = 4$ $4 - 4 = 0$	One as a Factor When 1 is one factor the product and other factor are the same. $4 \times 1 = 4, \quad 4 \div 1 = 4$ $4 \div 4 = 1$
Addition and subtraction are opposites. $4 + 6 = 10, \quad 10 - 6 = 4$	Multiplication and division are opposites. $4 \times 6 = 24, \quad 24 \div 6 = 4$
The Distributive (Common Factor) Property for Multiplication over Addition If 2 addends have a common factor we may multiply then add, or add the other factors and then multiply by the common factor: $(4 \times 3) + (5 \times 3) = 12 + 15$ or $(4 \times 3) + (5 \times 3) = (4 + 5) \times 3$	

Write the numerals 1–8. Next to each, write the letter for the property used in the equation. For example, write (1, a).

1. $10 + 6 = 6 + 10$
2. $3 \times 20 = 3 \times (2 \times 10) = (3 \times 2) \times 10$
3. $379 \times 1 = 379$
4. $120 + 6 = (100 + 20) + 6 = 100 + 26$
5. $4 \times 10 \times 2 = 4 \times 2 \times 10$
6. $3 \times 120 = (3 \times 100) + (3 \times 20)$
7. $129 + 0 = 129$
8. $0 \div 6 = 0$

a. Commutative (addition)
b. Commutative (multiplication)
c. Associative (addition)
d. Associative (multiplication)
e. Zero as an addend
f. One as a factor
g. Distributive
h. Zero as a factor

Solve the equations.

9. $100 + 24 = 120 + a$
10. $3 \times 20 = b \times 10$
11. $76 + (4 - 4) = 76 + c$
12. $1900 + 99 = 1990 + d$
13. $5 \times 70 = e \times 10$
14. $160 \times 17 = (100 + 60) \times f$
15. $23 \times 100 = g \times 10$
16. $9 \times 8000 = h$
17. $232 + 60 = 290 + i$
18. $137 \times 20 = (137 \times 10) + (137 \times j)$
19. $20 \times 10 = 2 \times k$
20. $96 + 24 = (100 + l)$
21. $18 \times 8 = (m \times 8) + (8 \times 8)$
22. $49 + 101 = n + 110$
23. $35 - 17 = (20 - 10) + (15 - p)$
24. $16 \times 0 \times 135 = q$

Copy and complete the number patterns.

25.
$$\begin{aligned} 1000 + 100 + 10 + 1 &= a \\ 2000 + 200 + 20 + 2 &= b \\ 3000 + 300 + 30 + 3 &= c \\ 4000 + 400 + 40 + 4 &= d \\ 10{,}000 + 1000 + 100 + 10 &= e \end{aligned}$$

26.

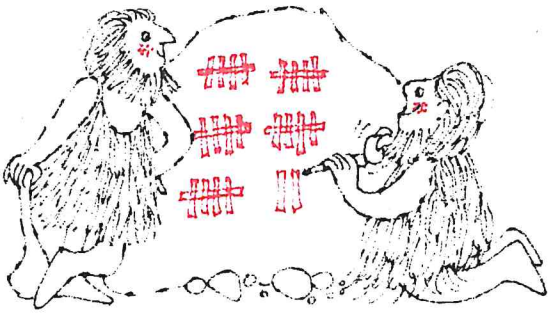
1000	200	30	4	1234
2000	400	10	3	*n*
3000	100	40	2	*a*
4000	300	20	1	*e*
10,000	*i*	*r*	*v*	*s*

Solve the equations.

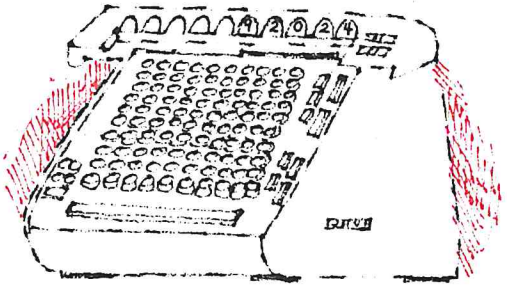
27. $(4 \times 10) + (5 \times 10) = a \times 10$
28. $240 = (b \times 10) + (4 \times 10)$
29. $12 \times 100 = (10 \times 100) + (c \times 100)$
30. $5700 = (d \times 100) + (7 \times 100)$

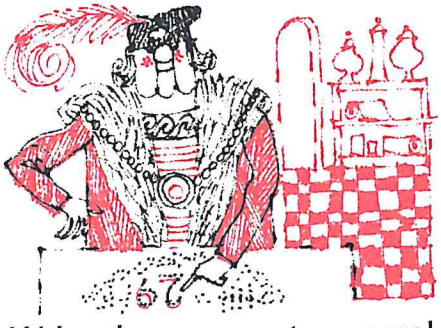
Ways of recording numbers

Old Ways of Recording Numbers



New Ways of Recording Numbers



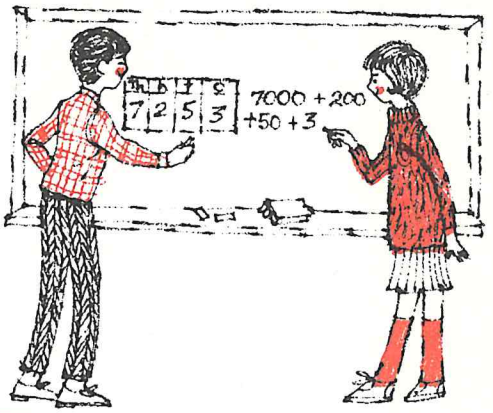


Write the compact numeral for the number shown in each illustration.

Ellen used the abacus to show 7253.

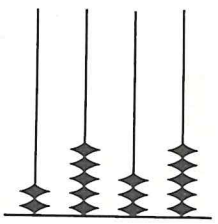
Ted used the place value chart to show 7253.

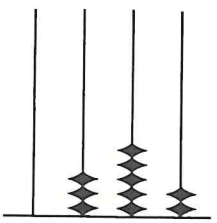
Maria used an expanded numeral to show 7253.

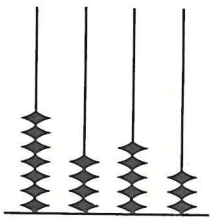


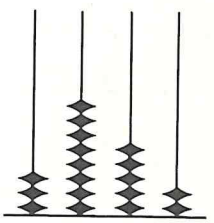
EXERCISES

Name the number shown on each abacus.

1. 

2. 

3. 

4. 

Copy and complete each place value chart for the number shown.

5. 5732

th	h	t	o
a	*b*	*c*	*d*

6. 4395

th	h	t	o
e	*f*	*g*	*h*

7. 8176

th	h	t	o
i	*j*	*k*	*l*

Solve each equation using expanded numerals.

8. $2138 = a + b + c + d$

9. $9460 = e + f + g + h$

10. $6875 = i + j + k + l$

11. $4321 = m + n + p + q$

Write compact numerals for these numbers.

12.

th	h	t	o
5	4	0	2

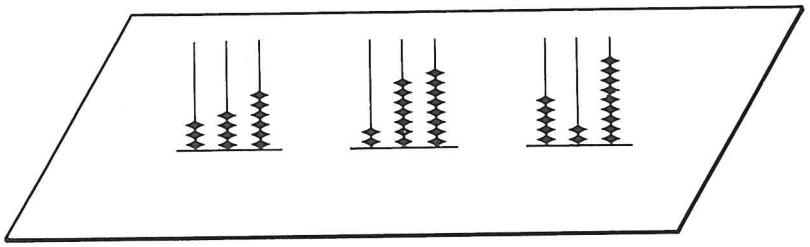
13.

th	h	t	o
3	7	8	1

14.

th	h	t	o
9	5	1	3

Counting to millions



On the abacus, no more than 9 counters will fit on each rod. Our example shows 346 millions, 278 thousands, 529 ones. The places are grouped in *periods* — the *ones*, the *thousands*, the *millions*. We use commas to separate one period from another.

EXERCISES

Rewrite each numeral using commas to show the periods.

1. 64318
2. 23720
3. 548692
4. 9253472
5. 47398561
6. 192463387

Write compact numerals for these numbers.

7.

8.

9.

10.

Name the number of millions in each numeral.

11. 471,359,110
12. 693,178,000
13. 893,472,618

Write a compact numeral for each number.

14. seventy-two thousand, five hundred eighty-three
15. eight hundred forty-five thousand, six hundred twenty-one
16. five million, two hundred thirty-seven thousand, sixty-two

In our system of numerals, we show the value of a digit by its place. In our system of measures we do not use place value, we name the unit.

Compare: 53 and 5 gallons 3 quarts

The 5 in 53 is in the tens' place, the 3 is in the ones' place.

In 5 gallons 3 quarts, the 5 names the number of gallons, the 3 names the number of quarts.

EXERCISES

Complete the sentences.

1. The most common units of length are the inch, the foot, the ___?___ and the ___?___.
2. The most common units of weight are the ounce, the ___?___ and the ___?___.
3. The most common units of liquid measure are the pint, the ___?___ and the ___?___.
4. The most common units of time are the ___?___, the ___?___ and the hour.

Write the names of the units to complete the sentences.

5. Thirteen pints is equivalent to 1 ___?___, 2 ___?___ and 1 ___?___.
6. Fourteen feet is equivalent to 4 ___?___, 2 ___?___.
7. There are 2000 pounds in a ___?___.
8. There are 60 ___?___ in one minute.

Write each of these in the smallest units.

9. 721 = ___?___ ones
10. 7 gallons 2 quarts 1 pint is equivalent to ___?___ pints.
11. 7 yards 2 feet 1 inch is equivalent to ___?___ inches.

Complete the sentences.

12. Five pints is equivalent to ___?___ quarts ___?___ pints.
13. Twenty-two cents is equivalent to ___?___ dimes ___?___ cents.

Exponents

Study the equations.

$$\begin{aligned} 2^3 &= 2 \times 2 \times 2 \\ &= (2 \times 2) \times 2 \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

$$\begin{aligned} 2^5 &= 2 \times 2 \times 2 \times 2 \times 2 \\ &= (2 \times 2) \times 2 \times 2 \times 2 \\ &= (4 \times 2) \times 2 \times 2 \\ &= (8 \times 2) \times 2 \\ &= 16 \times 2 \\ &= 32 \end{aligned}$$

The small numerals 3 and 5 show how many times 2 is used as a factor. They are **exponents.** An exponent names the **power** of the factor. The *third power* of 2, (2^3), is $2 \times 2 \times 2 = 8$. The *fifth power* of 2, (2^5), is $2 \times 2 \times 2 \times 2 \times 2 = 32$.

EXERCISES

Name the products.

1. $2^2 = a$
2. $4^2 = b$
3. $5^2 = c$
4. $1^2 = d$
5. $3^2 = e$
6. $7^2 = f$
7. $8^2 = g$
8. $2^3 = h$
9. $9^2 = i$
10. $6^3 = j$
11. $10^2 = k$
12. $1^4 = l$
13. $3^3 = m$
14. $2^4 = n$
15. $3^4 = p$
16. $0^2 = q$

Solve the equations.

17. $16 = 2 \times 2 \times 2 \times 2 = 2^a$
18. $9 = 3 \times 3 = 3^b$
19. $25 = 5^c$
20. $36 = 6^d$
21. $16 = 4^e$
22. $81 = 9^f$
23. $27 = 3^g$
24. $8 = 2^h$
25. $64 = 8^i$
26. $49 = 7^j$
27. $32 = 2^k$
28. $100 = 10^l$
29. $81 = 3^m$
30. $1000 = 10^n$

Complete the number patterns.

31.
$1^2 = 1$
$2^2 = 1 + a$
$3^2 = 1 + 3 + b$
$4^2 = 1 + 3 + 5 + c$
$5^2 = 1 + 3 + 5 + 7 + d$
$6^2 = 1 + 3 + 5 + 7 + 9 + e$
$7^2 = 1 + 3 + 5 + 7 + 9 + 11 + f$

32.
$1^2 + 1 = 2^2 - g$
$2^2 + 2 = 3^2 - h$
$3^2 + 3 = 4^2 - i$
$4^2 + 4 = 5^2 - j$
$5^2 + 5 = 6^2 - k$
$6^2 + 6 = 7^2 - l$
$7^2 + 7 = 8^2 - m$

thousands	hundreds	tens	ones
10^3	10^2	10^1	1

The first place to the left of the ones' place is for the **first power** of ten ($\mathbf{10^1}$), the second place is for the **second power** ($\mathbf{10^2}$), the third place is for the **third power** ($\mathbf{10^3}$) and so on.

EXERCISES

Solve the equations.

1. $10^1 = a$ **2.** $10^5 = b$ **3.** $10^2 = c$ **4.** $10^6 = d$ **5.** $10^3 = e$ **6.** $10^7 = f$ **7.** $10^4 = g$ **8.** $10^8 = h$

From your answers, complete this sentence.

9. In a power of 10, the exponent names the number of ___?___ in the compact numeral.

Solve the equations.

10. $(7 \times 10^4) = a$
11. $(6 \times 10^3) = b$
12. $(5 \times 10^2) = c$
13. $(3 \times 10) = d$
14. $(8 \times 10^4) = e$
15. $(3 \times 10^3) = f$
16. $(2 \times 10^2) = g$
17. $(9 \times 10) = h$
18. $(7 \times 10^4) + (6 \times 10^3) + (5 \times 10^2) + (3 \times 10) + 4 = i$
19. $(8 \times 10^4) + (3 \times 10^3) + (2 \times 10^2) + (9 \times 10) = j$
20. $(9 \times 10^4) + (2 \times 10^3) + (2 \times 10^2) + (6 \times 10) = k$

Write a compact numeral for each of these.

21. $(3 \times 10^5) + (4 \times 10^4) + (2 \times 10^3) + (6 \times 10^2) + (5 \times 10) + 4$
22. $(6 \times 10^6) + (3 \times 10^5) + (4 \times 10^4) + (2 \times 10^3) + (2 \times 10) + 8$
23. $(2 \times 10^5) + (9 \times 10^4) + (9 \times 10^3) + (3 \times 10^2) + (6 \times 10) + 1$
24. $(3 \times 10^6) + (4 \times 10^5) + (7 \times 10^4) + (9 \times 10^3) + (1 \times 10^2) + (4 \times 10) + 7$
25. $(4 \times 10^5) + (3 \times 10^4) + (2 \times 10^3) + (5 \times 10^2) + (8 \times 10) + 6$

Counting with a base of five

If man had learned in sets of five rather than ten, his counting would have begun like this:

sets	numerals	
	words	digits
{ ● }	one	1
{ ● ▲ }	two	2
{ ● ▲ ★ }	three	3
{ ● ▲ ★ ■ }	four	4
{ ● ▲ ★ ■ ◆ }	one five	10_{five}

We call this a **base five** numeral system because the basic set contains five members. The numeral for the basic set is 10_{five}. The *five* shows that the base is five. 10_{five} means "one set of five and zero ones."

EXERCISES

We can use tally marks to help us count in the system of fives. The fifth tally mark is drawn across the others to show the set of five, as shown here.

/	//	///	////	###	### /	### //	### ///	### ////	### ###
1	2	3	4	10_{five}	11_{five}	12_{five}	13_{five}	14_{five}	20_{five}

As we count in the base five system we name the numbers like this:

/	//	///	////	###	### /	### //
one	two	three	four	one five	one five one	one five two

Write base five numerals for the numbers shown by these tallies, first in digits, then in words.

1. ### / **2.** ### /// **3.** ### ### / **4.** ### ### /// **5.** ### ### ###

Draw tally marks for these base five numerals.

6. 12_{five} **7.** 20_{five} **8.** 24_{five} **9.** three fives **10.** two fives one

On this abacus, only four counters can fit on each rod. Five counters in one place may be exchanged for one counter in the place immediately to the left. We count as shown here.

one
1

two
2

three
3

four
4

one five
10

one five
one
11

one five
two
12

one five
three
13

one five
four
14

two fives

20

EXERCISES

Draw an abacus for each of these base five numerals.

1. two

2. three fives four

3. two fives three

4. four fives

5. three fives two

6. four fives three

Draw an abacus for each of these base five numerals.

7. 13_{five} **8.** 24_{five} **9.** 32_{five} **10.** 40_{five} **11.** 43_{five}

The second place to the left of the ones' place is for the second power of five, that is, for *five-fives*.

Write base five numerals for each of these numbers, shown on a base five abacus.

12.

13.

14.

15.

16.

Write each of these base five numerals in words.

17. 22_{five} **18.** 114_{five} **19.** 43_{five} **20.** 234_{five} **21.** 342_{five}

(The answer to 18 is written: one five-fives one five four.)

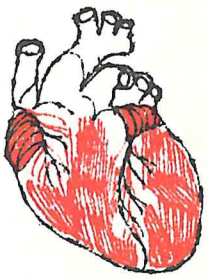
Comparing base ten and base five

This chart compares base ten and base five. Study it carefully, then copy and complete the 2 columns on the right.

BASE TEN			BASE FIVE		
grouping	compact numeral	numeral in words	grouping	compact numeral	numeral in words
○	1	one	○	1	one
○○	2	two	○○	2	two
○○○	3	three	○○○	*a*	three
○○○○	4	four	○○○○	*b*	four
○○○○○	5	five	(○○○○○)	10_{five}	one five
(○○○○○) ○	6	six	(○○○○○) ○	11_{five}	one five one
(○○○○○) ○○	7	seven	(○○○○○) ○○	*c*	one five two
(○○○○○) ○○○	8	eight	(○○○○○) ○○○	13_{five}	*d*
(○○○○○) ○○○○	9	nine	(○○○○○) ○○○○	*e*	one five four
(○○○○○) (○○○○○)	10	ten	(○○○○○) (○○○○○)	20_{five}	two fives
(○○○○○) (○○○○○) ○	11	eleven	(○○○○○) (○○○○○) ○	*f*	two fives one
(○○○○○) (○○○○○) ○○	12	twelve	(○○○○○) (○○○○○) ○○	22_{five}	*g*
(○○○○○) (○○○○○) ○○○	13	thirteen	(○○○○○) (○○○○○) ○○○	*h*	*i*
(○○○○○) (○○○○○) ○○○○	14	fourteen	(○○○○○) (○○○○○) ○○○○	24_{five}	two fives four
(○○○○○) (○○○○○) ○○○○○	15	fifteen	(○○○○○) (○○○○○) (○○○○○)	*j*	*k*

1. The area of Brazil is 3,286,270 square miles.

2. Gail's telephone number is 201–249–4673. The first 3 digits give the area code. Can you tell where Gail lives?



3. If you live to be 70 years of age, your heart will have beaten about 2,500,000,000 times. The heart is a very efficient engine.

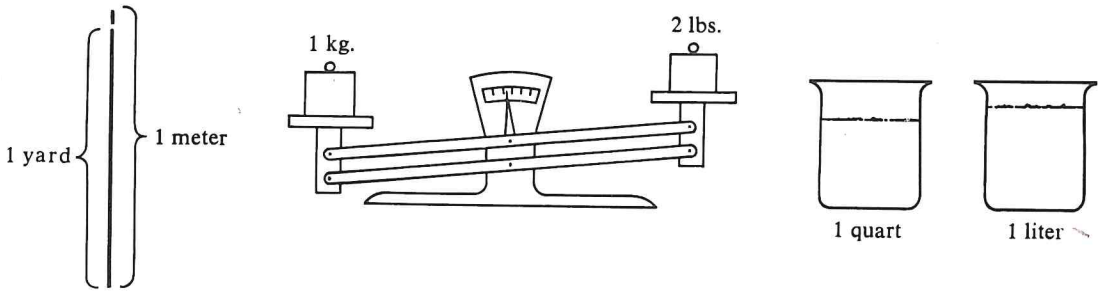
4. Mr. Smith's social security number is 281–34–4997. His driver's license number is Q–9031–78567–02375.

5. If you use a zip code number, your letter will be delivered more quickly.

6. The number of Mr. Smith's bank account is 8–0841–2376.

For some of these numbers, place value is not used. The digits are just read one after another. For example, Mr. Smith's bank account number would be read "Eight dash zero eight four one dash two three seven six."

The metric system of measures



Another system of measures that is used in many parts of the world is the **metric system.** Like our system of numeration, it uses 10 as the base. Like our system of measures, it may name the unit. It also may use place value.

basic units in the metric system		
length	the meter	— about 1 yard 3 inches
weight	the kilogram	— about 2 pounds 3 ounces
capacity	the liter	— about 1 quart 2 ounces

EXERCISES

Name the approximate number of yards in each of the following.

1. 3 meters **2.** 6 meters **3.** 11 meters **4.** 23 meters

Name the approximate number of inches in each of the following. The first one is done for you.

5. 3 meters
117 inches

6. 4 meters **7.** 7 meters **8.** 10 meters

Name the approximate number of pounds in each of these.

9. 1 kilogram **10.** 5 kilograms **11.** 20 kilograms **12.** 100 kilograms

Name the approximate number of kilograms in each of these.

13. 2 pounds **14.** 4 pounds **15.** 10 pounds **16.** 18 pounds

Name the approximate number of quarts in each of these.

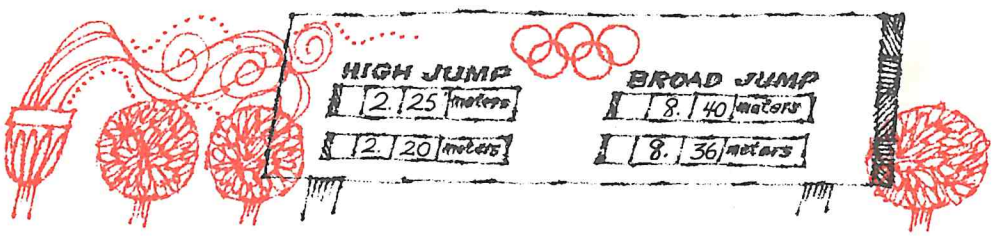
17. 2 liters **18.** 13 liters **19.** 27 liters **20.** 92 liters

The basic unit of length in the metric system is the meter (m.). Other units are shown in this chart.

1000 m.	$\frac{1}{10}$ m.	$\frac{1}{100}$ m.
kilometer	decimeter	centimeter

We may use a point to separate meters from decimeters and centimeters in the same way that we use a point to separate dollars from dimes and cents. For example, "7.53 meters" means "7 meters, 5 decimeters, 3 centimeters," "7 meters 53 centimeters," or "753 centimeters" in the same way that \$7.53 means "7 dollars, 5 dimes, 3 cents," "7 dollars and 53 cents" or "753 cents."

EXERCISES



At the Olympic Games one high jumper jumped 2.25 m.; another jumped 2.20 m.

1. How many centimeters did the first jumper jump?
2. How many centimeters more than the other jumper did he jump?

At the Olympic Games one broad jumper jumped 8.40 m.; another jumped 8.36 m.

3. How many centimeters longer was the first jump?

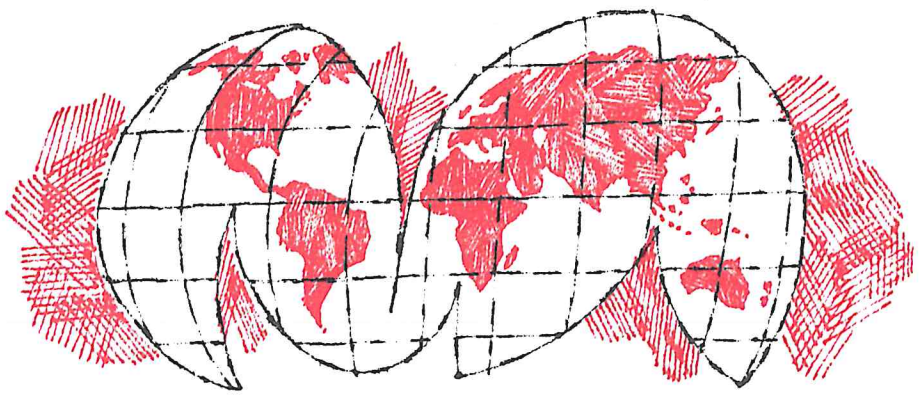
Name the number of meters in each.

4. 35 kilometers
5. 273 kilometers
6. 12 kilometers
7. 3 kilometers, 423 meters
8. 35 kilometers, 256 meters
9. 15 kilometers, 789 meters
10. 48 kilometers, 623 meters

Population of continents and countries

This table shows the populations* of the 7 continents. As you can see, there are a great many people in the world.

continent	population
Africa	296,000,000
North America	281,000,000
South America	157,000,000
Asia	1,829,200,000
Europe	634,900,000
Australia	10,965,000
Antarctica	0

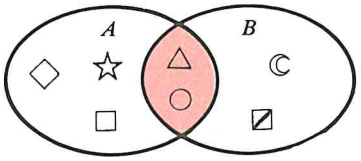


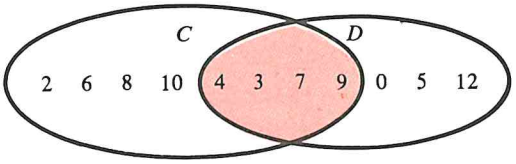
See if you can find each of the countries and cities listed in this table on a map or globe.

country	population	capital city
France	48,133,000	Paris
England	53,500,708	London
Spain	31,077,000	Madrid
New Zealand	2,574,600	Wellington
United States	192,072,000	Washington, D.C.
Italy	51,507,000	Rome
Japan	96,160,000	Tokyo

*1964

Use the diagrams to help you name the cardinal numbers.





1. n(A)
2. n(B)
3. n($A \cup B$)
4. n($A \cap B$)
5. n(C)
6. n(D)
7. n($C \cup D$)
8. n($C \cap D$)

Name the cardinal numbers, and answer the questions.

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $B = \{0, 2, 4, 6, 8, 10\}$

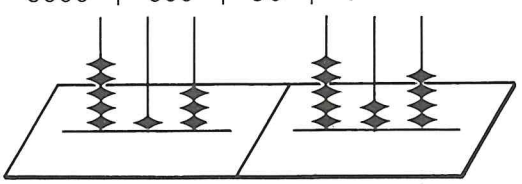
9. n(A)
10. n(B)
11. $A \cup B$
12. $A \cap B$
13. n($A \cup B$)
14. n($A \cap B$)
15. What is a universal set for A?
16. Is A a superset of $\{1, 3, 5, 7\}$?
17. Is $\{0, 4, 5, 6, 7, 10\}$ a subset of A?

Name the finite sets.

18. $A = \{10, 20, 30, 40, 50\}$
19. $C = \{1, 2, 3, 4, 5, \ldots\}$
20. $E = \{\triangle\ \bigcirc\ \diamond\ \mathbb{C}\ \square\}$
21. $B = \{\text{number of days in a year}\}$
22. $D = \{\text{number of hairs on your head}\}$
23. $F = \{\text{names for all the numbers}\}$

Write a compact numeral for each of these.

24. $200 + 70 + 3$
25. $8000 + 600 + 50 + 4$
26.



27. $15 + 10$
28.

th	h	t	o
9	7	4	2

29.

30. ten thousand, five hundred fifty-four
31. seven hundred forty-six thousand, three hundred eight
32. twenty-three million, four hundred seventy-six thousand, nine hundred eighty-two

LOOKING BACK

Set A is a **subset** of set B if *all* the members of A are in B. If A is a subset of B, B is a **superset** of A.

Write 4 subsets of each set.

1. {11, 12, 13, 14, 15}

2. {a, b, c, d, e}

The **intersection** of sets A and B is the set that contains all the objects that are in *both* A *and* B. The **union** of sets A and B is the set that contains all the objects that are *either* in A *or* in B, *or* both.

If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, name the sets.

3. $A \cap B$

4. $A \cup B$

Our system of numeration gives each digit **place value, face value,** and **total value.**

Name the total value of the 8 in each numeral.

5. 86

6. 864

7. 8645

8. 86,459

Exponents may be used to show the **power** of a factor. Using exponents and powers of ten, we may write numerals.

Write a compact numeral for each.

9. $(3 \times 10^4) + (4 \times 10^3) + (5 \times 10^2) + (9 \times 10) + 8$

10. $(6 \times 10^5) + (5 \times 10^3) + (8 \times 10^2) + (9 \times 10) + 2$

Systems of numerals may be used with bases other than ten. For example, we could have a system with **base five.**

Use digits to write these base five numerals.

11. one five four.

12. two fives three

13. three fives two

14. one five-fives two fives three

15. two five-fives four

The table with operation (+) shows:

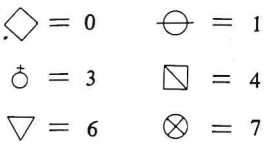
$2 + 0 = 2$ or $0 + 2 = 2$

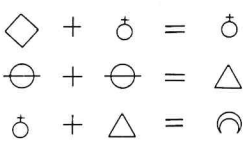
$3 + 2 = 5$ or $2 + 3 = 5$

$4 + 4 = 8$ or $4 + 4 = 8$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
4	4	5	6	7	8

Tony made the addition table at the right using the following symbols:





+	◇	⊖	△	♁	▧
◇	◇	⊖	△	♁	▧
⊖	⊖	△	♁	▧	∩
△	△	♁	▧	∩	▽
♁	♁	▧	∩	▽	⊗
▧	▧	∩	▽	⊗	☆

EXERCISES

Use the table given above to complete the equations.

1. △ + ⊖ = *a*

2. △ + △ = *b*

3. ▧ + ♁ = *c*

4. ⊖ + ♁ = *d*

5. ♁ + ⊖ = *e*

6. ▧ + △ = *f*

7. ⊖ + ▧ = *g*

8. ▧ + ⊖ = *h*

9. △ + ⊖ = *i*

Complete the equations, then answer the question.

10. ◇ + ♁ = ♁ + *a* **11.** △ + ⊖ = *b* + △ **12.** ♁ + ▧ = ▧ + *c*

13. What property of addition do exercises 10–12 remind you of?

Complete the equations, then answer the question.

14. (♁ + ◇) + ▧ = ♁ + (*a* + ▧) **15.** ⊖ + (△ + ◇) = (⊖ + △) + *b*

16. What property of addition do exercises 14–15 remind you of?

Joining and separating sets

Joining sets

$A = \{\triangle\ \bigcirc\ \triangle\ \bigcirc\}$
$B = \{\bigstar\ \bigstar\ \blacksquare\}$
$A \cup B = \{\triangle\ \bigcirc\ \triangle\ \bigcirc\ \bigstar\ \bigstar\ \blacksquare\}$
$n(A) + n(B) = n(A \cup B)$
$4 + 3 = 7$

Addition is the renaming of 2 **addends** as a **sum.**

Separating sets

$A = \{1, 2, 3, 4, 5, 6, 7\}$
$B = \{1, 2, 3\}$
$C = \{4, 5, 6, 7\}$
$n(A) - n(B) = n(C)$
$7 - 3 = 4$

Subtraction is the renaming of a sum and an addend as an addend. The missing addend is usually called the **difference.**

EXERCISES

Name the cardinal numbers of the sets.

$A = \{a, b, c, d, e\}$

1. $n(A)$

$B = \{x, y, z\}$

2. $n(B)$

3. $n(A \cup B)$

$C = \{f, a, s, t, e, r\}$

4. $n(C)$

$D = \{p, o, u, n, d\}$

5. $n(D)$

6. $n(C \cup D)$

$E = \{s, p, r, e, a, d\}$

7. $n(E)$

$F = \{d, e, a, r\}$

8. $n(F)$

$G = \{s, p\}$

9. $n(G)$

10. $n(E) - n(F)$

$H = \{\text{months of the year}\}$

11. $n(H)$

$I = \{\text{months whose names begin with M}\}$

12. $n(I)$

$J = \{\text{months whose names do not begin with M}\}$

13. $n(J)$

14. $n(H) - n(I)$

$S = \{\text{odd whole numbers between 15 and 21}\}$

15. $n(S)$

$T = \{\text{odd whole numbers between 15 and 21 which are also even}\}$

16. $n(T)$

17. $n(S \cup T)$

18. $n(S) - n(T)$

$A = \{▲ ● ■ ◆ c ★ ▲\}$

$C = \{a, b, c\}$

$n(A) = 7$, $n(C) = 3$, so $n(A) > n(C)$.

A has 4 more elements than C.
$7 - 3 = 4$.

We subtract to find *how many more* elements there are in one set *compared* with another.

EXERCISES

Write a sentence using $<$ or $>$ to compare the cardinal numbers of the sets in each pair.

1. $A = \{\text{Bob, Barbara, Bill, Ben, Bonnie}\}$ and $B = \{\text{Karl, Ken, Kathy}\}$
2. $C = \{4, 8, 7, 2, 6, 5\}$ and $D = \{0, 1, 9, 3, 11\}$
3. $E = \{p, e, a, r\}$ and $F = \{p, e, a, c, h\}$
4. $G = \{\text{feet in a yard}\}$ and $H = \{\text{inches in a foot}\}$

5–8. Write an equation to compare the cardinal numbers of the sets in each pair above.

Write the numeral to complete each sentence.

9. A set of 4 separated from a set of 6 leaves a set of ___?___.
10. A set of 5 joined with a set of 3 gives a set of ___?___.
11. The union of a set of 2 with a set of 7 gives a set of ___?___.
12. There are ___?___ more members in a set of 9 than a set of 4.
13. A set of 5 separated from a set of ___?___ leaves a set of 1.
14. The union of a set of ___?___ with a set of 10 gives a set of 16.

Study the sets, then complete the sentences.

$S = \{4, 5, 7, 2, 3, 6, 8\}$ $\quad$ $T = \{0, 1, 9, 11, 12\}$

15. $n(S) = a$ $\quad$ 16. $n(T) = b$ $\quad$ 17. $S \cup T = c$ $\quad$ 18. $n(S \cup T) = d$
19. $n(S) + n(T) = e$ $\quad$ 20. $n(S) - n(T) = f$
21. Which of these statements is true, $n(S) < n(T)$ or $n(S) > n(T)$?

Sentences

Study this chart.

	word sentence	number sentence
Open	**He** was a famous inventor.	$5 + 4 = n$ $n < 6 \quad n > 6$
Closed A. True B. False	**Edison** was a famous inventor. **George Washington** was a famous inventor.	$5 + 4 = \mathbf{9}$ $\mathbf{5} < 6 \quad \mathbf{7} > 6$ $5 + 4 = \mathbf{10}$ $\mathbf{9} < 6 \quad \mathbf{6} > 6$

We do not know whether an open word sentence is true or false until we replace the *pronoun* (*he* in our sentence) by the name of a person or thing (a *noun*). We do not know whether an open number sentence is true until we replace the *placeholder* (*n* in our sentence) by the name of a number (a *numeral*).

EXERCISES

Replace the pronouns by nouns to make each statement true.

1. **He** is a friend of mine.
2. **It** has black and white keys and makes music.
3. **They** sometimes live in igloos.
4. **It** is "man's best friend."
5. **It** rises in the east and sets in the west.
6. **It** is my favorite food.

Make these true sentences false.

7. A snake has no legs.
8. Marconi invented radio.
9. $8 - 3 < 9$
10. $3 + 5 = 8$
11. $13 > 7$
12. Four is an even number.

Make these false sentences true.

13. $3 + 5 = 9$
14. A numeral is the same as a number.
15. Zero is greater than ten.
16. $7 < 6$
17. The cardinal number of $\{5, 6, 7\}$ is 5.
18. A hen has 4 legs.

We solve a number sentence when we replace the *placeholder* by a numeral which makes the sentence *true*. The **solution** of the open equation $5 + 4 = n$ is 9.

EXERCISES

Replace each placeholder by a numeral to make these open equations true.

1. $6 + 1 = a$
2. $9 - 3 = b$
3. $8 + 1 = c$
4. $8 - 6 = d$
5. $10 + 1 = e$
6. $7 + f = 15$
7. $9 + 3 = 3 + g$
8. $10 + h = 8 + 10$
9. $14 - 7 = 6 + i$

Solve the equations.

10. $6 + 2 = a$
11. $5 + 5 = b$
12. $3 + c = 9$
13. $7 - 3 = d$
14. $6 + 6 = e$
15. $5 - f = 1$
16. $5 + 8 = g$
17. $14 - 7 = h$
18. $i - 7 = 5$
19. $11 - 3 = j$
20. $16 - 9 = k$
21. $l + 10 = 16$

Complete the sentences.

22. We use ___?___ to solve an equation when the solution is a missing sum.

23. We use ___?___ to solve an equation when the solution is a missing addend.

Write *T* or *F* to tell whether each number sentence is true or false.

24. $6 + 9 = 15$
25. $8 + 8 > 7 + 9$
26. $(5 + 8) + 1 = 5 + 9$
27. $7 + 8 = 14$
28. $3 + (4 + 5) = (3 + 4) + 5$
29. $7 + 6 > 9 + 3$
30. $8 + 3 > 10$
31. $8 + 4 < 4 + 8$
32. $13 - 6 > 14 - 5$
33. $9 + 4 < 6 + 6$
34. $16 - 9 < 17 - 9$
35. $6 + 3 + 8 < 5 + 4 + 7$

Name a number that will make each of these inequalities true.

36. $6 + 5 < a$
37. $7 + 7 > 10 + b$
38. $13 - 9 \neq c$
39. $13 - 5 > 13 - e$
40. $18 - f < 10$
41. $3 + 3 > 5 + g$

Reviewing the properties of addition

Solve the equations, then complete the sentence.

1. $6 + 5 = 5 + a$

2. $12 + 8 = b + 12$

3. $15 + c = 9 + 15$

4. $d + 10 = 10 + 11$

5. **The Commutative Property of Addition:** The order of the addends does not affect the ___?___.

Solve the equations, then complete the sentence.

6. $5 + 0 = a$ **7.** $978 + 0 = b$ **8.** $12 - 0 = c$ **9.** $19 - 19 = d$

10. **Zero as an Addend:** If zero is an addend, then the sum and the other addend are ___?___.

Copy and complete these equations, then complete the sentence.

11.
$$\begin{aligned} 14 &= 6 + 8 \\ &= (5 + 1) + a \\ &= 5 + (1 + b) \\ &= 5 + c \end{aligned}$$

12.
$$\begin{aligned} (8 + 6) + 4 &= 8 + (6 + g) \\ &= 8 + h \\ &= i \end{aligned}$$

13. **The Associative Property of Addition:** The way addends are grouped does not affect the ___?___.

Solve the equations, then complete the sentences.

14. $4 + 8 = 12$ so $12 - 8 = a$

15. $19 + 6 = 25$ so $25 - 6 = b$

16. $13 - 7 = 6$ so $7 + 6 = c$

17. $31 - 9 = 22$ so $22 + 9 = d$

18. **The Opposites Property:** Subtraction is the opposite of ___?___. Subtraction undoes ___?___ *and* addition undoes ___?___.

Since addition and subtraction are opposite operations, one can be used to check the other. For example, $6 + 4 = 10$, so $10 - 6 = 4$.

Name the sums or missing addends and check your results by using the opposites property.

19. $\begin{array}{r} 8 \\ +\ 7 \\ \hline \end{array}$ **20.** $\begin{array}{r} 12 \\ -\ 6 \\ \hline \end{array}$ **21.** $\begin{array}{r} 9 \\ +\ 5 \\ \hline \end{array}$ **22.** $\begin{array}{r} 15 \\ -\ 11 \\ \hline \end{array}$ **23.** $\begin{array}{r} 10 \\ +\ 9 \\ \hline \end{array}$ **24.** $\begin{array}{r} 16 \\ -\ 9 \\ \hline \end{array}$ **25.** $\begin{array}{r} 4 \\ +\ 13 \\ \hline \end{array}$

The simplest addition-subtraction facts are those in which both addends are greater than 0 and less than 10, for example, $3 + 6 = 9$. They are also the most important because they are the ones we must know to complete *any* addition or subtraction. There are 45 of these facts and we must know them thoroughly.

EXERCISES

List the 5 pairs of addends for the sums.

1. 10 **2.** 2 or 12 **3.** 4 or 14 **4.** 6 or 16 **5.** 8 or 18

List the 4 pairs of addends for the sums.

6. 11 **7.** 3 or 13 **8.** 5 or 15 **9.** 7 or 17 **10.** 9

Use the 45 pairs of addends you have just listed to complete these charts showing the number of facts for the sums 2 through 18.

11.

sum	2	3	4	5	6	7	8	9	10
number of facts	1	*a*	*c*	*x*	*n*	*v*	*r*	*s*	*u*

12.

sum	11	12	13	14	15	16	17	18
number of facts	*v*	*x*	*z*	*a*	*c*	*e*	*n*	*r*

The set of equations $\{6 + 4 = 10, 4 + 6 = 10, 10 - 4 = 6, 10 - 6 = 4\}$ is a **Family of Facts.** To solve the equation $6 + n = 10$ we may use any member of the family. The solution of any of these equations is 4.

Write the family of facts for each set of 2 addends and a sum.

13. $\{4, 3, 7\}$ **15.** $\{9, 4, 13\}$ **17.** $\{8, 7, 15\}$ **19.** $\{8, 8, 16\}$

14. $\{5, 6, 11\}$ **16.** $\{7, 5, 12\}$ **18.** $\{9, 9, 18\}$ **20.** $\{6, 8, 14\}$

Equations and functions

In the equation $x + 4 = y$ there are two placeholders. We can choose any number for x, then name the number for y. If we replace x by 1, our equation becomes $1 + 4 = y$. To solve this equation we replace y by 5. If we replace x by 2, y is replaced by 6. If we name the numbers for x and y that go together, we shall form a set of number pairs:

$$\{(1, 5), (2, 6), (3, 7), (4, 8), \ldots\}.$$

This set of number pairs, with the rule *add 4*, is a **function.**

EXERCISES

Complete the sentences.

1. If $x + 2 = y$ and x is 3, then y is ___?___; if x is 0, then y is ___?___.
2. If $x - 3 = y$ and x is 5, then y is ___?___; if x is 3, then y is ___?___.
3. If $x + 10 = y$ and x is 6, then y is ___?___; if x is 10, then y is ___?___.
4. If $x - 9 = y$ and x is 18, then y is ___?___; if x is 9, then y is ___?___.

Study the equations with 2 placeholders, then copy and complete the number charts.

5. $x + 1 = y$

x	2	4	7	9	10	1
y	3	5	n	r	c	s

6. $x - 6 = y$

x	8	9	13	6	15	12
y	2	x	a	e	n	i

7. $x - 4 = y$

x	5	7	11	13	4	17
y	1	r	c	s	a	n

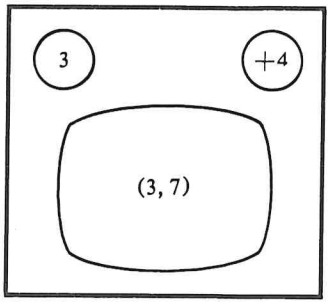
*8. $x + y = 12$

x	5	6	9	2	10	7
y	7	r	e	a	c	s

Study the rule, then complete each set of ordered pairs.

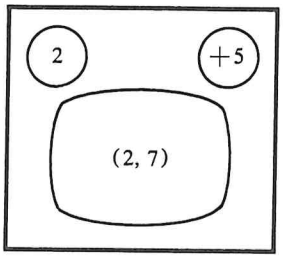
9. *Add 3:* $\{(2, a), (4, b), (7, c), (9, d), (0, e), (3, f)\}$
10. *Add 6:* $\{(1, g), (7, h), (3, i), (9, j), (8, k), (10, l)\}$
11. *Subtract 2:* $\{(4, m), (7, n), (2, o), (5, p), (11, q), (13, r)\}$
12. *Subtract 5:* $\{(8, s), (10, t), (11, u), (13, v), (15, w), (20, x)\}$
13. *Add 9:* $\{(4, y), (6, z), (0, a), (11, b), (8, c), (10, d)\}$

We may use a TV to show functions. Tom tuned in 3 on the number knob and the rule *add 4* on the rule knob. The screen shows the number pair (3, 7).

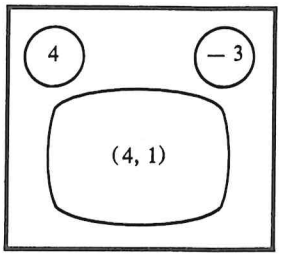


EXERCISES

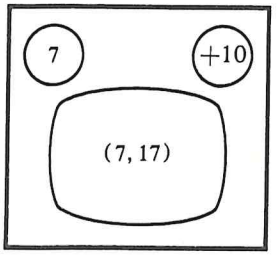
Study the function TVs and complete the sets of number pairs.



1. $\{(2, 7), (3, a), (6, b), (8, c), (9, d), (0, e)\}$

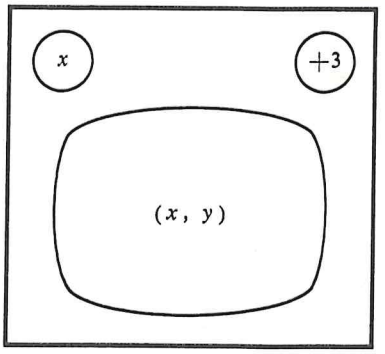


2. $\{(4, 1), (7, f), (6, g), (9, h), (13, i), (10, j)\}$



3. $\{(7, 17), (9, k), (6, l), (2, m), (0, n), (5, p)\}$

On this function TV the number knob shows x, the rule knob shows $+ 3$, so the function equation is $x + 3 = y$.

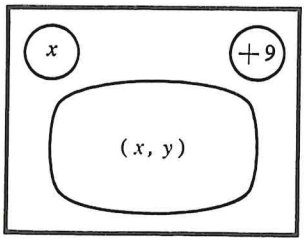


Using the rule shown on the TV, complete this set of pairs.

4. $\{(1, a), (2, b), (3, c), (4, d), (5, e), (6, f)\}$

Write the equation for the function shown on each TV. Then complete the sets of ordered pairs with first members $\{0, 1, 2, 3, 4, 5, 6\}$.

5.



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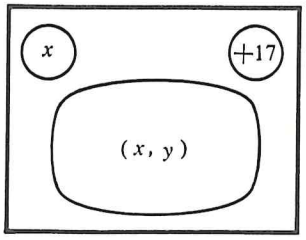
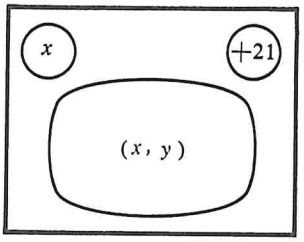


7.



Extending addition

Once we have learned the basic facts, we can use them to name any sum. We use the associative property to add 3 addends.

Study these examples.

$17 + 6 = (10 + 7) + 6$ $= 10 + (7 + 6)$ $= 10 + 13$ $= 10 + (10 + 3)$ $= (10 + 10) + 3$ $= 23$	$9 + 8 + 6 = (9 + 8) + 6$ $= 17 + 6$ $= 23$	$47 + 6 = (40 + 7) + 6$ $= 40 + (7 + 6)$ $= 40 + 13$ $= 40 + (10 + 3)$ $= (40 + 10) + 3$ $= 53$

EXERCISES

Name the sums.

1. $4 + 8 + 3 = a$ **2.** $7 + 8 + 4 = b$ **3.** $9 + 2 + 9 = c$

4. $13 + 4 = d$ **5.** $26 + 8 = e$ **6.** $37 + 8 = f$

7. $28 + 9 + 5 + 4 + 8 = g$ **8.** $38 + 6 + 7 + 4 + 9 = h$

Complete the number charts.

9.

+	8	6	2	3	9	5
17	*a*	*n*	*c*	*x*	*e*	*r*

10.

+	7	8	6	5	4	3
39	*n*	*a*	*s*	*v*	*i*	*c*

11.

+	2	9	7	5	4	0
46	*x*	*n*	*i*	*u*	*z*	*r*

12.

+	2	5	7	9	8	6
93	*s*	*e*	*r*	*v*	*w*	*x*

Name the sums.

13.
$$\begin{array}{r} 7 \\ 8 \\ 3 \\ 6 \\ 9 \\ 2 \\ +\ 4 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 12 \\ 6 \\ 8 \\ 9 \\ 7 \\ 4 \\ +\ 9 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 22 \\ 3 \\ 8 \\ 9 \\ 0 \\ 6 \\ +\ 8 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 27 \\ 4 \\ 8 \\ 9 \\ 6 \\ 3 \\ +\ 8 \\ \hline \end{array}$$

17.
$$\begin{array}{r} 39 \\ 7 \\ 8 \\ 3 \\ 6 \\ 5 \\ +\ 2 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 42 \\ 9 \\ 3 \\ 8 \\ 4 \\ 6 \\ +\ 1 \\ \hline \end{array}$$

19.
$$\begin{array}{r} 54 \\ 6 \\ 9 \\ 3 \\ 8 \\ 4 \\ +\ 5 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 87 \\ 7 \\ 8 \\ 6 \\ 5 \\ 4 \\ +\ 3 \\ \hline \end{array}$$

We may extend subtraction using the basic facts and the associative property.

Study these examples.

$17 - 4 = (10 + 7) - 4$ $= 10 + (7 - 4)$ $= 10 + 3$ $= 13$	$17 - 4 = (13 + 4) - 4$ $= 13 + (4 - 4)$ $= 13 + 0$ $= 13$	$22 - 6 = (10 + 12) - 6$ $= 10 + (12 - 6)$ $= 10 + 6$ $= 16$

EXERCISES

Name the differences.

1. $16 - 3 = a$ **5.** $98 - 5 = e$ **9.** $45 - 8 = i$ **13.** $73 - 9 = m$

2. $17 - 5 = b$ **6.** $79 - 9 = f$ **10.** $22 - 9 = j$ **14.** $74 - 9 = n$

3. $18 - 6 = c$ **7.** $15 - 9 = g$ **11.** $68 - 9 = k$ **15.** $275 - 9 = p$

4. $13 - 3 = d$ **8.** $32 - 7 = h$ **12.** $51 - 3 = l$ **16.** $376 - 9 = q$

Complete each sentence.
Write $<$, $>$, or $=$ to replace each ◯.

17. $16 - 8$ ◯ $4 + 4$ **19.** $26 + 9$ ◯ $42 - 5$ **21.** $39 + 4$ ◯ $9 + 34$

18. $97 - 7$ ◯ $82 + 7$ **20.** $73 + 8$ ◯ $72 + 9$ **22.** $27 + 8$ ◯ 4×9

Name the differences.

23. $54 - 3 - 2 - 5 - 9 - 3 = a$

24. $71 - 8 - 4 - 8 - 6 - 4 - 5 = b$

25. $39 - 9 - 8 - 7 - 6 - 5 = c$

26. $62 - 9 - 8 - 7 - 6 - 5 - 4 = d$

27. $50 - 1 - 3 - 5 - 7 - 9 = e$

28. $44 - 3 - 7 - 4 - 6 - 5 - 5 = f$

29. $54 - (3 + 2 + 5 + 9 + 3) = g$

30. $71 - (8 + 4 + 8 + 6 + 4 + 5) = h$

31. $39 - (9 + 8 + 7 + 6 + 5) = i$

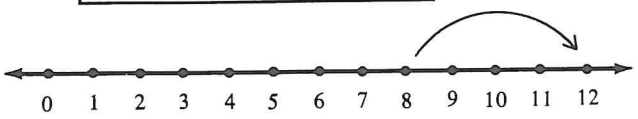
32. $62 - (9 + 8 + 7 + 6 + 5 + 4) = j$

33. $50 - (1 + 3 + 5 + 7 + 9) = k$

Number patterns and the number line

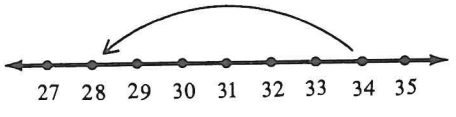
We may show addition and subtraction on the number line.

start	move	stop
8	4	12



$8 + 4 = 12$

start	move	stop
34	6	28



$34 - 6 = 28$

EXERCISES

Copy the number line and show these additions or subtractions.

1. $53 + 6 = a$ **3.** $55 + 8 = c$ **5.** $48 + 9 = e$ **7.** $52 + 12 = g$

2. $59 - 7 = b$ **4.** $53 - 5 = d$ **6.** $60 - 4 = f$ **8.** $64 - 9 = h$

Show all the following subtractions on one number line.

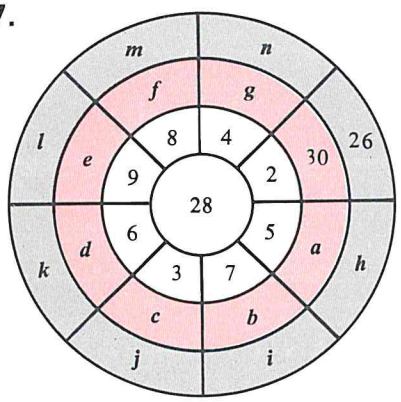
9. $43 - 3 = a$ **10.** $43 - 5 = b$ **11.** $43 - 7 = c$ **12.** $43 - 9 = d$

Show all the following additions on one number line.

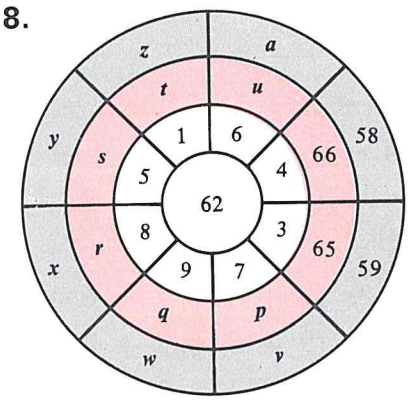
13. $76 + 6 = a$ **14.** $76 + 7 = b$ **15.** $76 + 8 = c$ **16.** $76 + 4 = d$

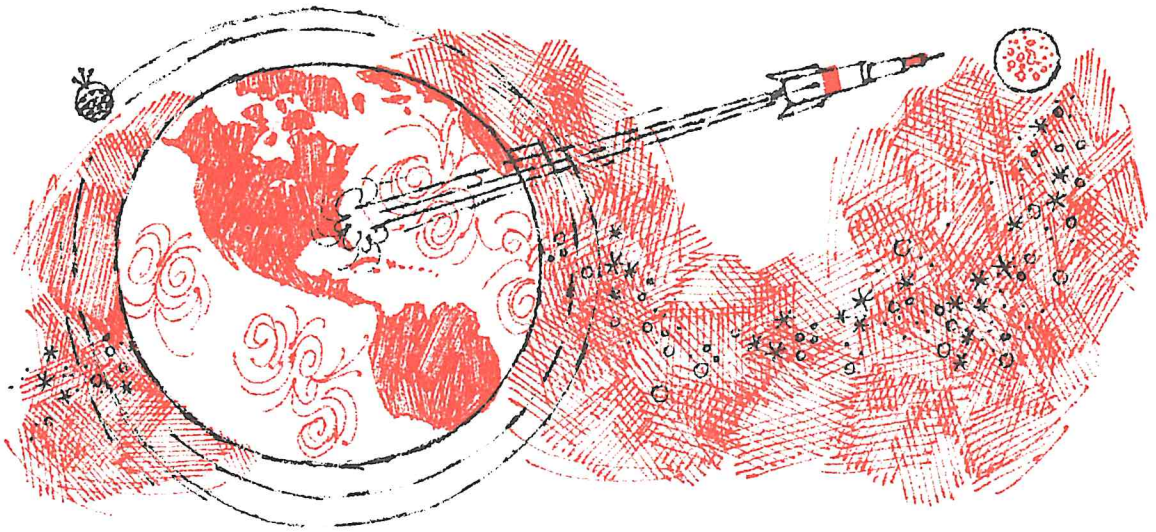
Complete the patterns by naming the sums in the red regions and the differences in the gray regions.

17.



18.





With spacecraft flying around the earth, the moon and Mars, we are meeting large numbers every day. Here are a few facts which may interest you.

1. Our sun has 9 planets including Earth. The earth travels around the sun at a speed of 66,600 miles per hour. The nearest the earth comes to the sun is 91,342,000 miles, while the farthest away it goes is 94,452,000 miles.

2. Pluto is the planet which is the greatest distance from the sun. At one point it is 4,600,000,000 miles from the sun, and about 4,700,000,000 miles from Earth. Pluto takes 90,677 days to go once around the sun compared to the 365 days it takes the earth.

3. Mercury is the smallest planet and the one nearest to the sun. It is about 36,200,000 miles from the sun, and completes one solar orbit in 88 days, at the speed of about 107,000 miles per hour.

4. Our own group of planets is quite small. If you ever look up at the Milky Way, you are looking at part of a collection of one hundred billion stars.

5. The closest star, not counting our own sun, is twenty-five thousand billion miles away.

In the pattern 1, 2, 2, 3, 4, 4, 5, 6, 6, . . . it appears that every even number occurs twice, so the pattern would continue 7, 8, 8, 9, 10, 10, 11, 12, 12, and so on.

EXERCISES

Name the next 10 members of each pattern.

1. 1, 1, 2, 3, 3, 4, 5, 5, 6, . . .
2. 1, 2, 3, 5, 6, 7, 9, 10, 11, . . .
3. 2, 1, 4, 3, 6, 5, 8, 7, 10, . . .
4. 3, 2, 1, 6, 5, 4, 9, 8, 7, 12, . . .
5. 1, 0, 2, 0, 3, 0, 4, 0, 5, . . .
6. 0, 5, 10, 5, 15, 5, 20, 5, 25, . . .

Name the missing members of these patterns.

7. 1, 4, a, 10, b, c, d, 22, 25, e, . . .
8. 5, 1, 6, f, 7, 3, 8, g, h, i, . . .
9. 60, 58, 56, j, k, 50, l, m, n, . . .
10. 60, 58, 55, p, q, 40, 33, r, s, t, . . .
11. 1, 2, 4, 7, u, v, 22, 29, w, x, . . .

Beginning with the first number in a pattern, we name each number by following a rule. For example, if the first number is 3 and the rule is *add 4 to the previous number*, we can name the numbers 3, 7, 11, 15, 19.

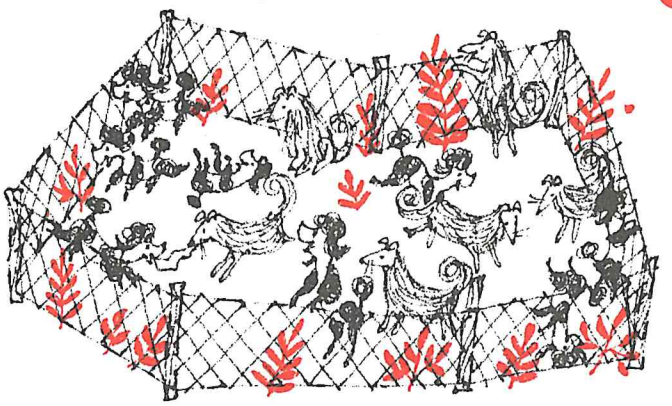
In each of these patterns we add to or subtract from the number that comes just before. Name the first 5 members of each pattern.

12.

first number	3	2	68	99	99	188	197	200
rule	$+5$	$+7$	$+9$	$+8$	-7	-6	-9	-8

13.

first number	2	7	23	102	276	43	168	327
rule	$+3$	$+4$	$+6$	-3	-9	$+5$	-4	-7



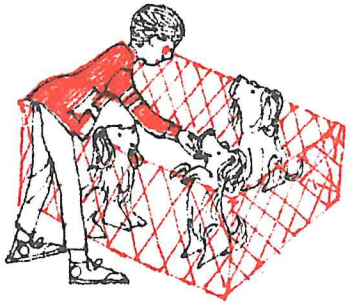
Linda used the 5-step method to solve the following problem.

At the kennel there are 8 poodles and 9 collies that are 6 weeks old. How many puppies are 6 weeks old?

The 5-step method

1. Ask, "What are the sets in the problem?" A set of **8** and a set of **9**.
2. Ask, "What is being done to the sets?" The sets are being joined.
3. Write an open equation using the cardinal numbers of the sets.
   $8 + 9 = n$
4. Solve the equation. $8 + 9 = 17$
5. Use the solution to answer the question in the problem. There are **17** puppies that are 6 weeks old.

EXERCISES



1. If there are 17 cocker spaniel puppies and 6 collie puppies, how many puppies in all?

2. If there are 25 puppies and 9 of them are beagles, how many are not beagles?

3. There are 4 brown and 18 black puppies in one pen. How many puppies all together?

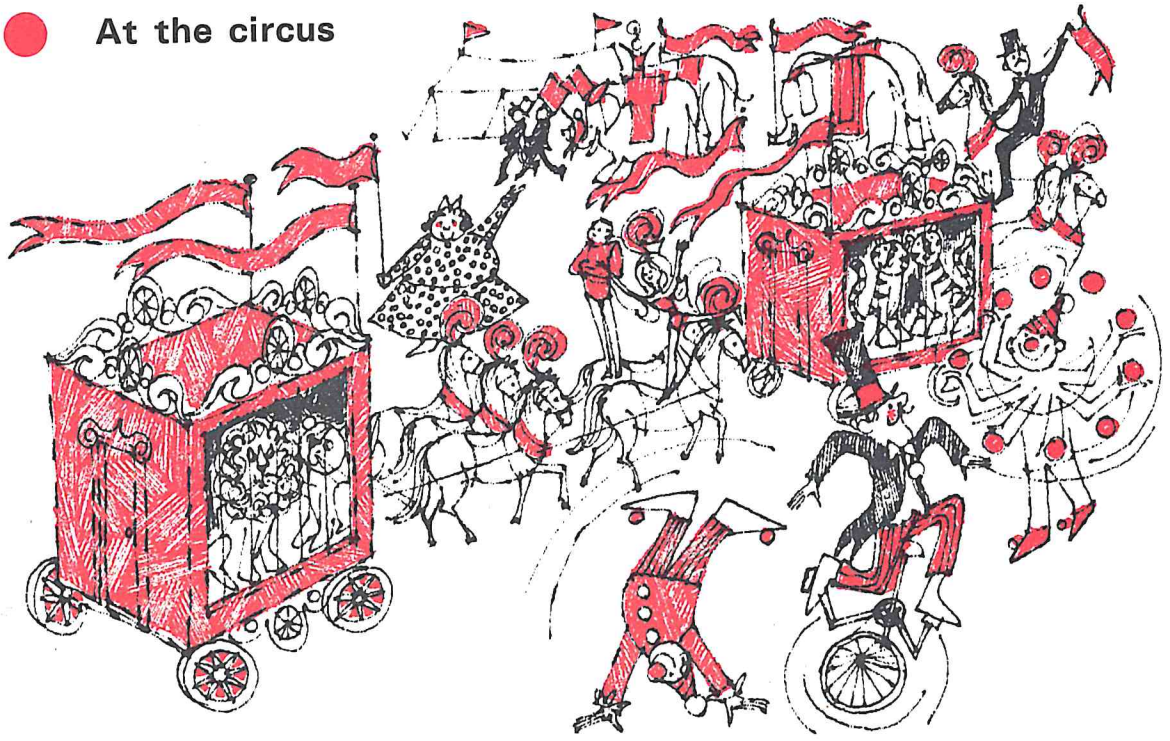
4. There were 9 fox terrier puppies. Seven were sold last week. How many fox terrier puppies are left?

5. Five collie and 17 beagle puppies were born on 1 day. How many puppies were born that day?

6. There are 23 puppies, some brown, some spotted. If 7 are spotted, how many are brown?

7. One day there were 19 puppies. If 6 were sold, how many were left?

At the circus



1. There were 6 bears in one ring, and 13 dogs in another. How many animals are there in all?

2. Twenty-three clowns were in a little house, and 17 in a little car. What was the total number of clowns?

3. In one ring there were 12 bareback riders, in the second 9 acrobats, and in the third 8 jugglers. How many circus performers in all 3 rings?

4. In the circus parade there were 7 elephants, 11 lions, 38 horses, and 22 monkeys. How many animals in all?

5. All together, there are 25 trapeze artists and 16 acrobats in the circus. How many more trapeze artists are there?

6. There are 21 brown horses and 7 white ones. How many more brown horses than white ones are there?

7. If there were 37 people sitting in one row, and 8 of them went to buy peanuts, how many were left?

8. The fat lady weighs 393 pounds, and one of the bareback riders weighs 118 pounds. How much heavier is the fat lady?

Study these methods for naming the sum 34 + 28.

Expanded numerals	Three-step	Short form
$34 = 30 + 4$ $+\ 28 = 20 + 8$ $50 + 12 = 62$	34 + 28 12 50 62	1 34 + 28 62

In each method we see that 4 ones + 8 ones = 12 ones, which we rename as 1 ten + 2 ones.

EXERCISES

Use expanded numerals to name the sums.

1. 26 + 53 **2.** 32 + 64 **3.** 45 + 48 **4.** 29 + 65 **5.** 53 + 17 **6.** 38 + 47

Use the 3-step method to name the sums.

7. 54 + 32 **8.** 46 + 23 **9.** 64 + 26 **10.** 39 + 43 **11.** 19 + 58 **12.** 58 + 36

Use the short form to name the sums.

13. 23 + 25 **15.** 57 + 32 **17.** 44 + 26 **19.** 65 + 37 **21.** 28 + 69 **23.** 54 + 48

14. 36 + 48 **16.** 73 + 25 **18.** 49 + 31 **20.** 66 + 35 **22.** 27 + 53 **24.** 49 + 42

Solve the problems.

25. If Jerry earned 85¢ and Frank earned 68¢, how much did the boys earn in all?

26. Elsie has 48 records in her collection and Janice has 76 in hers. How many records do the girls have all together?

Extending addition

Study these methods for naming the sum 432 + 285.

Expanded numerals	Four-step	Short form
$\begin{array}{rl} 432 = & 400 + 30 + 2 \\ +\ 285 = & 200 + 80 + 5 \\ \hline & 600 + 110 + 7 = 717 \end{array}$	$\begin{array}{r} 432 \\ +\ 285 \\ \hline 7 \\ 110 \\ 600 \\ \hline 717 \end{array}$	$\begin{array}{r} {\scriptstyle 1} \\ 432 \\ +\ 285 \\ \hline 717 \end{array}$

EXERCISES

Use expanded numerals to name the sums.

1. $\begin{array}{r} 326 \\ +\ 259 \\ \hline \end{array}$ **2.** $\begin{array}{r} 548 \\ +\ 325 \\ \hline \end{array}$ **3.** $\begin{array}{r} 714 \\ +\ 457 \\ \hline \end{array}$ **4.** $\begin{array}{r} 838 \\ +\ 597 \\ \hline \end{array}$ **5.** $\begin{array}{r} 962 \\ +\ 338 \\ \hline \end{array}$

Use the 4-step method to name the sums.

6. $\begin{array}{r} 596 \\ +\ 263 \\ \hline \end{array}$ **7.** $\begin{array}{r} 931 \\ +\ 425 \\ \hline \end{array}$ **8.** $\begin{array}{r} 779 \\ +\ 482 \\ \hline \end{array}$ **9.** $\begin{array}{r} 973 \\ +\ 548 \\ \hline \end{array}$ **10.** $\begin{array}{r} 989 \\ +\ 827 \\ \hline \end{array}$

Use the short form to name the sums.

11. $\begin{array}{r} 465 \\ +\ 226 \\ \hline \end{array}$ **13.** $\begin{array}{r} 638 \\ +\ 257 \\ \hline \end{array}$ **15.** $\begin{array}{r} 798 \\ +\ 798 \\ \hline \end{array}$ **17.** $\begin{array}{r} 845 \\ +\ 797 \\ \hline \end{array}$ **19.** $\begin{array}{r} 994 \\ +\ 456 \\ \hline \end{array}$

12. $\begin{array}{r} 345 \\ +\ 123 \\ \hline \end{array}$ **14.** $\begin{array}{r} 920 \\ +\ 911 \\ \hline \end{array}$ **16.** $\begin{array}{r} 419 \\ +\ 105 \\ \hline \end{array}$ **18.** $\begin{array}{r} 607 \\ +\ 410 \\ \hline \end{array}$ **20.** $\begin{array}{r} 912 \\ +\ 716 \\ \hline \end{array}$

Solve the problems.

21. At the Animal Show there were 226 dogs and 195 cats. How many dogs and cats were in the show?

22. If 849 people attend the Animal Show on the first day and 786 came on the second day, how many people attended on these 2 days?

Study the patterns.

$$\begin{array}{rcl} 100 + 20 + 3 & = & 123 \\ 300 + 40 + 8 & = & 348 \\ \underline{200 + 50 + 4} & = & \underline{254} \\ 600 + 110 + 15 & = & 725 \end{array} \qquad \begin{array}{rcl} 100 + 300 + 200 & = & 600 \\ 20 + 40 + 50 & = & 110 \\ \underline{3 + 8 + 4} & = & \underline{15} \\ 123 + 348 + 254 & = & 725 \end{array}$$

EXERCISES

Complete the patterns.

1.
$$\begin{array}{rcl} 200 + 30 + 5 & = & 235 \\ 400 + 60 + 3 & = & a \\ \underline{100 + 20 + 7} & = & \underline{b} \\ 700 + 110 + 15 & = & c \end{array}$$

2.
$$\begin{array}{rcl} 700 + 80 + 6 & = & g \\ 800 + 60 + 7 & = & h \\ \underline{600 + 70 + 8} & = & \underline{i} \\ j + k + l & = & m \end{array}$$

3.

300	60	5	365
400	30	7	n
100	50	2	x
800	c	a	e

4.

700	80	6	u
800	60	7	e
600	70	8	i
n	s	x	a

5.

800	40	7	e
800	50	8	s
800	60	9	n
u	v	w	x

6.
$$\begin{array}{rcl} 400 + 300 + 800 & = & 1500 \\ 80 + 30 + 50 & = & 160 \\ \underline{5 + 3 + 7} & = & \underline{15} \\ 485 + d + e & = & f \end{array}$$

7.
$$\begin{array}{rcl} 700 + 700 + 700 & = & n \\ 50 + 30 + 80 & = & a \\ \underline{6 + 9 + 6} & = & \underline{p} \\ q + r + s & = & t \end{array}$$

8.

200	400	900	1500
80	70	20	a
3	9	7	e
283	n	v	x

9.

600	600	600	c
80	80	80	u
7	8	9	n
a	i	s	r

10.

500	700	800	2000
60	40	70	a
9	9	9	i
569	m	r	s

Adding greater numbers

Study these examples.

A	B	C
4365	4365	4365
+ 3528	+ 3568	+ 8796
7893	7933	13161

In example A we rename 10 ones as 1 ten; in example B we rename 10 ones as 1 ten and 10 tens as 1 hundred; in example C we rename 10 ones as 1 ten, 10 tens as 1 hundred, and 10 hundreds as 1 thousand.

EXERCISES

Name the sums.

1. 5237 + 6458	**3.** 7366 + 2593	**5.** 8766 + 1821	**7.** 5487 + 3296	**9.** 7329 + 3292
2. 4697 + 2285	**4.** 5784 + 3678	**6.** 5849 + 4683	**8.** 8629 + 4734	**10.** 7679 + 6859

Name the sums.

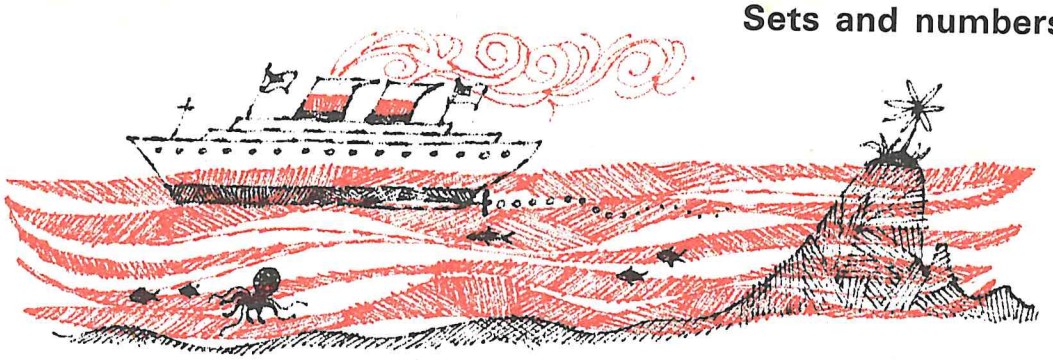
11. 53252 + 23416	**13.** 38465 + 20927	**15.** 45762 + 43985	**17.** 12058 + 73966	**19.** 83954 + 78789
12. 12345 + 67890	**14.** 98765 + 40321	**16.** 80906 + 19094	**18.** 123589 + 68476	**20.** 468794 + 329857

Name the sums.

21. 28	**22.** 47	**23.** 123	**24.** 325	**25.** 1357	**26.** 3621
56	65	456	437	2468	4587
43	31	789	995	3579	8720
95	19	876	871	+ 4681	+ 6069
+ 52	+ 58	+ 543	+ 302		

Solve the problem.

27. The attendances for the first three Mustang football games were 2376, 3092, and 2954. What was the total attendance?



ocean	area (square miles)	average depth (feet)
Pacific	63,801,668	14,048
Atlantic	31,839,306	12,880
Indian	28,356,306	13,002
Arctic	5,440,197	3,953

The chart above shows the area and average depth of the four major oceans in the world.

EXERCISES

1. What is the total of the areas of the Pacific and Arctic Oceans?
2. What is the total of the areas of the Atlantic and Indian Oceans?
3. How much greater in area is the Pacific Ocean than the Atlantic?
4. How much greater in area is the Indian Ocean than the Arctic?
5. What is the total of the depths of the Pacific and Indian Oceans?
6. What is the total of the depths of the Atlantic and Arctic Oceans?
7. How much deeper is the Pacific Ocean than the Arctic?
8. There are some places in the Pacific Ocean that are 35,000 feet deep. How much deeper is that than the average depth?

Extending subtraction

Study these methods for naming the difference 64 − 28.

Expanded numerals	Short form
$64 = 60 + 4 = 50 + 14$ $-28 = -20 - 8 = -20 - 8$ $30 + 6 = 36$	$\overset{5}{\cancel{6}}\overset{14}{\cancel{4}}$ $-\ 28$ 36

Notice that in each method we rename 1 ten as 10 ones.

EXERCISES

Use expanded numerals to name the differences.

1. 76 − 43 **2.** 38 − 18 **3.** 43 − 27 **4.** 56 − 39 **5.** 84 − 55 **6.** 92 − 27

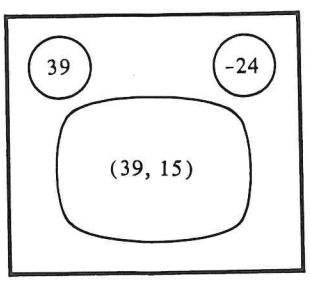
Use the short form to name the differences.

7. 54 − 32 **10.** 97 − 79 **13.** 60 − 36 **16.** 91 − 62 **19.** 65 − 37 **22.** 85 − 52

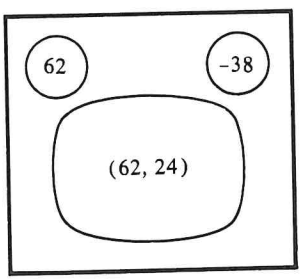
8. 43 − 20 **11.** 32 − 17 **14.** 100 − 52 **17.** 76 − 19 **20.** 48 − 29 **23.** 53 − 26

9. 61 − 49 **12.** 60 − 21 **15.** 95 − 47 **18.** 92 − 53 **21.** 68 − 47 **24.** 92 − 78

Study the function TVs, then complete each set of number pairs.



25. {(39, 15), (47, *a*), (79, *b*), (32, *c*), (24, *d*), (91, *e*)}



26. {(62, 24), (78, *f*), (42, *g*), (72, *h*), (95, *i*), (77, *j*)}

Study this example.

	Step 1	Step 2	Step 3
3 2 4 − 1 6 8	1 14 3 2 4 − 1 6 8 6	2 11 14 ~~3~~ ~~2~~ ~~4~~ − 1 6 8 5 6	2 11 14 ~~3~~ ~~2~~ ~~4~~ − 1 6 8 1 5 6

Step 1: Rename 1 ten as 10 ones. $(10 + 4) - 8 = 6$

Step 2: Rename 1 hundred as 10 tens. $(\mathbf{100} + \mathbf{10}) - 60 = 50$

Step 3: $200 - 100 = 100$

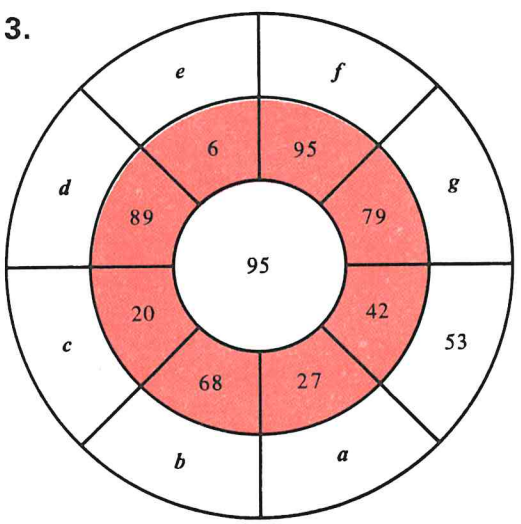
EXERCISES

Name the differences.

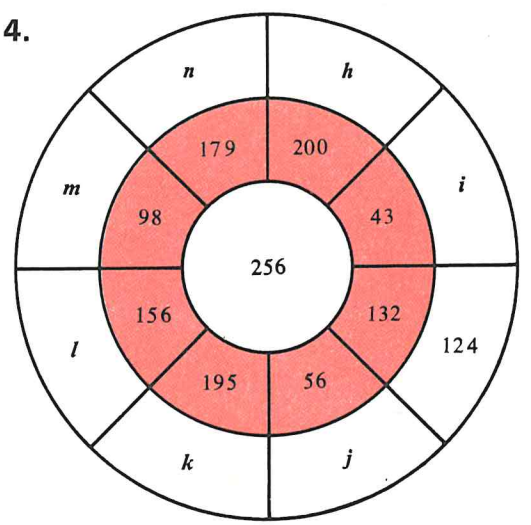
1. 456 − 239	**3.** 648 − 385	**5.** 754 − 294	**7.** 621 − 170	**9.** 692 − 378	**11.** 429 − 379
2. 526 − 378	**4.** 942 − 575	**6.** 820 − 358	**8.** 900 − 547	**10.** 644 − 589	**12.** 447 − 349

Complete each pattern by naming the difference between the number in the center and each number in the red region.

13.



14.



Subtracting greater numbers

Study this example.

	Step 1	Step 2	Step 3	Step 4
4 5 3 2 − 2 6 5 7	2 12 4 5 ~~3~~ ~~2~~ − 2 6 5 7 5	4 **12** 12 4 ~~5~~ ~~3~~ ~~2~~ − 2 6 5 7 7 5	3 14 12 12 ~~4~~ ~~5~~ ~~3~~ ~~2~~ − 2 6 5 7 8 7 5	3 14 12 12 ~~4~~ ~~5~~ ~~3~~ ~~2~~ − 2 6 5 7 1 8 7 5

Step 1: Rename 1 ten as 10 ones. $(10 + 2) - 7 = 5$

Step 2: Rename 1 hundred as 10 tens. $(\mathbf{100} + \mathbf{20}) - 50 = 70$

Step 3: Rename 1 thousand as 10 hundreds. $(1000 + 400) - 600 = 800$

Step 4: $\mathbf{3000} - 2000 = 1000$

EXERCISES

Name the differences.

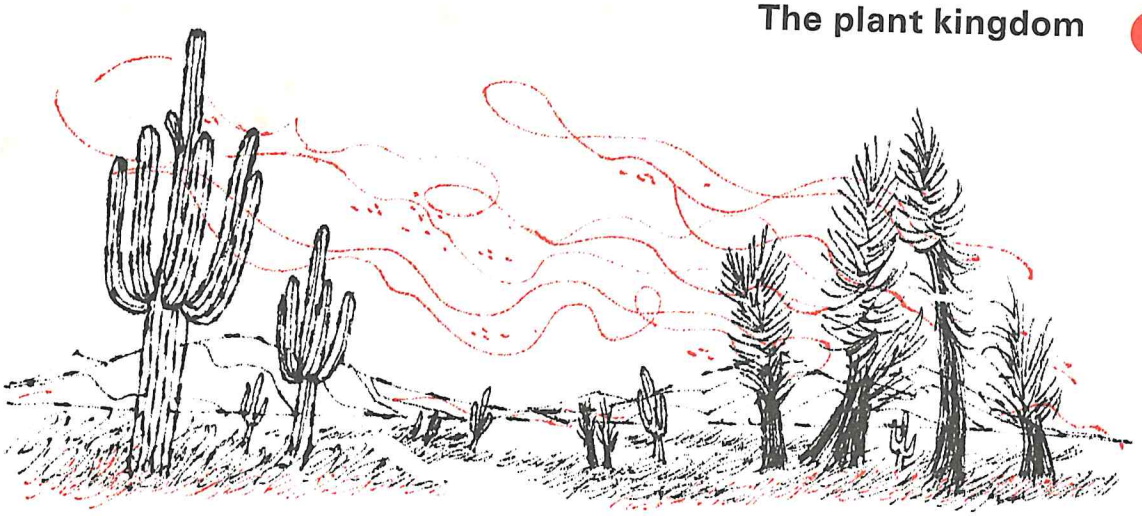
1. 5365 − 2249	**3.** 7926 − 3541	**5.** 8720 − 5293	**7.** 4227 − 1935	**9.** 7108 − 6824
2. 6291 − 3747	**4.** 8365 − 3458	**6.** 6342 − 1978	**8.** 5222 − 3876	**10.** 7025 − 6827

Name the differences.

11. 43541 − 21421	**13.** 76425 − 52180	**15.** 59734 − 16873	**17.** 60257 − 21745	**19.** 84157 − 16874
12. 92541 − 54684	**14.** 63027 − 33789	**16.** 51121 − 19939	**18.** 90000 − 72356	**20.** 100000 − 46819

Name the sums or differences.

21. 987654321 + 123456789	**23.** 987654321 − 123456789	**25.** 999999 + 999999
22. 97531 − 13579	**24.** 97531 + 13579	**26.** 909090909 + 101010101



Did you know that the largest cacti in the world are found in Arizona, New Mexico and California? Some have grown to a height of 52 feet.

1. There are some tree orchids in the swamps of Uganda that grow as tall as 11 feet.

2. The sequoia (big tree) named "General Sherman" in Sequoia National Park, California, is the largest living thing on earth. It is almost 275 feet tall, and the distance around its base is over 100 feet.

3. Bamboo in Ceylon grows as tall as 100 feet and is considered to be the fastest growing plant on earth. It may grow as much as 15 inches a day.

4. There is a California redwood tree in Humboldt State Park that is almost 370 feet tall. It is thought to be the tallest living plant in the world.

5. There is a huckleberry bush in Pennsylvania that covers an area of about 132,000 square feet. It covers more area than any other single plant that is known.

6. The oldest living things on earth are bristlecone pines found in California. Several of them are more than 4000 years old, and one, nicknamed "Great-granddad Pickaback," is over 4600 years old.

Working with money

Name	penny	nickel	dime	quarter	half-dollar	dollar	five dollars	ten dollars
value in cents	1¢	5¢	10¢	25¢	50¢	100¢	500¢	1000¢
value in dollars	\$.01	\$.05	\$.10	\$.25	\$.50	\$1.00	\$5.00	\$10.00

The **point** in \$2.50 separates the dollars from the cents. We read \$2.50 as "two dollars and fifty cents."

EXERCISES

Name the total value in cents.

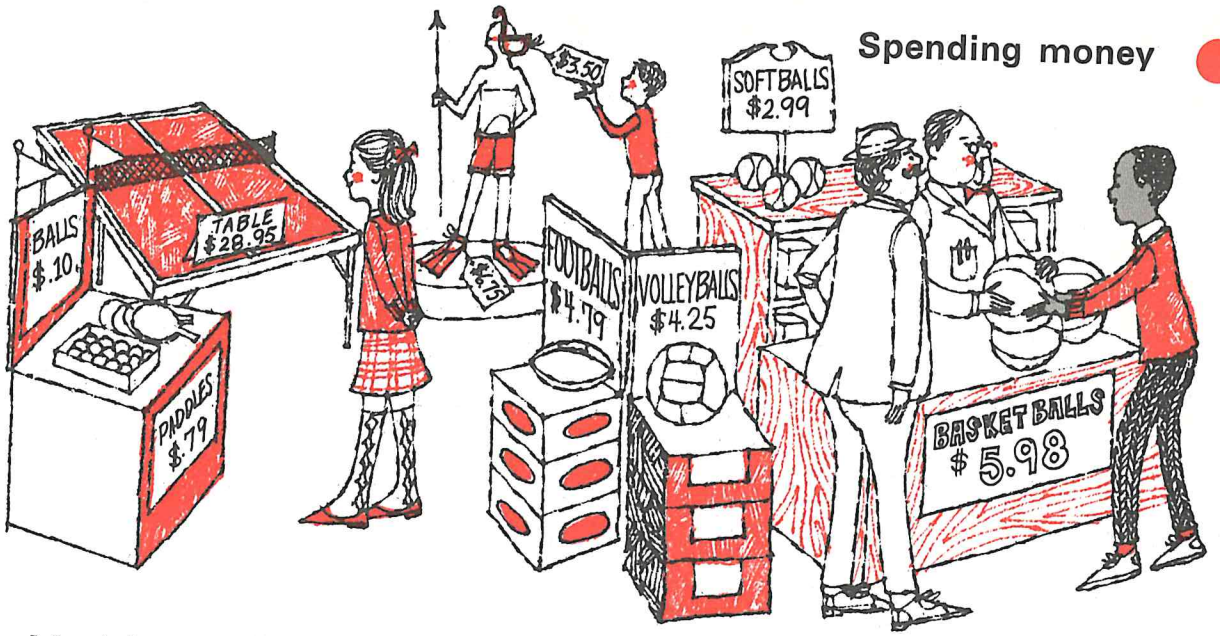
1. 1 quarter, 4 dimes, 2 pennies
2. 2 half-dollars, 3 quarters
3. 6 dimes, 3 nickels, 7 pennies
4. 6 dollars, 4 quarters, 1 dime, 8 pennies
5. 7 dollars, 2 half-dollars, 2 quarters, 6 dimes, 3 nickels, 6 pennies

Name the sums.

6. \$2.19 + .56
7. \$3.56 + 2.75 + 9.90
8. \$3.52 + .69
9. \$21.25 + 3.48 + 8.69
10. \$32.48 + 27.94
11. \$349.99 + 27.29 + 146.59
12. \$59.98 + 6.79
13. \$99.50 + 4.38 + 167.79
14. \$42.57 + 3.25 + 5.50 + 19.29 + 6.48 + 12.99

Name the differences.

15. \$9.75 − 4.82
16. \$18.39 − 8.54
17. \$64.47 − 28.99
18. \$147.50 − 88.27
19. \$984.23 − 689.97



Mr. Adams took Cathy, Steve, and Sam to the sporting goods store to buy equipment for the Junior Sports League. They have $120 to spend.

1. How much would it cost to buy a basketball, a football, and a volleyball?

2. What would be the price of a ping pong table, 2 paddles and 4 balls?

3. How many basketballs could they buy without spending more than $20?

4. They decided to buy 2 sets of swim fins and 4 snorkels. How much would this cost?

5. Would it be cheaper to buy 2 basketballs or 4 softballs?

6. How much would 2 ping pong tables cost?

7. How much more is a basketball than a football?

8. What is the difference in cost between 2 softballs and 2 snorkels?

*9. If Mr. Adams spent all the money on ping pong tables, how many could he buy? How much change would he get?

Measures

Many different measures are used every day. You should know these measures and how they are related.

land measures	
12 inches (12″)	in 1 foot (1 ft.)
3 feet (3′)	in 1 yard (1 yd.)
5280′	in 1 mile (1 mi.)

time measures	
60 seconds	in 1 minute
60 minutes	in 1 hour
24 hours	in 1 day
7 days	in 1 week
52 weeks	in 1 year
365 days	in 1 year
100 years	in 1 century

liquid measures	
1 cup	in 1 half-pint
2 half-pints	in 1 pint (1 pt.)
2 pints	in 1 quart (1 qt.)
4 quarts	in 1 gallon (1 gal.)

weight	
16 ounces (oz.)	in 1 pound (1 lb.)
2000 pounds	in 1 ton

EXERCISES

1. How many inches are in 1 yard?
2. How many feet are in 3 miles?
3. If an airplane flies at 36,000 feet, *about* how many miles is this?
4. Columbus landed in America in 1492. *About* how many centuries ago was this?
5. In order to keep the measurement of time accurate, every 4th year has 366 days in it. These are called leap years and the extra day is February 29. Find out when the next leap year will be.
6. Have you ever seen a sign in an elevator like this, *Weight Limit 3 tons?* How many pounds would this be?
7. How many pounds will a 10-ton truck hold?
8. How many cups are in 1 pint?
9. How many pints are in 1 gallon?
10. How many half-pints are in 1 gallon?

Adding and subtracting with measures

To name the total of 2 yards, 1 foot, 6 inches and 4 yards, 1 foot, 3 inches, we add the numbers in labeled columns as in the example. The total is 6 yards, 2 feet, 9 inches.

yd.	ft.	in.
2	1	6
+ 4	1	3
6	2	9

Study these examples.

ft.	in.
2	8
+ 3	6
5	14

The sum is 5′ 14″ or 6′ 2″.

gal.	pt.
4	10
~~5~~	~~2~~
− 2	5
2	5

The difference is 2 gal. and 5 pt.

EXERCISES

Name the sums. State the totals in the simplest way.

1.

yd.	ft.
4	2
+ 2	1

2.

lb.	oz.
4	12
+ 3	12

3.

wk.	days
3	5
+ 15	6

4.

gal.	qt.
8	3
+ 9	2

5.

yd.	ft.	in.
2	2	7
6	1	4
+ 7	2	9

6.

gal.	qt.	pt.
3	2	1
4	3	1
+ 6	3	0

7.

hr.	min.	sec.
3	24	34
2	17	23
+ 3	42	18

Name the differences.

8.

ft.	in.
4	10
− 2	7

9.

yd.	ft.
6	1
− 3	2

10.

wk.	days
8	4
− 5	6

11.

lb.	oz.
2	8
−	12

12.

yd.	ft.	in.
3	1	10
− 1	2	7

13.

gal.	qt.	pt.
4	1	1
− 2	3	1

14.

hr.	min.	sec.
4	23	13
− 2	45	36

Practicing addition and subtraction

Name the sums.

1. 28 + 56	**3.** 122 + 68	**5.** 475 + 311	**7.** 587 + 732	**9.** 272 + 638	**11.** 6538 + 7289
2. 37 + 44	**4.** 378 + 93	**6.** 369 + 276	**8.** 946 + 756	**10.** 876 + 987	**12.** 8964 + 7087

Name the differences.

13. 63 − 29	**15.** 256 − 92	**17.** 357 − 175	**19.** 470 − 193	**21.** 1372 − 968	**23.** 8459 − 3681
14. 91 − 43	**16.** 147 − 69	**18.** 922 − 648	**20.** 807 − 355	**22.** 625 − 577	**24.** 5120 − 3754

Name the sums.

25. 46 27 35 + 12	**26.** 294 32 87 + 169	**27.** 543 296 + 728	**28.** 869 352 + 519	**29.** 435 694 378 + 922	**30.** 1346 828 3739 + 4917

Name the differences.

31. 5432 − 3321	**33.** 6205 − 1573	**35.** 10407 − 7998	**37.** 26042 − 19064	**39.** 70302 − 54794
32. 8903 − 2887	**34.** 4831 − 2954	**36.** 32290 − 1762	**38.** 18469 − 13975	**40.** 69543 − 50505

Complete the sentences.
Write $<$, $>$, or $=$ to replace each ◯.

41. $62 + 48 \bigcirc 100$

42. $129 - 64 \bigcirc 64$

43. $9999 \bigcirc 7777 + 2322$

44. $238 + 382 \bigcirc 899 - 279$

45. $920 + 911 \bigcirc 2000$

46. $1752 - 257 \bigcirc 1500$

47. $543 + 321 \bigcirc 323 + 541$

48. $1234 + 4321 \bigcirc 5050 + 505$

49. $1111 \bigcirc 999 + 122$

50. $3333 \bigcirc 3000 + 300 + 33$

51. $10{,}000 - 6789 \bigcirc 3321$

52. $519 \bigcirc 213 + 312$

Solve the equations.

1. $4 + 9 = a$ **3.** $13 + c = 19$ **5.** $27 + 38 = e$ **7.** $28 + g = 72$

2. $12 - b = 8$ **4.** $46 - 8 = d$ **6.** $46 - 19 = f$ **8.** $h - 41 = 78$

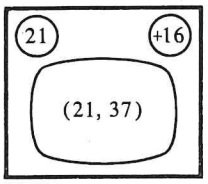
Name the sums.

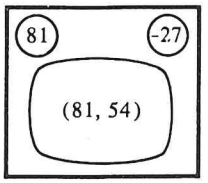
9. $64 + 72$ **10.** $85 + 76$ **11.** $135 + 95$ **12.** $478 + 241$ **13.** $986 + 338$ **14.** $587 + 452 + 895$

Name the differences.

15. $94 - 58$ **16.** $127 - 73$ **17.** $522 - 363$ **18.** $904 - 287$ **19.** $1476 - 882$ **20.** $7063 - 4865$

Study the function TVs, then complete the sets of number pairs.





21. $\{(21, 37), (33, a), (60, b), (47, c), (17, d), (108, e)\}$

22. $\{(81, 54), (30, f), (57, g), (74, h), (27, i), (105, j)\}$

Complete the table for each function.

23. $x - 3 = y$

x	4	5	7	3	15
y	1	a	r	c	s

24. $x + 15 = y$

x	2	4	15	21	93
y	e	n	s	a	i

Name the sums or differences.

25.

ft.	in.
3	7
+ 4	5

27.

hr.	min.
17	35
+ 44	52

29.

min.	sec.
15	33
+ 17	54

26.

yd.	ft.
7	1
− 4	2

28.

qt.	pt.
6	0
− 3	1

30.

yd.	ft.	in.
8	1	6
− 2	2	10

LOOKING BACK

An addition or subtraction with the addends less than 10 is a **basic fact.** Basic facts may be learned in **families.** For example, the equations $6 + 3 = 9$, $3 + 6 = 9$, $9 - 3 = 6$, and $9 - 6 = 3$ form a family of addition-subtraction facts.

Solve the equations.

1. $9 + 5 = a$ **2.** $14 - 8 = b$ **3.** $17 - c = 9$ **4.** $d - 4 = 6$

Write the equations to complete each family.

5. $3 + 2 = 5$ **6.** $12 - 6 = 6$ **7.** $16 - 7 = 9$ **8.** $9 + 4 = 13$

A rule, such as *subtract 7*, and a set of ordered pairs form a **function.** We may show a function with a TV set, or an equation, like $x + 3 = y$.

Complete the sets of number pairs.

9. $x + 13 = y$ $\{(5, 18), (3, a), (7, b), (0, c), (12, d), (8, e)\}$

10. $x - 4 = y$ $\{(7, 3), (10, f), (4, g), (15, h), (23, i), (40, j)\}$

Addition and subtraction are extended through an understanding of the basic facts, place value, and the properties of addition.

Name the sums or differences.

11. $\begin{array}{r} 18 \\ +\ 9 \\ \hline \end{array}$ **12.** $\begin{array}{r} 38 \\ +\ 82 \\ \hline \end{array}$ **13.** $\begin{array}{r} 128 \\ +\ 364 \\ \hline \end{array}$ **14.** $\begin{array}{r} 906 \\ -\ 358 \\ \hline \end{array}$ **15.** $\begin{array}{r} 2425 \\ -\ 1498 \\ \hline \end{array}$ **16.** $\begin{array}{r} 53502 \\ -\ 34594 \\ \hline \end{array}$

Measurement involves comparing basic units to the objects being measured.

Name units which match each of these.

17. 3 ft. **18.** 60 sec. **19.** 5280′ **20.** 4 qt. **21.** 2000 lb.

1. This is called a *Magic Square*. Each row, column, or diagonal gives a sum of 15. Notice that the only numbers used here are the numbers 1 to 9. Can you make other 3 by 3 magic squares?

4	9	2
3	5	7
8	1	6

2. Here is a 4 by 4 magic square using the numbers from 1 to 16. It was created by a famous German named Dürer. The middle boxes in the bottom row give the date of the square (1514). What is the sum of each row, column or diagonal? Try to make other 4 by 4 magic squares.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

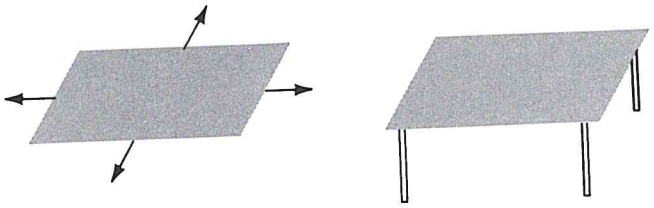
3. Leonard Euler, a famous Swiss mathematician, made a magic square where each vertical or horizontal row had a sum of 260. If you stop halfway the sum is 130. Write the numerals to complete the magic square. Find as many 4 by 4 magic squares as you can in the large magic square.

1	48	31	50	33	*c*	63	18
30	51	46	*n*	62	19	14	35
r	2	49	32	15	34	17	64
52	29	4	45	20	61	36	*a*
5	44	25	56	*v*	40	21	60
28	53	8	41	24	57	*i*	37
s	6	55	26	39	10	59	22
54	27	42	7	58	23	38	*x*

4. Benjamin Franklin made this magic square. Each set of 8 numbers in a line has a sum of 260. If you follow up 4 squares and down 4 squares on the red lines, the sum is 260. Any 4 numbers next to one another, which form a square, have a sum of 130.

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

Plane figures



A set of points, in a flat surface, that goes on and on in all directions without end is a **plane**. We may think of the table top as part of a plane.

Curves are sets of connected points. A line is a special kind of curve.

line ($\overleftrightarrow{AB}$)

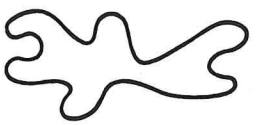
line segment ($\overline{AB}$)

ray ($\overrightarrow{AB}$)

The line segment has two end points, A and B; the ray has one, A; the line has no end points.

curve

closed curve



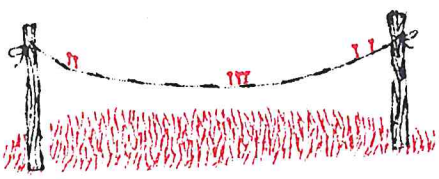
simple closed curve

These are plane figures since all their points lie in one plane.

EXERCISES

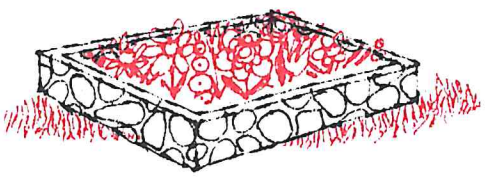
Name the set of points suggested by each picture.

1.



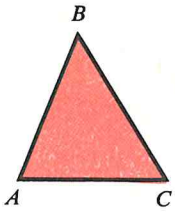
2.

3.

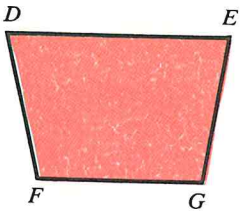


4.

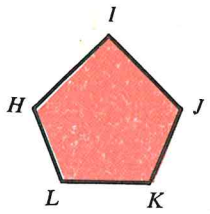
Special simple closed curves



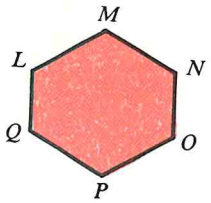
triangle



quadrilateral



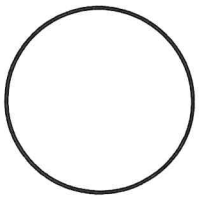
pentagon

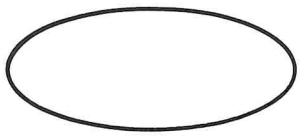


hexagon

These figures are **polygons.** They are simple closed curves formed by the union of line segments.
The line segments are called **sides.**
The corner points are called **vertices.** They are labeled with capital letters.
The points inside the polygons, shown red, are called **regions.**

circle





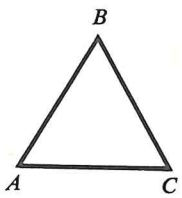
ellipse

The circle and ellipse are not polygons since they are not formed by line segments.

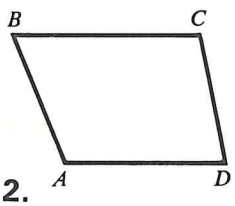
EXERCISES

Name the polygons by the letters at each vertex.

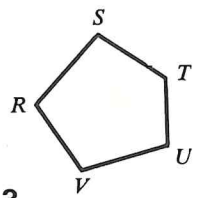
1.



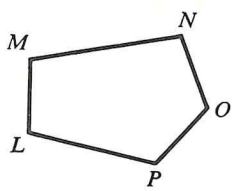
2.



3.



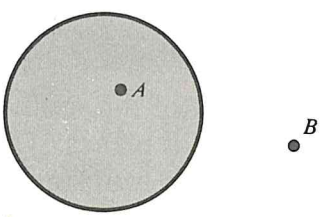
4.



Name the number of sides and vertices for a

5. triangle
6. quadrilateral
7. pentagon
8. circle

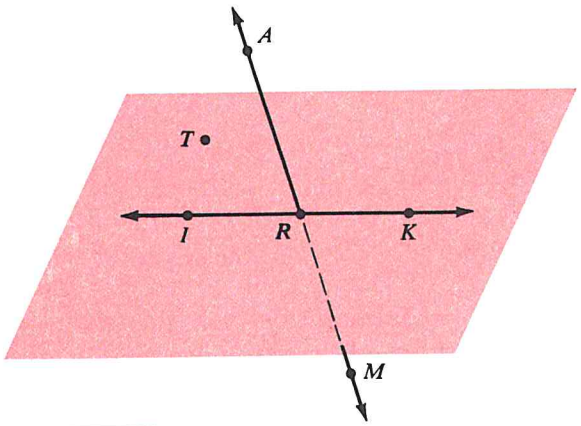
9. Point A is in the circular region while point B is not. Can you join A and B without intersecting the circle? Do you think this will be true for any simple closed curve?



10. What are the 3 different sets of points formed by a simple closed curve in a plane?

Points, lines, and planes

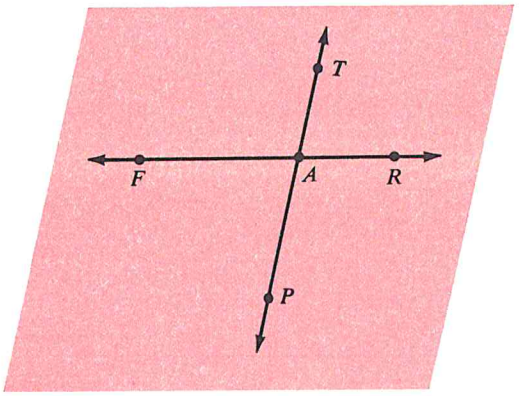
A line and a point not on the line will fix a plane. For example, there is only one plane which contains both $\overleftrightarrow{IK}$ and point T.
$\overleftrightarrow{AM}$ is not in the red plane.
It intersects the plane in the point R.



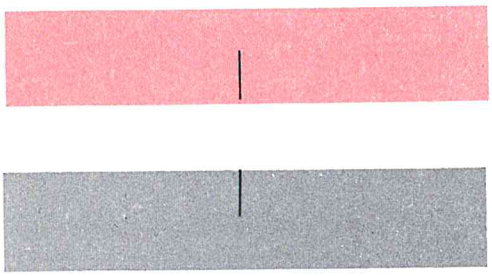
EXERCISES

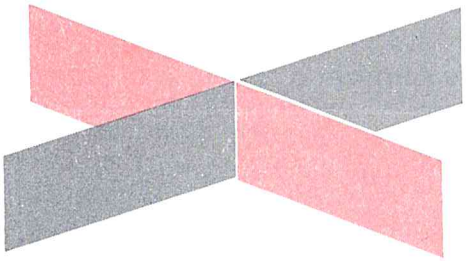
1. Can any 3 points not on one line lie in more than one plane?
2. Can a triangle lie in more than one plane?

$\overleftrightarrow{FR}$ and $\overleftrightarrow{PT}$ lie in the red plane. $\overline{FA}$ and $\overline{FR}$ are line segments. $\overrightarrow{AF}$ and $\overrightarrow{RF}$ are rays.

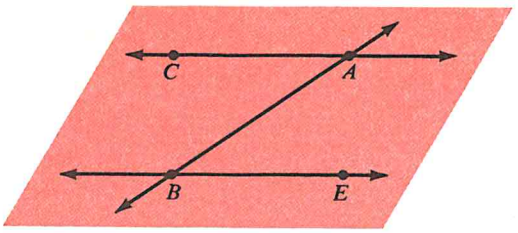


3. Name 4 other line segments in the red plane.
4. Name 6 other rays in the red plane.
5. Will $\overleftrightarrow{FR}$ and $\overleftrightarrow{PT}$ have any points in common besides A?
6. Cut 2 pieces of cardboard halfway through. Place them together to show 2 intersecting planes. What is the intersection of these 2 planes?

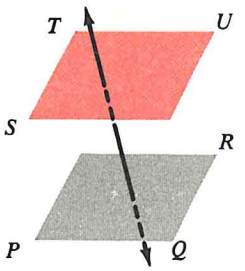




7. Think of the walls, floor, and ceiling in your classroom as parts of planes. Can you name examples of 2 planes meeting to form a line? of 3 planes meeting at a point? Can you name 2 planes that do not intersect?



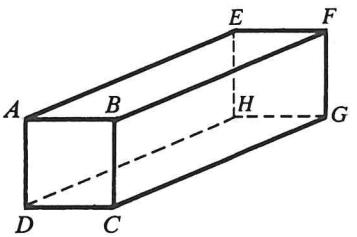
Lines which are in the same plane and do not intersect are said to be **parallel.** $\overleftrightarrow{CA}$ is parallel to $\overleftrightarrow{BE}$.



Planes which do not intersect are called **parallel** planes. The red plane *STU* is parallel to the gray plane *PQR*.

EXERCISES

Study the figure.

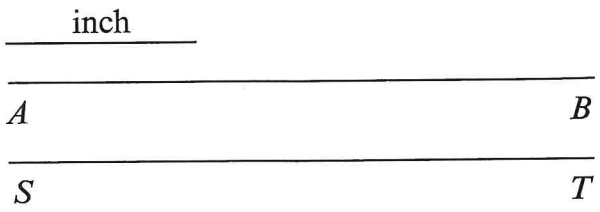


1. Name a plane parallel to plane *ABC*.
2. Name a plane parallel to plane *DHG*.
3. Name 2 line segments parallel to $\overline{AE}$.
4. Name 3 line segments parallel to $\overline{AB}$.
5. Name 3 planes which have the point *E* in common.
6. Name the intersection of planes *ADH* and *DCG*.
7. How many planes form this figure?

Parallel lines and parallel planes are important. Can you think of some examples where parallel lines and parallel planes are necessary? Would railroad tracks be one example? Can you think of some examples where you would want to have lines or planes which are not parallel? For example, why is it sometimes useful to have the roof of a house not parallel to the ground?

8. Can a triangle have 2 sides parallel?
9. Can a quadrilateral have 2 sides parallel?
10. Can a quadrilateral have 2 pairs of parallel sides?
11. Can a pentagon have 2 sides parallel?
12. Can a pentagon have 2 pairs of parallel sides?

Finding lengths



$m(\overline{AB}) = 3$ $m(\overline{ST}) = 3$

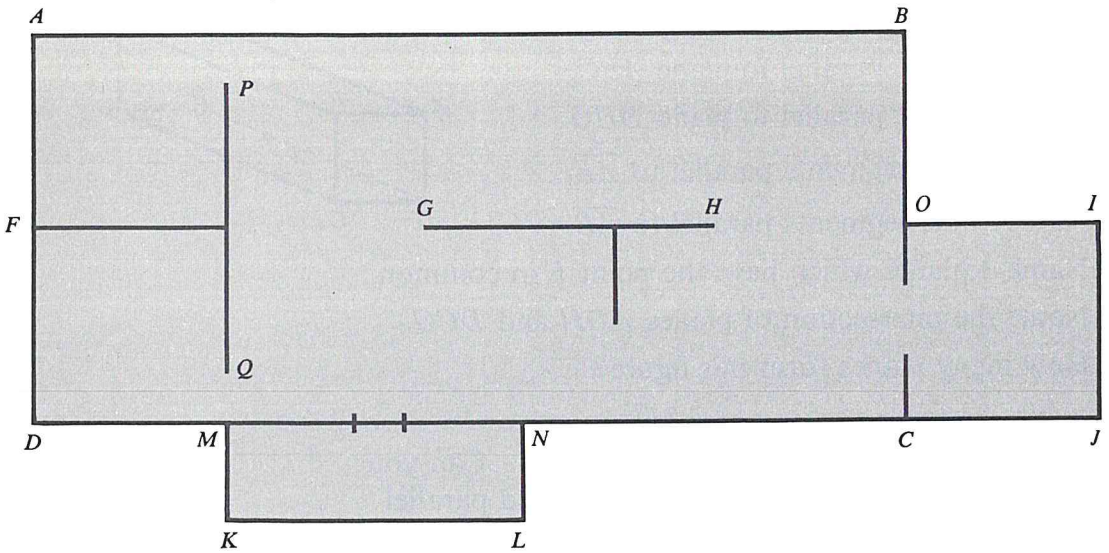
The length of $\overline{AB}$ is 3″.
The length of $\overline{ST}$ is 3″.

To name the measure of $\overline{AB}$ we match a unit of length with $\overline{AB}$. We ask, "*How many* units match $\overline{AB}$?" The **measure** is a number.

If two line segments have the same length, they are **congruent.** $\overline{AB}$ and $\overline{ST}$ are congruent.

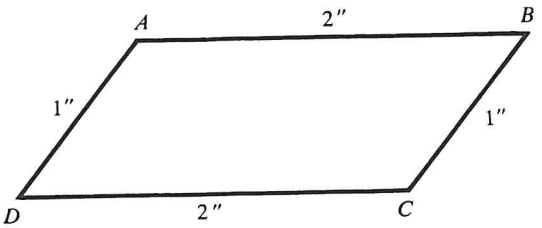
EXERCISES

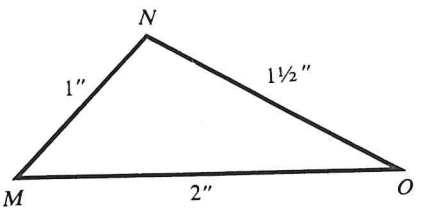
Measure the line segments. Then solve the equations and answer the questions.



1. $m(\overline{AB}) = a$
2. $m(\overline{IJ}) = b$
3. $m(\overline{AD}) = c$
4. $m(\overline{KL}) = d$
5. $m(\overline{GH}) = e$
6. $m(\overline{NJ}) = f$
7. $m(\overline{MN}) = g$
8. $m(\overline{BO}) = h$
9. $m(\overline{OI}) = i$

10. Name the pairs of congruent line segments.
11. If you wish to follow a path from A to C, is it shorter to go through B or through D?
12. Can you measure the length of a line? of a ray?
13. Name some units (other than inches) for measuring lengths.

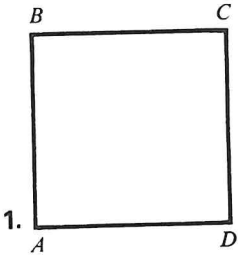


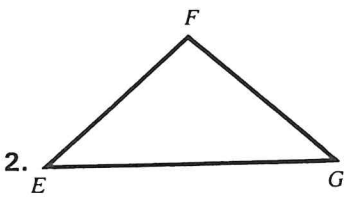


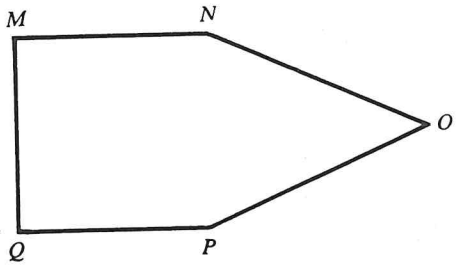
The distance around a polygon is called its **perimeter.** The measure of the perimeter may be found by adding the measures of the sides. The perimeter of $ABCD$ is $6''$; the perimeter of MNO is $4\frac{1}{2}''$.

EXERCISES

Measure the sides, then name the perimeter of each figure.







Name the perimeter of

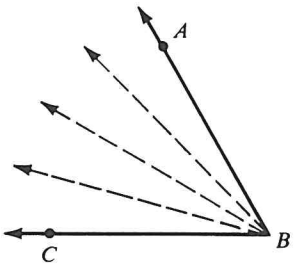
4. a triangle whose sides are $6''$, $8''$, and $12''$
5. a quadrilateral whose sides are 7 feet (we write $7'$), $11'$, $8'$, and $12'$
6. a quadrilateral whose sides are $22'$, $17'$, $22'$, and $17'$
7. a pentagon whose sides are $15''$, $8''$, $9''$, $13''$, and $16''$

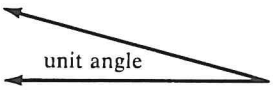
Answer the questions.

8. If the perimeter of a quadrilateral is $84'$ and the length of 3 sides are $23'$, $18'$, and $14'$, what is the length of the 4th side?
9. If a quadrilateral has 4 congruent sides and its perimeter is $60''$, how long is each side?
10. A quadrilateral has 2 pairs of congruent sides. If two sides are each $20''$ long and the perimeter of the quadrilateral is $100''$, how long is each of the other sides?

Angles

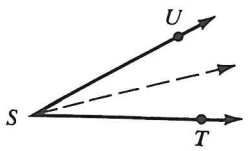
An **angle** is the union of two rays with a common end point. The rays are the *sides* of the angle, and the common end point is the *vertex*. The angle on the right is named $\angle ABC$ or $\angle CBA$. An angle is measured by comparing it with a *unit angle*. The **measure** of an angle tells how many unit angles it contains. The measure of $\angle ABC$ is 4. We write, $m(\angle ABC) = 4$.
The **size** of an angle is the measure together with the unit: The *size* of $\angle ABC$ is 4 units. Two angles of the same size are **congruent.**



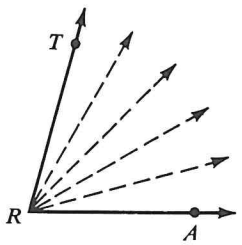


EXERCISES

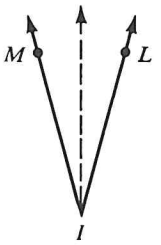
Compare the unit angle with each angle, and name the measures.



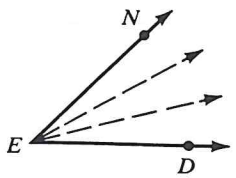
1. $m(\angle TSU) = a$



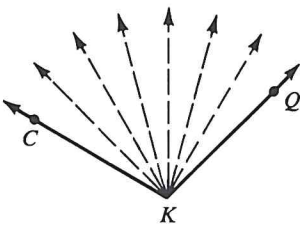
4. $m(\angle TRA) = d$



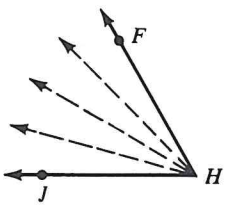
7. $m(\angle MIL) = g$



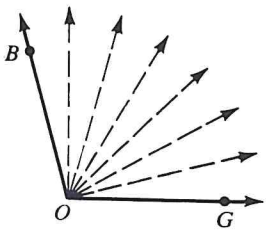
2. $m(\angle NED) = b$



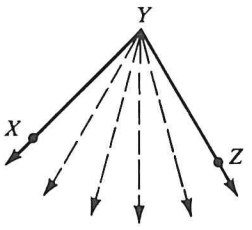
5. $m(\angle CKQ) = e$



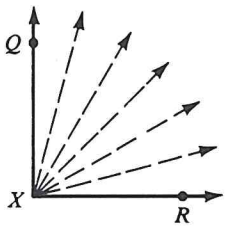
8. $m(\angle FHJ) = h$



3. $m(\angle BOG) = c$

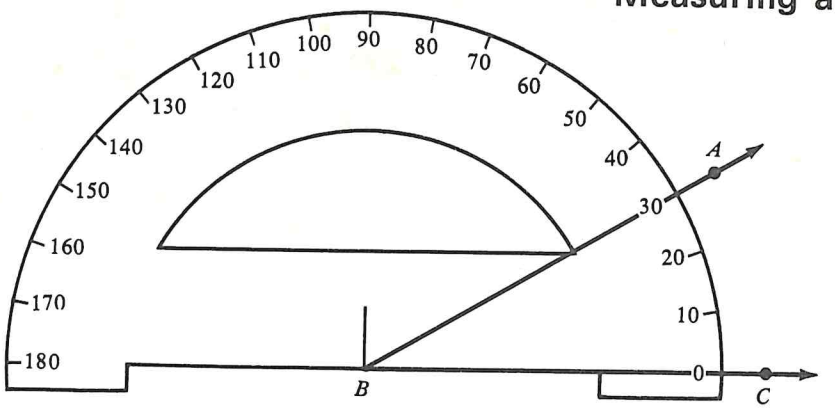


6. $m(\angle XYZ) = f$



9. $m(\angle QXR) = g$

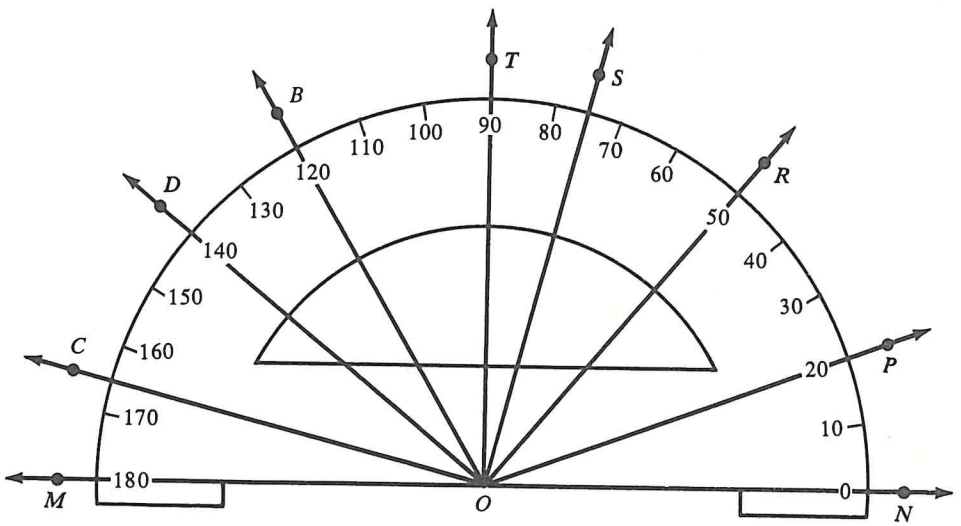
10. Name the pairs of congruent angles.



The most common unit for measuring angles is a very small angle called a **degree.** The instrument used to measure angles in degrees is called a **protractor.** It shows many degrees placed side by side so that we may measure any angle easily.
The *measure* of $\angle ABC$ is 30. The *size* of $\angle ABC$ is 30 degrees, also written 30°.

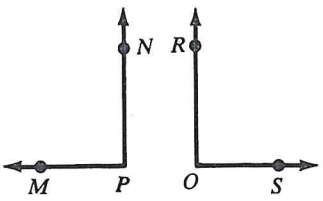
EXERCISES

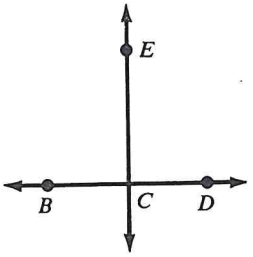
Study the protractor, then name the number of degrees in each angle.

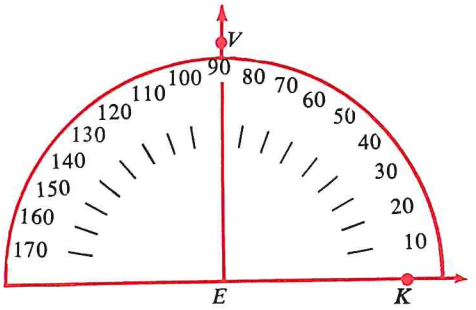


1. $m(\angle PON) = a$
2. $m(\angle TOP) = b$
3. $m(\angle DON) = c$
4. $m(\angle SON) = d$
5. $m(\angle MOR) = e$
6. $m(\angle MON) = f$
7. $m(\angle BON) = g$
8. $m(\angle RON) = h$
9. $m(\angle BOR) = i$
10. $m(\angle CON) = j$
11. $m(\angle TON) = k$
12. $m(\angle MOT) = l$

Right angles



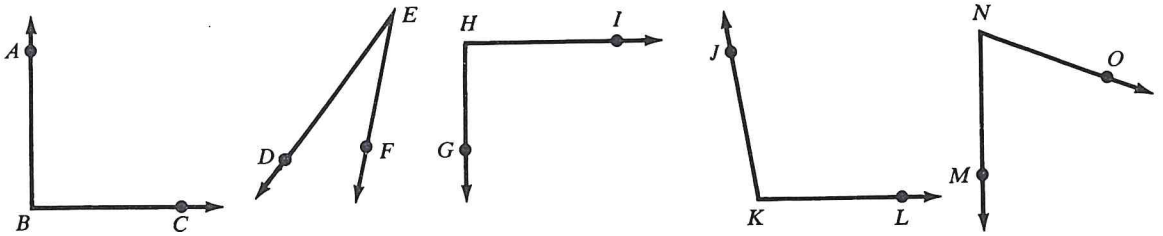




Angles MPN and ROS are **right angles.** When two rays meet to form right angles, we say that they are **perpendicular** to each other. $\overleftrightarrow{EC}$ is perpendicular to $\overleftrightarrow{BD}$. The size of any right angle is 90°. $\angle VEK$ is a right angle since $m(\angle VEK) = 90$, in degrees.

EXERCISES

1. Use a protractor to find the size of each angle.



2. Name the right angles in exercise 1.

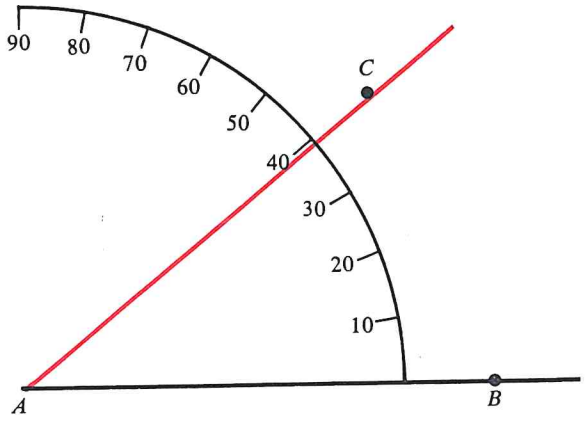
Your protractor can be used to draw angles.

Step 1: Draw $\overline{AB}$.

Step 2: Place the edge of your protractor along the line so that the center point lies on A.

Step 3: Find the measure of the angle you wish to draw and mark the point C.

Step 4: Join the points C and A.

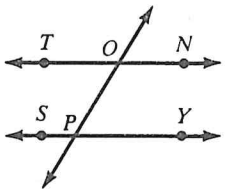


3. Following these steps and using a degree as your unit, draw these angles.

a. $m(\angle XYZ) = 90$ **b.** $m(\angle ABC) = 120$ **c.** $m(\angle EFG) = 65$

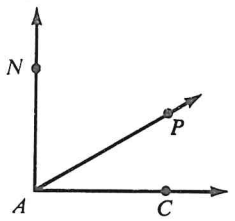
Use your protractor to name

1. $m(\angle TOP)$
2. $m(\angle OPY)$
3. $m(\angle SPO)$
4. $m(\angle PON)$
5. the congruent angles



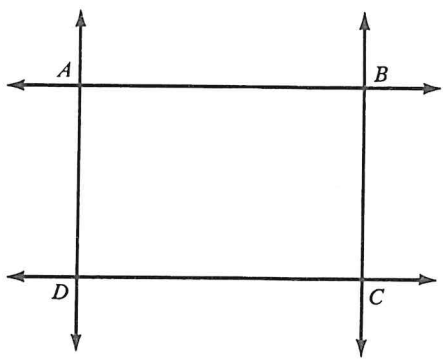
Use your protractor to find the measures, then answer the questions.

6. $m(\angle PAC)$
7. $m(\angle PAN)$
8. $m(\angle CAN)$
9. Is it true that $m(\angle CAN) = m(\angle PAC) + m(\angle PAN)$?
10. Is $\overline{AN}$ perpendicular to $\overline{AC}$?



Name the measures, then answer the questions.

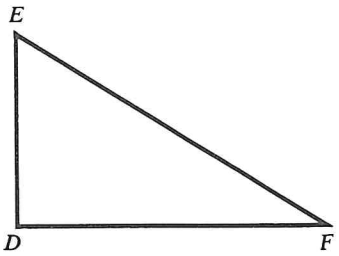
11. $m(\overline{AD})$
12. $m(\overline{BC})$
13. $m(\overline{AB})$
14. $m(\overline{DC})$
15. $m(\angle ADC)$
16. $m(\angle DCB)$
17. $m(\angle DAB)$
18. $m(\angle ABC)$
19. Is $\overline{AD}$ perpendicular to $\overline{DC}$? to $\overline{AB}$?
20. Is $\overline{BC}$ perpendicular to $\overline{DC}$? to $\overline{AB}$?
21. Are $\overline{AB}$ and $\overline{DC}$ parallel? congruent?
22. Are $\overline{AD}$ and $\overline{BC}$ parallel? congruent?
23. How can you find the shortest distance between parallel lines?



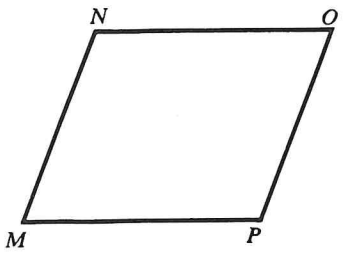
Answer the questions.

24. What do we call two lines in the same plane that are always the same distance apart?
25. Are all right angles congruent to one another?
26. What can you say about 2 lines which are perpendicular to the same line?

Polygons



A triangle which has one right angle is called a **right triangle.** DEF is a right triangle since $\angle EDF$ is a right angle.

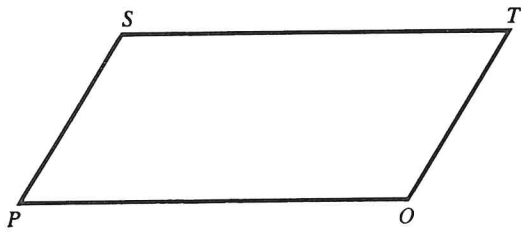


A quadrilateral whose opposite sides are parallel is called a **parallelogram.** $MNOP$ is a parallelogram since $\overline{MN}$ is parallel to $\overline{PO}$ and $\overline{PM}$ is parallel to $\overline{ON}$.

EXERCISES

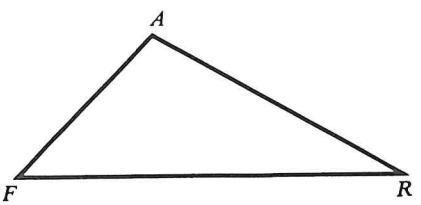
Use your ruler to name

1. $m(\overline{ST})$
2. $m(\overline{SP})$
3. $m(\overline{PO})$
4. $m(\overline{TO})$
5. the perimeter of $PSTO$
6. the congruent line segments in $PSTO$
7. the parallel line segments in $PSTO$
8. Do you think that the opposite sides of all parallelograms are congruent?



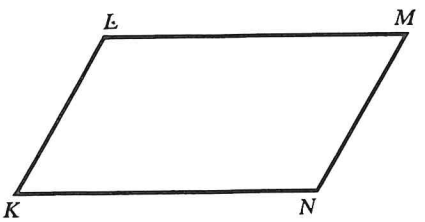
Use your ruler to name

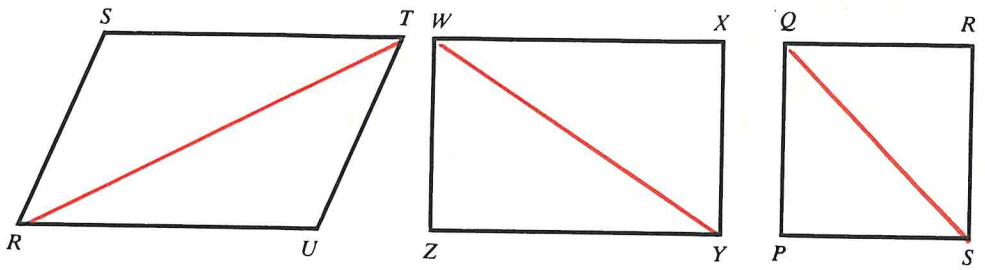
9. $m(\overline{FA})$
10. $m(\overline{AR})$
11. $m(\overline{RF})$
12. the perimeter of FAR



Use your protractor to name

13. $m(\angle LKN)$
14. $m(\angle LMN)$
15. $m(\angle KLM)$
16. $m(\angle KNM)$
17. the congruent angles of parallelogram $KLMN$





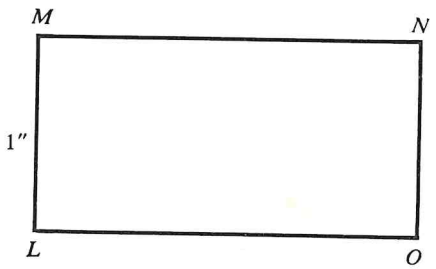
parallelogram **rectangle** **square**

A **rectangle** is a parallelogram that has right angles. A **square** is a rectangle with all its sides congruent. Any line segment that joins vertices and is not a side is a **diagonal.** $\overline{RT}$, $\overline{WY}$, and $\overline{QS}$ are diagonals.

EXERCISES

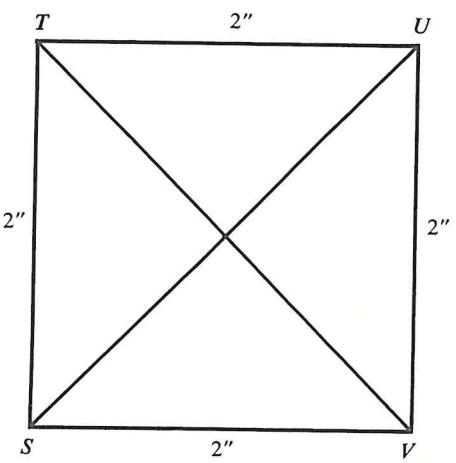
For *LMNO*, name

1. $m(\overline{LM}) = a$
2. $m(\overline{NO}) = b$
3. $m(\overline{MN}) = c$
4. $m(\overline{LO}) = d$
5. $m(\angle MNO) = e$
6. $m(\angle MLO) = f$
7. $m(\angle LON) = g$
8. $m(\angle LMN) = h$
9. the congruent line segments
10. the congruent angles
11. the pairs of parallel sides



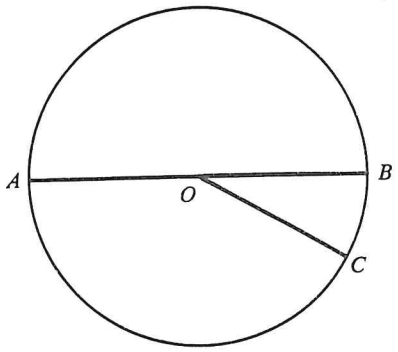
For *STUV*, name

12. $m(\overline{TU}) = a$
13. $m(\overline{SV}) = b$
14. $m(\overline{TS}) = c$
15. $m(\overline{UV}) = d$
16. $m(\angle TSV) = e$
17. $m(\angle SVU) = f$
18. $m(\angle VUT) = g$
19. $m(\angle UTS) = h$
20. the congruent line segments
21. the congruent angles
22. the pairs of parallel sides
23. the diagonals



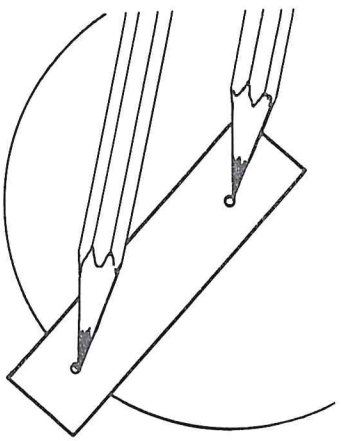
Circles

Each point in a circle is the same distance from a point in the circular region called the **center.** Points A, B, and C are all the same distance from O. $\overline{OA}$ is **a radius** of the circle; $\overline{OB}$ and $\overline{OC}$ are also **radii.** The length of any radius is called **the radius** of the circle. $\overline{AB}$ is **a diameter** of the circle. The length of $\overline{AB}$ is **the diameter** of this circle.



EXERCISES

You can use a strip of cardboard to draw circles. Make 2 holes in the strip, and place pencils in the holes. Keep one pencil fixed and move the other all the way around. Notice that each point on the simple closed curve you draw is the same distance from the fixed point.

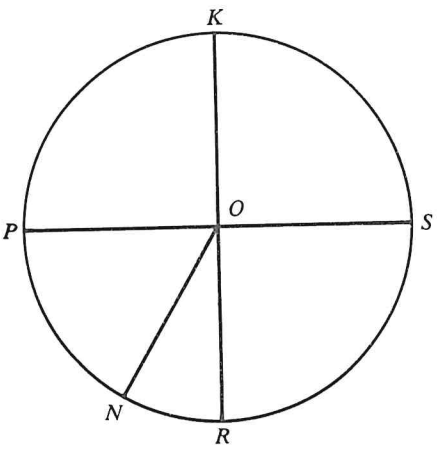


Draw a circle with radius

1. $1''$ **2.** $\frac{1}{2}''$ **3.** $\frac{3}{4}''$ **4.** $2''$ **5.** $1\frac{1}{2}''$

For the circle at the right name these measures in inches or degrees.

6. $m(\overline{OP})$ **7.** $m(\overline{ON})$ **8.** $m(\overline{KR})$ **9.** $m(\overline{PS})$ **10.** $m(\angle POK)$ **11.** $m(\angle KOS)$



For the circle on the right name

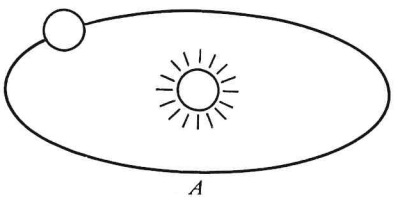
12. the center

13. a radius

14. two diameters

15. the radius

16. the diameter



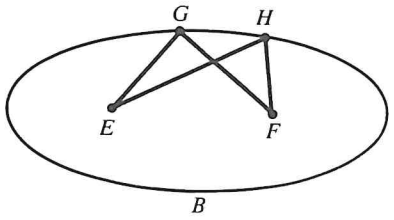
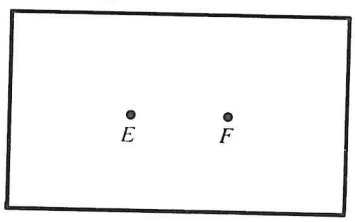
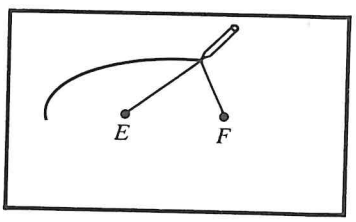


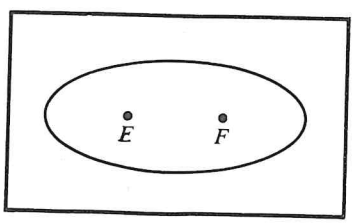
Figure A shows the path of the earth around the sun. This path forms a special closed curve called an **ellipse.** For any 2 points on the ellipse the sum of the distances from 2 special fixed points in its region is the same. In figure B, for example, $m(\overline{EG}) + m(\overline{FG}) = m(\overline{EH}) + m(\overline{FH})$.

EXERCISES

An ellipse can be drawn like this.

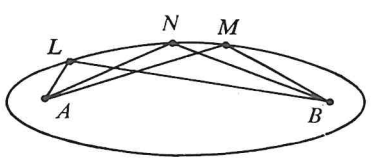






Fix 2 points, E and F. Take a piece of string longer than $\overline{EF}$, and fix one end of the string on E and the other on F. Use a pencil to pull the string taut and then, keeping the string tight at all times, move the pencil around the string. Draw several ellipses by changing the distance between E and F. What effect does this have on the shape of the ellipse?

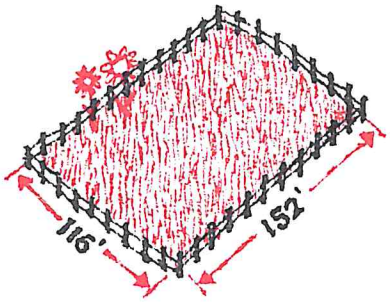
Name these measures, in inches, for this ellipse.



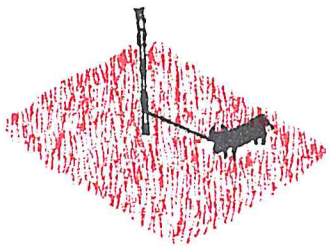
1. $m(\overline{AL})$
2. $m(\overline{BL})$
3. $m(\overline{AL}) + m(\overline{BL})$
4. $m(\overline{AM})$
5. $m(\overline{BM})$
6. $m(\overline{AM}) + m(\overline{BM})$
7. $m(\overline{AN})$
8. $m(\overline{BN})$
9. $m(\overline{AN}) + m(\overline{BN})$
10. Are $m(\overline{AL}) + m(\overline{BL})$, $m(\overline{AM}) + m(\overline{BM})$, and $m(\overline{AN}) + m(\overline{BN})$ all equal?

Using sets and numbers

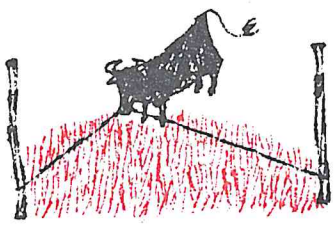
1. Farmer Harris wants to build a fence around his rectangular field. If the field is 116′ wide and 152′ long, how much wire will he need for the fence?



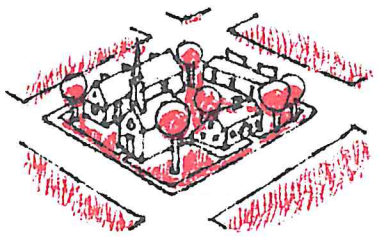
2. The bull is tied to a stake with a 40′ rope. As the bull moves around, what is the shape of the region over which he can graze?



3. If the bull is tied to a ring on a rope which is fastened to 2 stakes, what is the shape of the region over which he can graze?

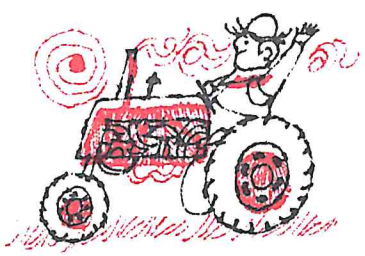


4. The square in the town where Farmer Harris shops is 90 yards along one side. If you walk around the square, how many yards would you walk?



5. How far does Farmer Harris travel if he goes from the farm to town, to the dairy, and back to the farm?

6. Last year Farmer Harris bought a tractor for $3750, milking machines for $878, and other equipment for $659. How much did he spend all together?



Name the sums.

1.	2.	3.	4.	5.	6.
85 + 76	432 + 248	328 + 949	876 + 429	4592 + 3146	8904 + 7898

Name the differences.

7.	8.	9.	10.	11.	12.
93 − 57	436 − 219	807 − 245	624 − 384	1953 − 1578	8207 − 3539

Solve the equations.

13. $8 + 17 = a$
14. $9 + 85 = b$
15. $17 + c = 29$
16. $128 + d = 365$
17. $31 - 18 = e$
18. $42 - f = 29$
19. $119 - g = 74$
20. $h - 246 = 398$
21. $317 + 27 = i$

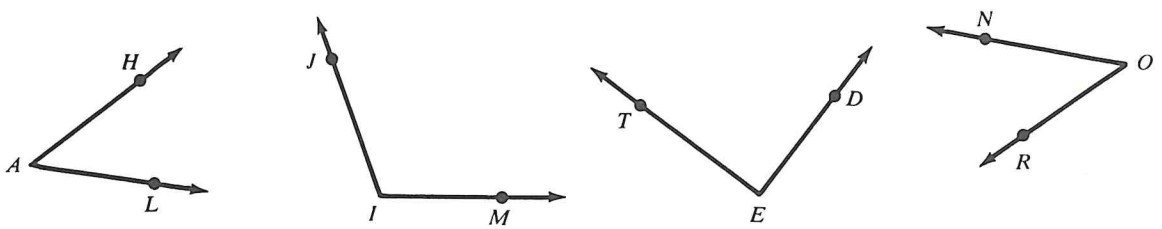
Complete each sentence by replacing the ◯ with $=$, $<$, or $>$.

22. $4 + 5 + 6 \bigcirc 7 + 3 + 4$
23. $123 + 987 \bigcirc 1100$
24. $235 + 476 \bigcirc 476 + 235$
25. $784 \bigcirc 700 + 60 + 34$
26. $1000 - 777 \bigcirc 333$
27. $878 + 344 \bigcirc 5000 - 3678$

Name the perimeter of

28. a triangle whose sides are 24″, 39″ and 33″
29. a square with side 28″
30. a rectangle whose sides are 18″ and 24″
31. a parallelogram whose sides are 57″ and 36″

Use your protractor to name



32. $m(\angle HAL) = a$
33. $m(\angle JIM) = b$
34. $m(\angle TED) = c$
35. $m(\angle RON) = d$
36. the congruent angles
37. the right angle

Sharpshooting

See how quickly you can answer the questions in A. Then see if you can answer the questions in B even more quickly.

A

1. How much more than \$3.97 is \$4.10?
2. Mike has 3 quarters and 2 dimes. Larry has 4 dimes and 8 nickels. Who has more money?
3. If Jane bought 8 apples and 9 pears, how many pieces of fruit did she buy?
4. There were 28 birds on the porch, then 17 flew away. How many are left?
5. $35 - 21 = n$
6. There are 9 boys and 11 girls at the party. How many children are there?
7. There are 37 cows and 52 sheep in the field. How many animals are there in all?
8. There are 54 watermelons in the store but 23 are broken. How many are not broken?
9. Name the next ten after 597.
10. Which is less, (4×2) or $(21 \div 3)$?

B

1. How many balls in 2 boxes of footballs if each box holds 36?
2. 162 cars are parked on the first floor and 98 on the second floor. How many cars are there in all?
3. Which is greater, $(16 - 9)$ or $(5 + 4)$?
4. We travelled 317 miles the third day and 265 miles on the fourth day. How many miles more did we travel on the third day?
5. Bart has 75 stamps. Bill has 78 stamps. Ben has an even number of stamps, fewer than Bill, more than Bart. How many stamps does Ben have?
6. What is the difference between 513 and 294?
7. $16 \times 5 = m$
8. How many days to the end of next month?
9. How many dozen in 72?
10. If you want to have 75 models and have 59, how many more must you get?

Complete the number patterns.

1.

8	9	7	9	*a*
7	6	4	8	*r*
7	4	5	8	*c*
9	4	6	1	*s*
e	*x*	*n*	*u*	*i*

2.

3000	100	70	5	*s*
4000	600	80	9	*v*
7000	800	20	1	*i*
3000	200	40	7	*a*
n	*c*	*x*	*e*	*r*

Complete the number patterns. Replace each $\square$ by a numeral, each $\triangle$ by $+$ or $-$.

3.

$10 + \square = \square - 22 = 55$ $\quad$ $\square + 65 = 87$ $\quad$ $44 \triangle \square = 29 + 14$

$31 + \square = \square + 29$ $\quad$ $15 \triangle \square = 57$

4.

$66 + 14 = \square + 43$ $\quad$ $\square - 22 = 102$ $\quad$ $87 = \square + 36$ $\quad$ $7 \triangle \square = 64$

$\square + 27 = 16$ $\quad$ $25 \triangle 16 = 29 + \square - 68$

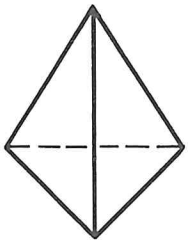
5.

$42 + 16 = \square + 14$ $\quad$ $\square - 23 = 14$ $\quad$ $\square + 16 = 65$ $\quad$ $15 + \square = 29$ $\quad$ $\square + 16 = 30$ $\quad$ $\square + 12 = 54 + 33 = \square - 19$

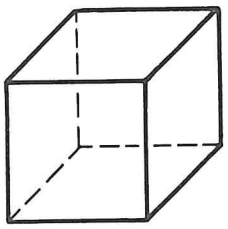
$\square + 14 = 16 + \square$ $\quad$ $\square + 13 = 29$ $\quad$ $\square + 18 = 63$ $\quad$ $24 + \square + 14 = \square$

$\square = 39 + \square + 17$ $\quad$ $\square + 17 = \square$

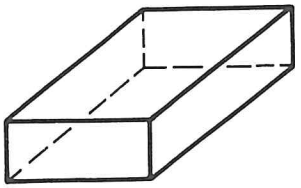
Figures not in a plane



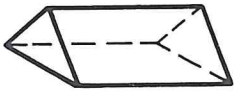
tetrahedron



cube



rectangular prism

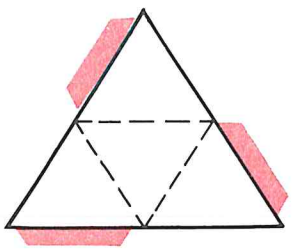


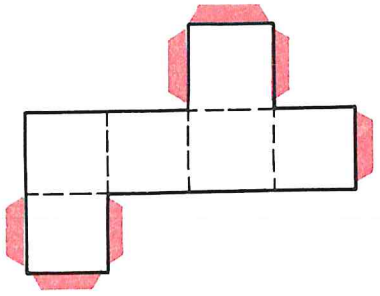
triangular prism

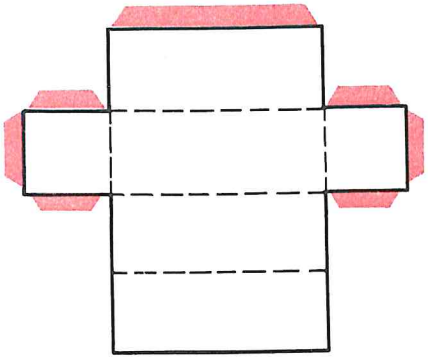
These are examples of simple *closed surfaces* which are formed by the union of 4 or more plane regions. The planes are called **faces** or **sides,** the line segments are called **edges,** and the corner points are called **vertices.**

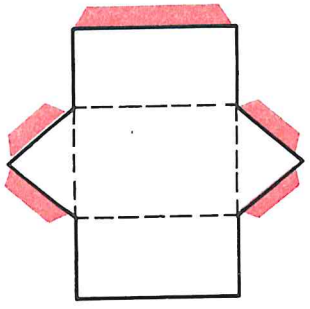
EXERCISES

1. Here are the patterns for making these figures. Make a copy of each on heavy paper. The paper should be folded along the dotted line segments. The red regions can be used for pasting.







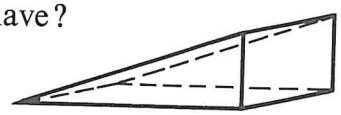


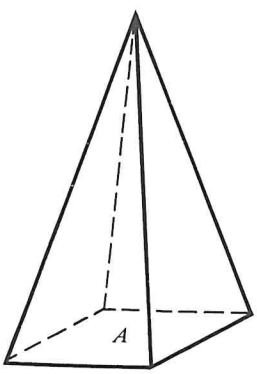
***2.** A closed surface is one which does not lie entirely in one plane. A simple closed surface separates space into 3 sets of points. Name the 3 sets.

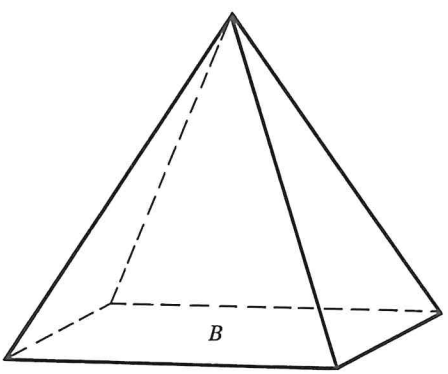
Use your closed surface models to help you complete this table.

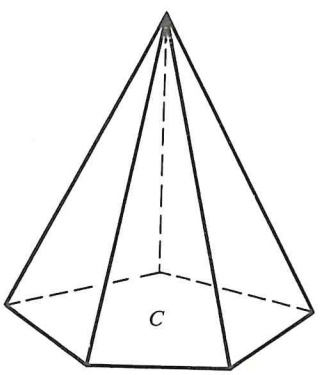
closed surface	no. of faces	no. of vertices	no. of edges	shape of faces
tetrahedron	*a*	*b*	*c*	triangles
cube	*d*	*e*	*f*	*g*
rectangular prism	*h*	*i*	*j*	*k*
triangular prism	*l*	*m*	*n*	triangles and *p*

1. Are all the edges of a cube congruent?
2. Is a cube a rectangular prism?
3. Closed surfaces whose edges and angles are all congruent are said to be **regular.** Is a cube a regular closed surface? How many congruent edges does a regular tetrahedron have?
4. Have you ever seen a carpenter use a wedge? What space figure looks like a wedge?



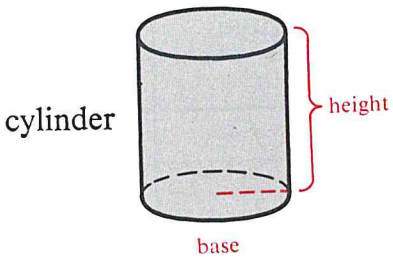


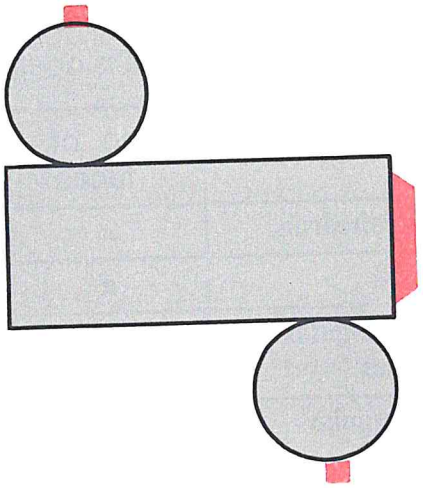


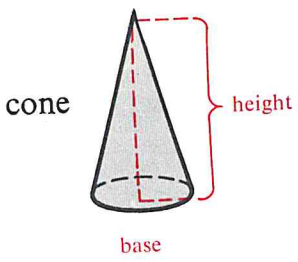


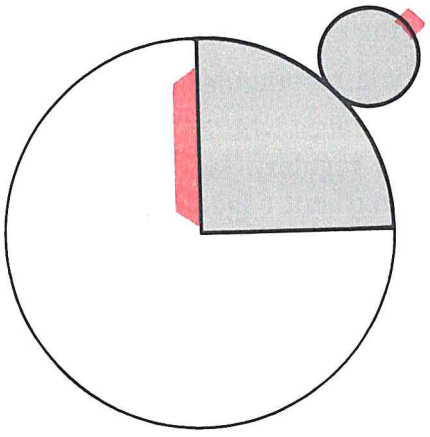
5. The figures above are called **pyramids.** A pyramid is labeled according to the shape of its *base*. The base of *A* is a square so it would be called a *square pyramid*. *B* is a *rectangular pyramid* and *C* is a *pentagonal pyramid*. Can a tetrahedron be called a triangular pyramid?
6. Make a table like the one at the top of the page, for the pyramids.
7. Will pyramids always have triangular faces? Why?

Curved surfaces







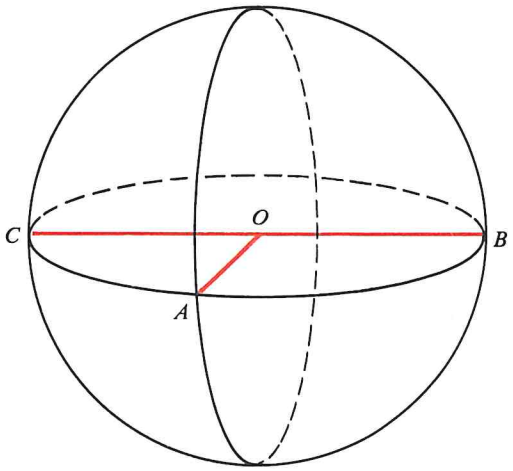


Not all simple closed surfaces are formed by plane regions. Cylinders and cones are surfaces which are curved. The base of a cylinder or cone is usually a circular region. The pattern for each figure is given on the right above.

EXERCISES

1. Make models of a cylinder and a cone using the patterns shown above.
2. Name some everyday examples of cylinders and cones.
3. If you place the round surface of a cylinder on your desk and roll it, in how many directions can it go?
4. If you place a cone on its side on your desk and roll it, what kind of region does it cover?

A **sphere** is a simple closed surface which is "completely round." Every point on the sphere is the same distance from a point called the center.

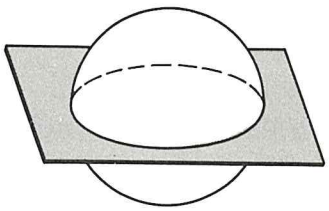


$\overline{OA}$ is **a radius,** the length of $\overline{OA}$ is **the radius.**

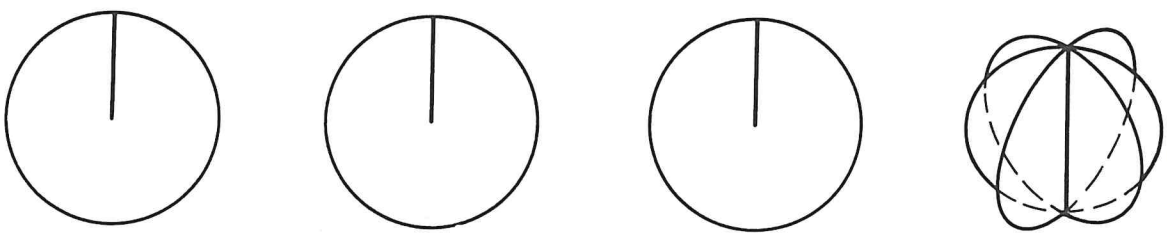
$\overline{CB}$ is **a diameter,** the length of $\overline{CB}$ is **the diameter.**

EXERCISES

1. Are all the radii of a sphere congruent?
2. Are all the diameters of a sphere congruent?
3. If the radius of a sphere is 6″, what is the diameter?

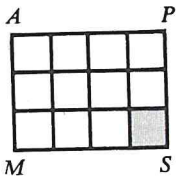


4. When a plane intersects a sphere in more than 1 point, what closed curve is formed?
5. If you place a sphere on your desk, in how many directions can it roll?
6. Cut out of cardboard several circular regions of the same radius. Cut along a radius of each circle and fit the regions together as shown below. What shape does this begin to form?



7. Do you think the center of the shape will be the same as the centers of your circular regions? Will its radius be the same as the radius of your circular regions?

Measuring regions



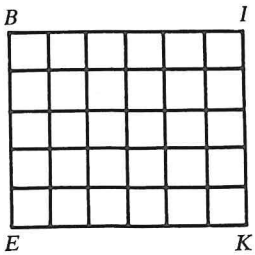
To measure the **area** of a region we compare a unit of area, called a *square unit*, with the region.

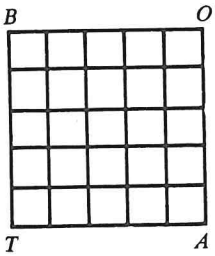
Since 12 square units will match the region of rectangle $MAPS$, we say that its *area* is 12 *square units*.

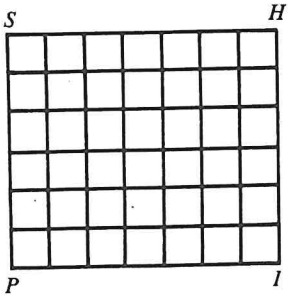
$\text{m}(MAPS) = 12$

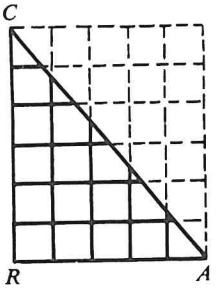
EXERCISES

How many square units are contained in each region?



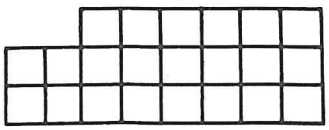




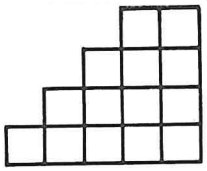


1. $\text{m}(BIKE)$ **2.** $\text{m}(BOAT)$ **3.** $\text{m}(SHIP)$ **4.** $\text{m}(CAR)$

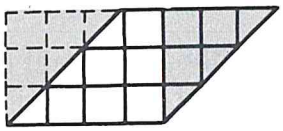
How many square units are contained in each region?



5.



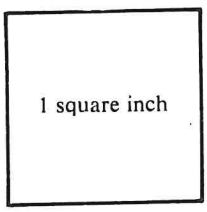
6.



7.

8.

The square inch is a common unit for measuring area. Draw a square inch. Cut it out and use it to find the area of each region.



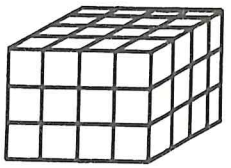
9. square with sides 3″

10. rectangle with sides 4″ and 6″

11. rectangle with sides $4\frac{1}{2}$″ and 2″

The volume of closed surfaces

To measure the **volume** of the space inside a space figure we compare a unit of volume, called a *cubic unit*, with the space. Since 48 cubic units will match the space of rectangular prism P we say that its *volume* is 48 *cubic units*.



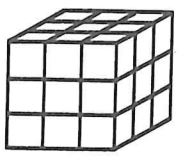
P

1 cubic unit

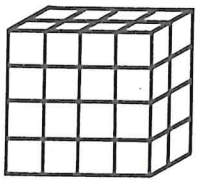
$m(P) = 48$

EXERCISES

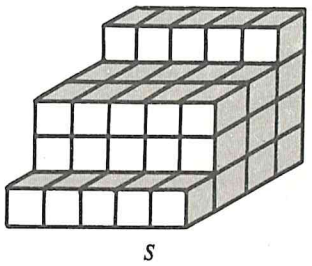
How many cubic units are contained in each of these space figures?



Q



R



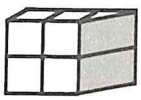
S

1. $m(Q) = a$

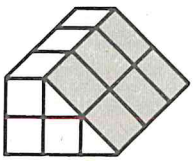
2. $m(R) = b$

3. $m(S) = c$

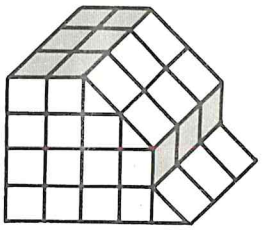
How many cubic units are contained in each of these space figures?

4. 

T

5. 

U

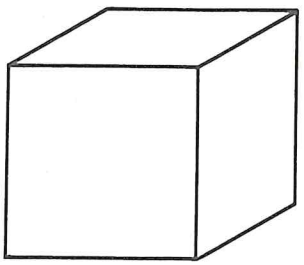
6. 

V

7. The cubic inch is a common unit for measuring volume. Use the pattern for the cube on page 82 to construct a cube with all edges of length 1″.

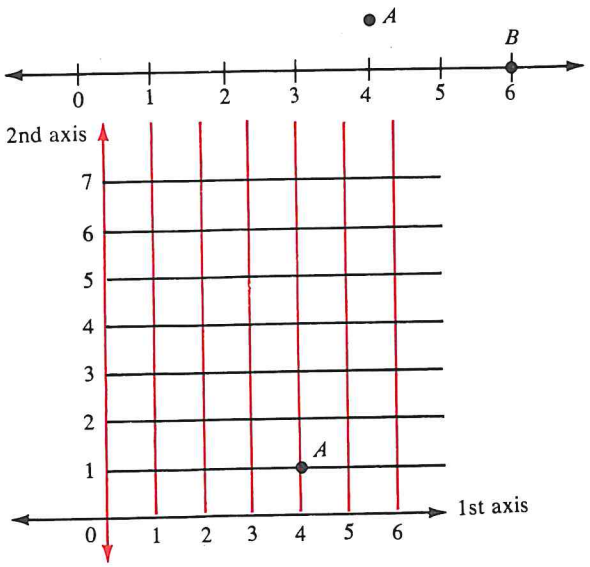
8. Construct a cube with each edge 3″. Compare it with your cubic inch and name its volume.

*9. Find boxes of various sizes and see if you can name their volumes by comparing them with a cubic inch.



1 cubic inch

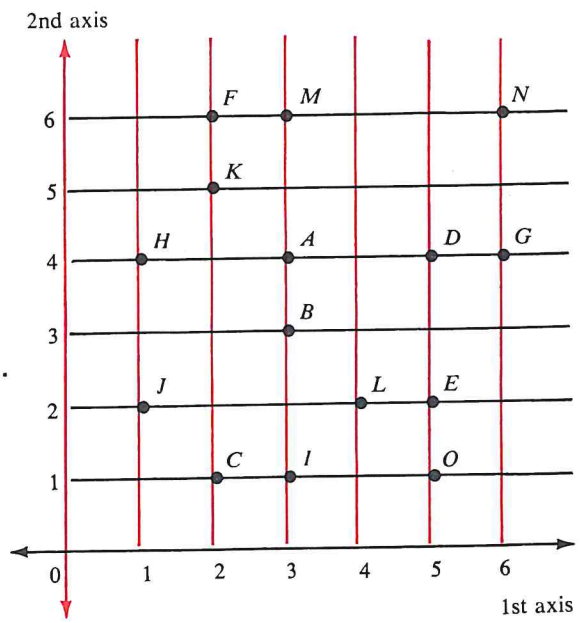
Using two number lines



The point B is labeled 6. The point A cannot be labeled by a single number since it is not on this number line. If we draw another number line perpendicular to the first one, we can now label point A using a pair of numbers. Since A is 4 units along the black number line (called the **first axis**) and 1 unit along the red number line (called the **second axis**), we write the number pair (4, 1).

EXERCISES

Study this figure. Name the letters which label the points.



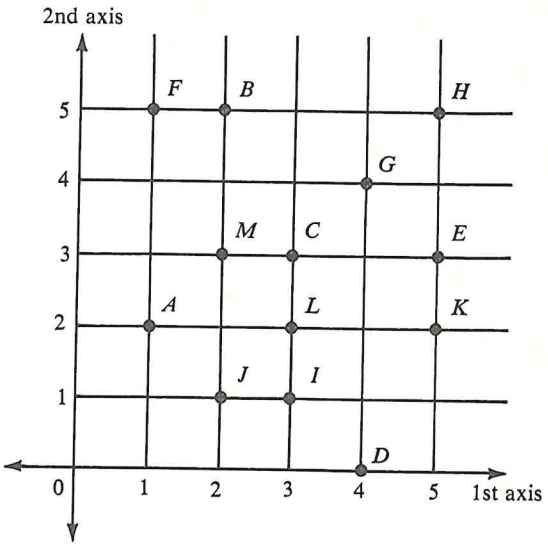
1. (2, 1)
2. (1, 2)
3. (3, 3)
4. (5, 2)
5. (6, 4)
6. (3, 6)
7. (5, 1)
8. (2, 6)
9. (3, 4)
10. (6, 6)

Name the number pairs for the points.

11. K
12. A
13. L
14. D
15. N
16. F
17. H
18. I
19. E
20. C

21. Using the diagram above, locate the points $\left(3\frac{1}{2}, 0\right)$, $\left(0, 2\frac{1}{2}\right)$, and $\left(3\frac{1}{2}, 2\frac{1}{2}\right)$.

A set of points in a plane labeled by number pairs is called a **number plane.** For each point there is a number pair. The point (2, 3) is reached by going 2 units along the first axis and 3 units along the second axis, from the starting point (0, 0). We call (2, 3) an **ordered number pair.** The order of the numbers in each pair is important. Notice that (2, 3) and (3, 2) name different points, *M* and *L*.

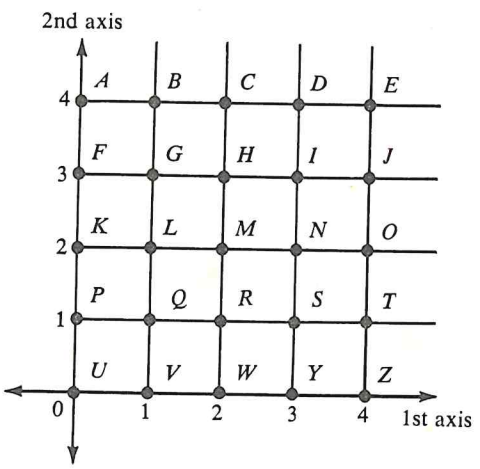


EXERCISES

Write a number pair to name the points in the number plane above labeled

1. *A*
2. *D*
3. *C*
4. *E*
5. *B*
6. *G*
7. *J*
8. *L*
9. *K*
10. *H*

We can use this number plane to play a spelling game. A number pair may be used in place of a letter. For example, the pairs (1, 4), (4, 2), (3, 0), give us the word *BOY*.



Use number pairs to write these words.

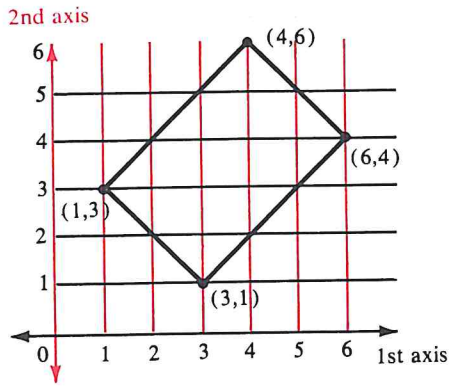
11. *SCHOOL*
12. *VACATION*
13. *FUN*
14. *WALTZ*
15. *QUICKLY*
16. *SUMMER*

From the diagram above, name these words.

17. (2, 0), (4, 2), (0, 0), (1, 2), (3, 4)
18. (3, 0), (4, 2), (0, 0)
19. (1, 2), (3, 3), (0, 2), (4, 4)
20. (4, 1), (4, 2)
21. (1, 0), (3, 3), (3, 1), (3, 3), (4, 1)
22. (2, 2), (0, 4), (2, 1), (3, 1)

Drawing shapes on the number plane

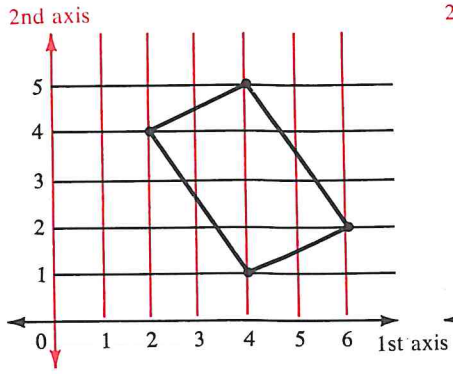
We may join points on the number plane to draw shapes. On this number plane we have joined the points (3, 1), (6, 4), (4, 6), and (1, 3) to draw a rectangle.



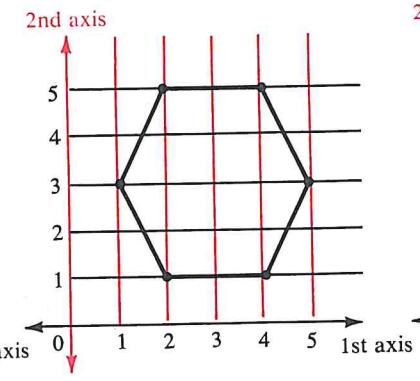
EXERCISES

Name the vertices of these polygons, using number pairs.

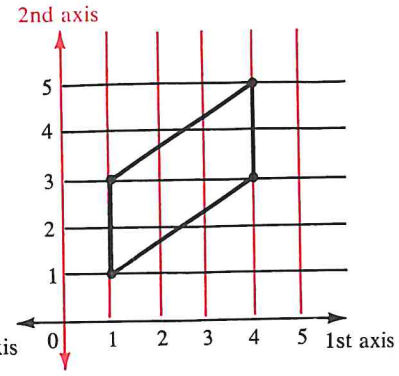
1.



2.



3.



Draw a number plane using 10 units on each axis.

4. Label the points (0, 0), (2, 2), (4, 4) and (8, 8). Name the figure you get when you join the points.

5. Label the points (3, 4), (7, 8), and (5, 0). Name the figure you get when you join the points.

6. Label the points (2, 5), (4, 7), and (7, 10). Name the figure you get when you join these points. Are (0, 3) and (6, 9) points on this figure?

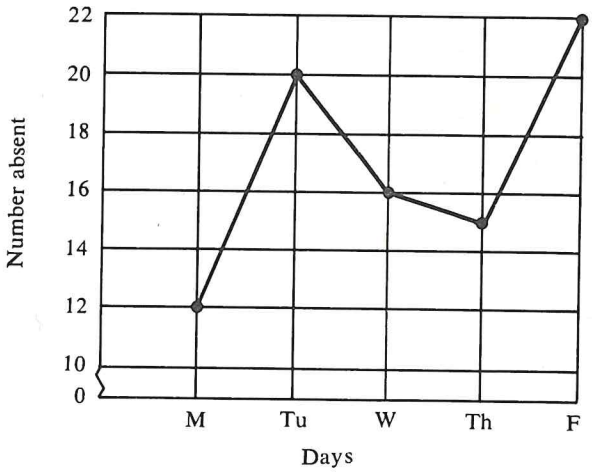
7. Find these points and join them in order. Then name the shape you have drawn.

(1, 0), (1, 6), (0, 6), (4, 10), (6, 8), (6, 9), (7, 9), (7, 7), (8, 6)
(7, 6), (7, 5), (10, 5), (10, 0), (6, 0), (6, 3), (5, 3), (5, 0), (1, 0)

Central School recorded the number of absences in a week in two ways, on a chart and on a number plane.

Day	Absent
M	12
T	20
W	16
Th	15
F	22

Information such as this is called **data.**



This picture of data is called a **graph.**

EXERCISES

Data can be shown as ordered pairs. For example, the pair $(M, 12)$ shows that on Monday 12 students were absent.

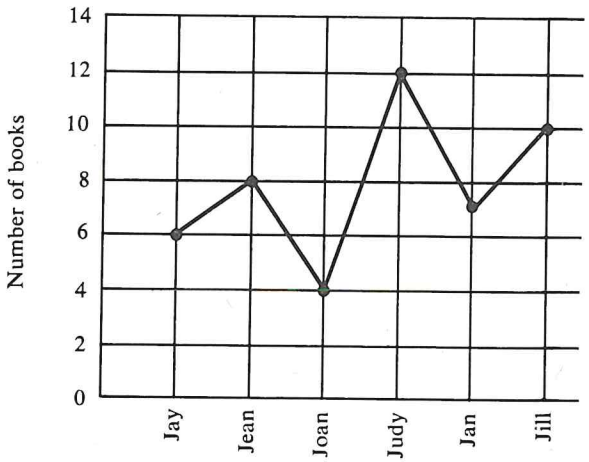
1. List the 4 other ordered pairs for the data above.

2. Six girls made a graph of the number of books they read in a month.

a. Which girl read the most books? the fewest?

b. How many books did Judy read?

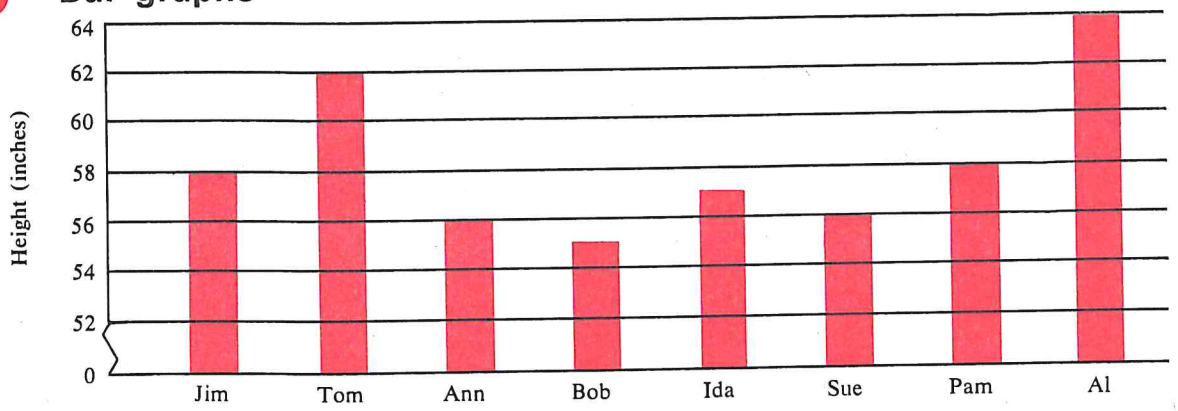
c. List the ordered pairs shown by the graph.



3. Make a graph of this data.

student	Jim	Tom	Ann	Bob	Ida	Sue	Pam	Al
test score	16	20	18	14	12	15	18	16

Bar graphs

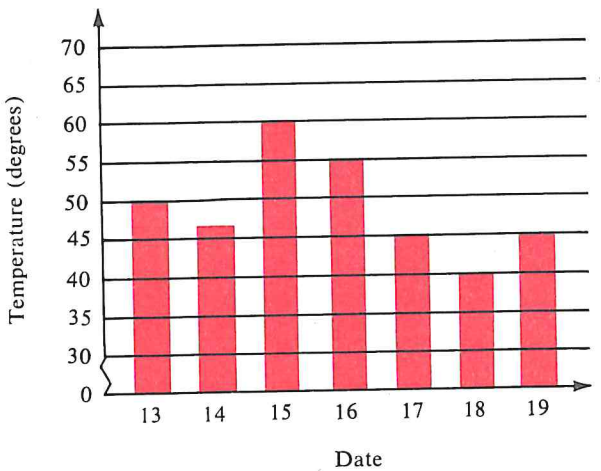


Another way to picture data is with a **bar graph.** This bar graph shows the height, in inches, of 8 students.

EXERCISES

1. Form ordered pairs like (*Jim*, 58) for the data on the bar graph above.

2. Study this bar graph, which shows the highest temperature reached on 7 days in October.



   a. What was the highest temperature for the week?

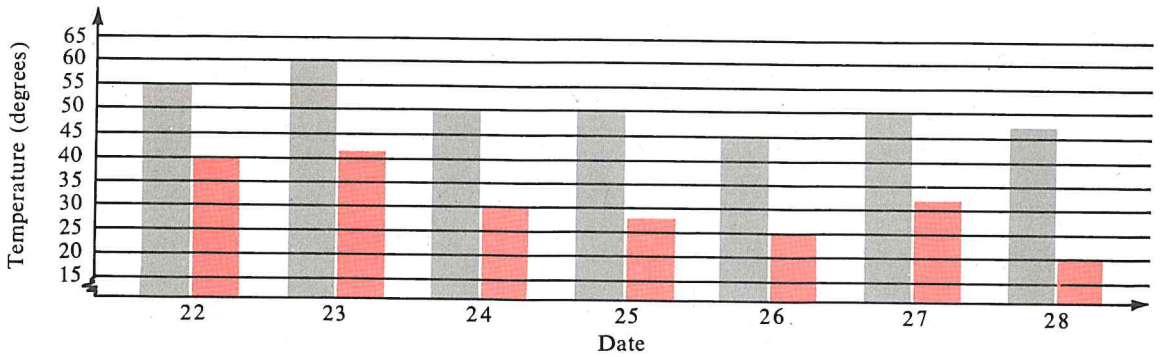
   b. On what date was the highest temperature 55°? 47°?

   c. Form ordered pairs like (13, 50) from the data on this bar graph.

3. Make a bar graph showing the data on this chart.

student	1	2	3	4	5	6	7	8
weight (lb.)	65	55	70	85	72	67	70	75

*4. Make a bar graph showing the heights, in inches, of the students in your row.



A *bar graph* can be used to show more than 1 set of data. This bar graph pictures the highest and lowest temperatures for each day of a week in November.

EXERCISES

Study the temperature bar graph.

1. What was the highest temperature on the 24th? the 27th?
2. What was the lowest temperature on the 23rd? the 26th?
3. On what date was the high temperature 47° and the low temperature 20°?
   On what date was the high temperature 50° and the low temperature 27°?
4. On what date did the temperature change the most? by how much?
5. List the ordered pairs showing the date and the highest temperature.
6. List the ordered pairs showing the date and the lowest temperature.

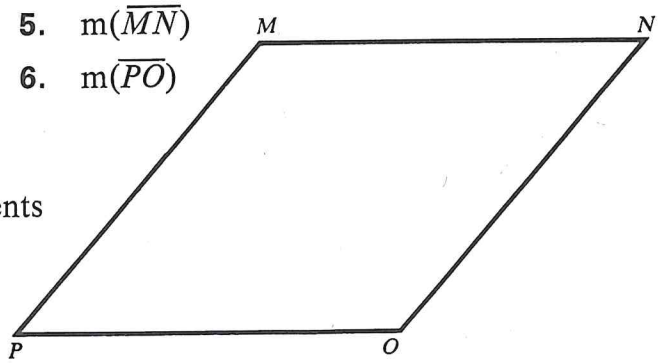
Make a double bar graph showing data about attendance of boys and girls at 6 school games.

game	1st	2nd	3rd	4th	5th	6th
girls	25	30	25	30	50	55
boys	20	30	40	35	35	65

CHECKING UP

Use your protractor or ruler to find these measures, in inches or degrees, for *MNOP*.

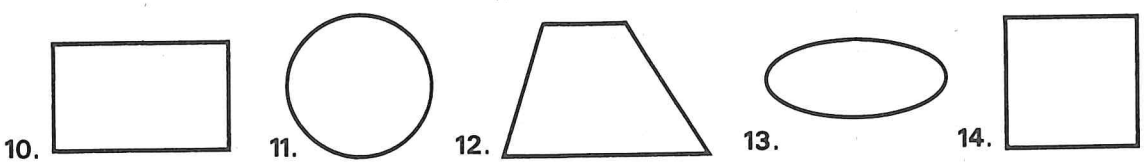
1. $m(\overline{MP})$
2. $m(\angle PMN)$
3. $m(\overline{NO})$
4. $m(\angle MNO)$
5. $m(\overline{MN})$
6. $m(\overline{PO})$



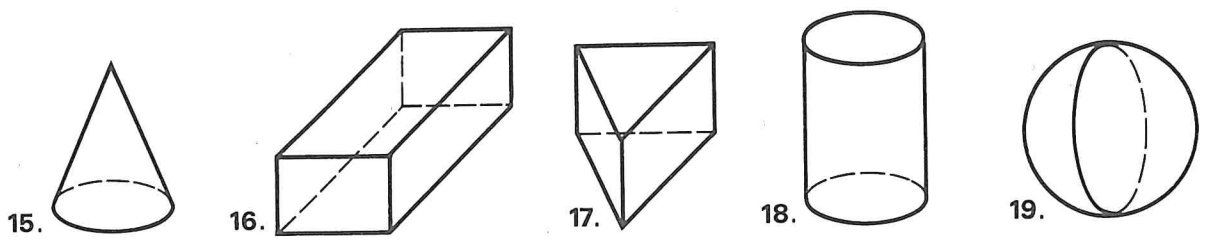
From your answers above, name

7. two pairs of parallel line segments
8. the perimeter of *MNOP*
9. the figure *MNOP*

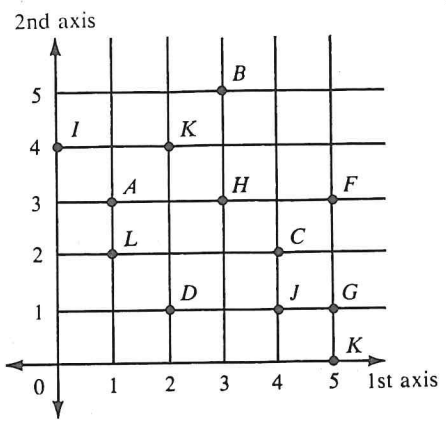
Name each plane figure.



Name each surface.



Study this number plane. Use number pairs to name these points.



20. *A*
21. *J*
22. *B*
23. *D*

Use letters to name these points.

24. (5, 3)
25. (2, 1)
26. (0, 4)

Geometry is the study of points. A curve is a set of points. A line is straight. A plane is a set of points.

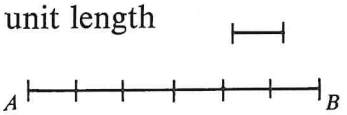
Copy and complete this chart by drawing the figure or completing the description.

	figure	formed by	looks like
1.	**angle**	two rays	∠
2.	**triangle**	? line segments	?
3.	**quadrilateral**	four ?	?
4.	**tetrahedron**	? plane regions	
5.	**cube**	? plane regions	?

To measure length, area, volume, or the size of an angle, we compare a sample unit with the line segment, region, space, or angle we are measuring.

Name the measure for each figure, using the given unit.

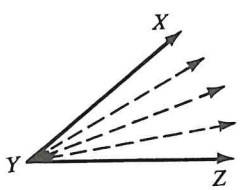
unit length



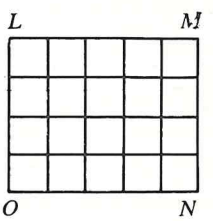
6. m($\overline{AB}$)

unit angle

7. m($\angle XYZ$)

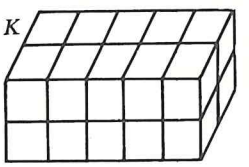


unit area



8. m($LMNO$)

unit volume



9. m(K)

Points on a number plane may be labeled by number pairs.

Draw a number plane like the one on page 88 and label the points.

10. (2, 3) **11.** (3 ,4) **12.** (5, 2) **13.** (1, 4)

LOOKING WAY BACK

List the elements in these sets.

1. A = {the odd numbers between 40 and 50}
2. B = {the whole numbers between 45 and 50}
3. C = {the first ten letters in the alphabet}
4. D = {the letters in the word *largest*}
5. $A \cup B$ **6.** $A \cap B$ **7.** $C \cup D$ **8.** $C \cap D$

Name these numbers in their shortest form.

9. $3000 + 400 + 6$ **10.** 273 thousands, 476 ones

11. four million, three hundred forty-seven thousand, nine hundred twenty-seven

12. $(3 \times 10^6) + (4 \times 10^5) + (6 \times 10^4) + (8 \times 10^3) + (5 \times 10^2) + (6 \times 10) + 8$

Complete these number charts.

13.

+	8	12	14	31	41	73
19	*a*	*b*	*c*	*d*	*e*	*f*

15.

+	*g*	*h*	*i*	*j*	*k*	*l*
27	49	62	31	52	41	83

14.

+	300	20	6	326
400	700	420	*a*	*b*
20	320	*c*	*d*	*e*
7	*f*	*g*	*h*	*i*
427	*j*	*k*	*l*	*m*

16.

+	400	60	7	467
300	700	360	307	*n*
80	*o*	*p*	*q*	*r*
5	*s*	*t*	*u*	*v*
385	*w*	*x*	*y*	*z*

Name the sums or differences.

17.
$$\begin{array}{r} 46829 \\ 38742 \\ +86531 \\ \hline \end{array}$$

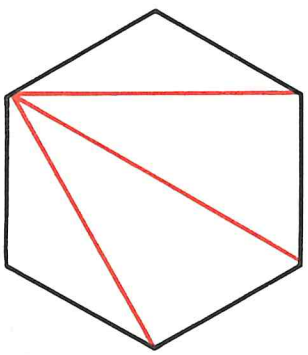
18.
$$\begin{array}{r} 529837 \\ -\ 254661 \\ \hline \end{array}$$

19.
$$\begin{array}{r} 82974 \\ 5462 \\ +\ 72807 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 82107 \\ -\ 71016 \\ \hline \end{array}$$

Make a model of a triangle and any other polygon using cardboard strips with pins to hold the corners together.

1. Is the shape of the triangle fixed?
2. Is the shape of the polygon fixed?
3. Have you seen applications of this property of triangles?
4. Any polygon may be changed to a set of triangles by drawing diagonals. For example, a 6-sided polygon (hexagon) may be changed to form 4 triangles by drawing 3 diagonals, as shown here:



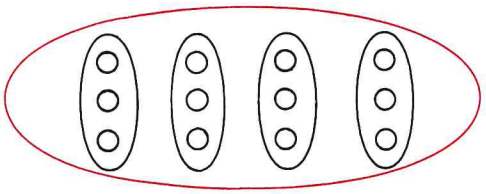
Complete this chart up to 10-sided polygons, showing the number of diagonals you draw and the number of triangles you form.

number of sides of polygon	number of diagonals	number of triangles
4	1	2
5	2	3
6	3	4
⋮	⋮	⋮

Euler's Formula

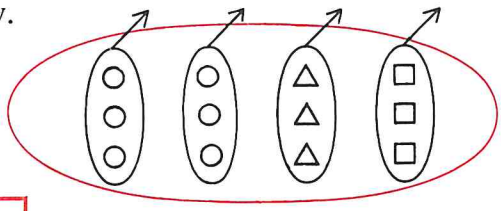
A famous Swiss mathematician, Leonard Euler, discovered this connection between the vertices, faces, and edges of any space figure: $V + F = E + 2$. V is the number of vertices, F is the number of faces, and E is the number of edges. For a cube: $V = 8$, $F = 6$, $E = 12$, $8 + 6 = 12 + 2 = 14$. Check this equation for other space figures.

One meaning of multiplication and division



4 sets of 3 form 1 set of **12**.

$$3 + 3 + 3 + 3 = \mathbf{12}$$
$$4 \times 3 = \mathbf{12}$$



The arrows show 4 subsets of 3 being taken away.
A set of **12** forms 4 sets of 3.

$$\mathbf{12} - (3 + 3 + 3 + 3) = \mathbf{12} - (4 \times 3) = 0$$
$$\mathbf{12} \div 3 = 4$$

EXERCISES

Complete the sentences and solve the equations.

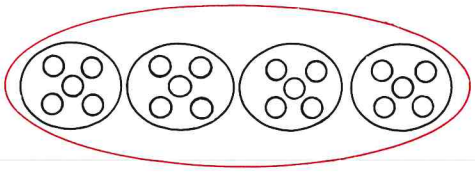
1. 2 sets of 5 form a set of a. $2 \times 5 = b$
2. 6 sets of 3 form a set of c. $6 \times 3 = d$
3. 3 sets of e form a set of 21. $3 \times f = 21$
4. g sets of 4 form a set of 24. $h \times 4 = 24$
5. A set of 15 forms 3 sets of i. $15 \div 3 = j$
6. A set of 42 forms 6 sets of k. $42 \div 6 = l$
7. A set of m forms 7 sets of 4. $n \div 4 = 7$

Solve the equations.

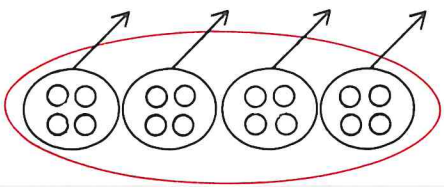
8. $2 + 2 + 2 + 2 = a$
9. $4 \times 2 = b$
10. $c \div 4 = 2$
11. $9 + 9 + 9 = d$
12. $3 \times 9 = e$
13. $f \div 3 = 9$
14. $32 - (8 + 8 + 8 + 8) = g$
15. $32 \div 8 = h$
16. $8 \times i = 32$
17. $18 - (9 + 9) = j$
18. $18 \div 9 = k$
19. $9 \times l = 18$

Write an equation for each set.

20.



21.



$$\begin{array}{r} 7 \\ 7 \\ +\ 7 \\ \hline 21 \end{array}$$

$3 \times 7 = 21$

factors: 3, 7 — product: 21

We can think of multiplication as **repeated addition.** Multiplication is the renaming of 2 **factors** as a **product.**

$$\begin{array}{r} 21 \\ -\ 7 \\ \hline 14 \\ -\ 7 \\ \hline 7 \\ -\ 7 \\ \hline 0 \end{array}$$

$21 - (3 \times 7) = 0$

$21 \div 7 = 3$

product: 21 — factors: 7, 3

We can think of division as **repeated subtraction.** Division may be the renaming of a **product** and **factor** as the **missing factor.**

EXERCISES

Solve the equations.

1. $3 \times 5 = a$
2. $7 \times 5 = b$
3. $9 \div 3 = c$
4. $24 \div 8 = d$
5. $6 \times e = 36$
6. $f \times 8 = 56$
7. $81 \div g = 9$
8. $h \div 4 = 7$
9. $30 \div 5 = i$
10. $42 \div j = 6$
11. $k \div 9 = 4$
12. $l \times 7 = 63$

Solve the equations.

13. $4 + 4 + 4 + 4 + 4 = a$
14. $5 \times 4 = b$
15. $c \div 4 = 5$
16. $d \div 5 = 4$
17. $27 - (9 + 9 + 9) = e$
18. $27 \div 9 = f$
19. $g \times 9 = 27$
20. $27 \div 3 = h$
21. $48 - (i \times 8) = 0$
22. $48 \div 8 = j$
23. $48 \div k = 8$
24. $6 \times 8 = l$

Rewrite the equations using the $\times$ sign.

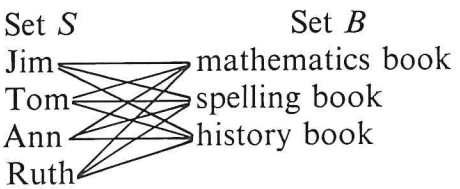
25. $6 + 6 + 6 = 18$
26. $8 + 8 + 8 + 8 = 32$
27. $9 + 9 + 9 + 9 + 9 = 45$
28. $10 + 10 = 20$

Rewrite the equations using the $\div$ sign.

29. $27 - (9 + 9 + 9) = 0$
30. $56 - (7 \times 8) = 0$
31. $36 - (6 \times 6) = 0$
32. $16 - (8 + 8) = 0$

Cross product of sets

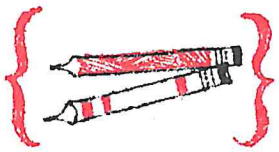
Each of the students can be *paired* with each of the books. The set of all possible pairings is called the **cross product** of the sets. The total number of possible pairings is $4 \times 3 = 12$.

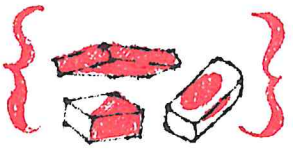


EXERCISES

Name the number of possible pairings.

1.





2.

3. $A = \{\text{Joan, Jerry, Kathy, Karl, Norma}\}$
$B = \{\text{book, ball, bike, bat, bread}\}$

4. $V = \{1, 2, 3, 4, 5, 6\}$
$T = \{2, 4, 6, 8, 10, 12, 0\}$

Match $L = \{\text{a, e, i, o, u}\}$ with each of these sets.
Name the total number of pairings in each case.

5. $M = \{\text{t, n, s}\}$ **6.** $N = \{\text{d, l, m, r, w, y}\}$ **7.** $P = \{0, 2, 4, 6, 8, 10, 12\}$

Complete the sentences.

8. The cross product of a set of 3 and a set of 6 is a set of ***a***.

9. The cross product of a set of 8 and a set of 5 is a set of ***b***.

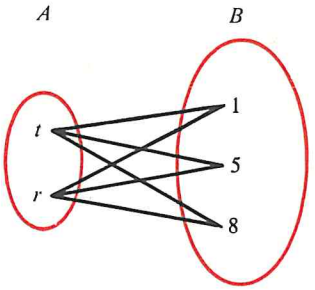
10. The cross product of a set of 7 and a set of 9 is a set of ***c***.

11. The cross product of a set of ***d*** and a set of 4 is a set of 32.

The cardinal number of set A is 2.
The cardinal number of set B is 3.
The cross product of A and B (written $A \otimes B$) can be shown as a set of pairs.

$A \otimes B = \{(t, 1), (t, 5), (t, 8), (r, 1), (r, 5), (r, 8)\}$

$n(A) = 2 \quad n(B) = 3 \quad n(A \otimes B) = 6$



EXERCISES

Write the cross product of each pair of sets.

1. $S = \{\text{Steve, Stan, Sylvia}\}$ and $T = \{1, 2, 3, 4\}$
2. $V = \{\text{t, o, d, a, y}\}$ and $N = \{1, \text{u, c, k}\}$

Name these cardinal numbers for the sets above.

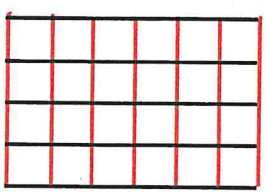
3. $n(S)$ 4. $n(T)$ 5. $n(S \otimes T)$ 6. $n(V)$ 7. $n(N)$ 8. $n(V \otimes N)$

Study the sets, then solve the equations.

$L = \{\text{Lora, Linda, Lisa}\}$
$C = \{\text{s, h, a, p, e}\}$
$Y = \{0, 1, 2, 3, 4, 5, 6\}$
$P = \{\blacksquare \blacktriangle \bigstar\}$

9. $n(L) = a$
10. $n(C) = b$
11. $n(Y) = c$
12. $n(P) = d$
13. $n(L \otimes C) = e$
14. $n(L \otimes Y) = f$
15. $n(C \otimes Y) = g$
16. $n(P \otimes C) = h$
17. $n(P \otimes Y) = i$

Study the figure, then complete each sentence.



18. The number of red line segments is ____?____.
19. The number of black line segments is ____?____.
20. The number of points of intersection is ____?____.

How many pairings can be formed from each pair of sets?

21. 2 boys and 7 chairs
22. 3 cats and 5 bowls of milk
23. 4 fish and 8 fish-hooks
24. 9 girls and 6 presents
25. 3 circles and 8 squares
26. 7 balls and 9 bats

Properties of multiplication

Solve the equations.

1. $2 \times 3 = 3 \times a$
2. $5 \times 4 = b \times 5$
3. $7 \times 3 = c \times 7$
4. $5 \times 8 = 8 \times d$

Complete the sentence.

5. **The Commutative Property of Multiplication**
The order of the factors does not affect the ___?___.

Solve the equations.

6. $(2 \times 3) \times 2 = a$
7. $2 \times (3 \times 2) = b$
8. $(2 \times 3) \times 2 = 2 \times (c \times 2)$
9. $(4 \times 2) \times 3 = d$
10. $4 \times (2 \times 3) = e$
11. $(4 \times 2) \times 3 = f \times (2 \times 3)$

Complete the sentence.

12. **The Associative Property of Multiplication**
The way in which factors are associated (grouped) does not affect the ___?___.

Solve the equations.

13. $3 \times (4 + 2) = 3 \times a$
$= b$
14. $(3 \times 4) + (3 \times 2) = 12 + c$
$= d$
15. $4 \times (3 + 5) = 4 \times e$
$= f$
16. $4 \times (3 + 5) = (g \times 3) + (h \times 5)$
$= i$

The Distributive Property
The sum of 2 addends with a common factor may be found by multiplying then adding: $(4 \times 3) + (5 \times 3) = 12 + 15 = 27$ *or* adding then multiplying: $(4 \times 3) + (5 \times 3) = (4 + 5) \times 3 = 27$.

The 4 equations $3 \times 4 = 12$, $4 \times 3 = 12$, $12 \div 4 = 3$, and $12 \div 3 = 4$ form a **family of facts.**

Name the family of facts for each of these sets of 2 factors and their product.

17. $\{7, 3, 21\}$
18. $\{5, 4, 20\}$
19. $\{9, 7, 63\}$
20. $\{8, 9, 72\}$

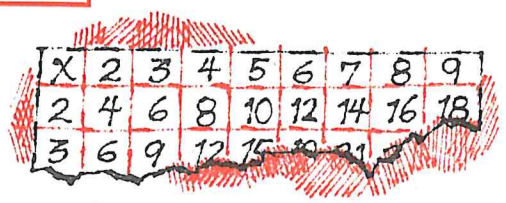
There are 36 basic facts to be learned. We call them basic because we use them for *all* the multiplications and divisions we do. They have factors greater than 1 and less than 10. Here is one way they can be arranged.

10 2×5	**20** 4×5	**30** 6×5	**40** 8×5	**21** 3×7	**81** 9×9	**12** 2×6 3×4	**32** 4×8
42 6×7	**72** 8×9	**63** 7×9	**4** 2×2	**14** 2×7	**24** 3×8 4×6	**54** 6×9	**64** 8×8
15 3×5	**25** 5×5	**35** 5×7	**45** 5×9	**6** 2×3	**16** 2×8 4×4	**36** 4×9 6×6	**56** 7×8
27 3×9	**8** 2×4	**18** 2×9 3×6	**28** 4×7	**48** 6×8	**9** 3×3	**49** 7×7	

EXERCISES

Complete this number pattern to show the 36 basic facts.

1.



Solve the equations.

2. $3 \times 0 = a$ **3.** $0 \times 3 = b$ **4.** $0 \times 0 = c$ **5.** $0 \div 7 = d$

Complete the sentence.

6. When 0 is one of the factors, the product is ____?____.

Solve the equations.

7. $3 \times 1 = a$ **8.** $5 \times 1 = b$ **9.** $9 \div c = 9$ **10.** $12 \div d = 1$

Complete the sentence.

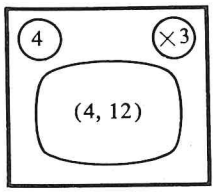
11. When 1 is one of the factors, the product and the other factor are the ____?____.

Equations and functions

Study these methods for naming the function.

Multiply by 3: {(1, 3)(2, 6)(3, 9)(4, 12). . . .}

Function TV



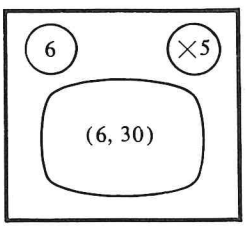
Equations

$3 \times m = n$

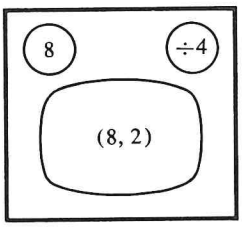
So, if $m = 4$, $n = 12$

EXERCISES

Study each function TV, then complete the set of number pairs.



1. {(6, 30), (2, 10), (4, ***a***), (7, ***b***), (5, ***c***), (8, ***d***)}



2. {(8, 2), (16, ***e***), (24, ***f***), (12, ***g***), (36, ***h***), (40, ***i***)}

Study each equation, then complete the number pattern.

3. $2 \times y = z$

y	3	5	8	6
z	6	***a***	***e***	***v***

4. $8 \times y = z$

y	2	4	5	9
z	16	***n***	***e***	***b***

5. $y \div 3 = z$

y	3	18	24	30
z	***c***	***n***	***i***	***s***

6. $y \div 4 = z$

y	4	16	28	0
z	***s***	***r***	***a***	***n***

7. $12 + y = z$

y	8	17	26	39
z	***i***	***e***	***c***	***r***

8. $y \times z = 36$

y	2	3	***u***	***v***
z	***s***	***a***	4	6

***9.** Suppose you have $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and the equation $6 \times y = z$. If A is the set for y, list the members of the set for z.

Multiplication, one factor greater than 10

Study these methods for naming the product 7×24.

Expanded numerals	Three-step	Short form
$24 = 20 + 4$ $\times\ 7 = \times\ 7$ $140 + 28 = 168$	24 $\times\ 7$ 28 140 168	24 $\times\ 7$ 168

Notice how the *distributive property* is used in each method.

EXERCISES

Solve the equations.

1. $7 \times 24 = 7 \times (20 + a)$
$= (7 \times 20) + (7 \times a)$
$= b + c$
$= d$

2. $8 \times 39 = 8 \times (30 + e)$
$= (8 \times 30) + (8 \times e)$
$= f + g$
$= h$

Use expanded numerals to name the products.

3. 18×9 **4.** 47×6 **5.** 68×3 **6.** 54×9 **7.** 38×6 **8.** 27×8 **9.** 35×7 **10.** 48×6

Complete the examples.

11. 48×7: $5a$, $2b0$, 336

12. 63×6: 18, $c60$, $3d8$

13. 29×4: $3e$, $f0$, 116

14. 54×7: 28, 350, $3gh$

15. 63×5: 15, $3ij$, 315

16. 87×9: $6k$, $l20$, 783

Use the short method to name the products.

17. 17×5 **18.** 46×7 **19.** 63×9 **20.** 82×5 **21.** 43×8 **22.** 69×7

Solve the problem.

23. Beth spends 45 minutes a day practicing ballet. How many minutes does she practice in 5 days?

Multiplication, one factor greater than 100

Study these methods for naming the product 7×324.

Four-step

$$\begin{array}{r} 324 \\ \times\ 7 \\ \hline 28 \\ 140 \\ 2100 \\ \hline 2268 \end{array}$$

Short form

$$\begin{array}{r} 324 \\ \times\ 7 \\ \hline 2268 \end{array}$$

EXERCISES

Solve the equations.

1. $4 \times 652 = 4 \times (600 + 50 + 2)$
$= (4 \times 600) + (4 \times a) + (4 \times b)$
$= \quad 2400 \quad + \quad c \quad + \quad d$
$= e$

2. $7 \times 295 = 7 \times (200 + 90 + 5)$
$= (7 \times f) + (7 \times g) + (7 \times h)$
$= \quad i \quad + \quad j \quad + \quad k$
$= l$

Use the 4-step method to name the products.

3. $\begin{array}{r} 157 \\ \times\ 3 \\ \hline \end{array}$ **4.** $\begin{array}{r} 461 \\ \times\ 5 \\ \hline \end{array}$ **5.** $\begin{array}{r} 583 \\ \times\ 7 \\ \hline \end{array}$ **6.** $\begin{array}{r} 729 \\ \times\ 9 \\ \hline \end{array}$ **7.** $\begin{array}{r} 804 \\ \times\ 6 \\ \hline \end{array}$ **8.** $\begin{array}{r} 567 \\ \times\ 8 \\ \hline \end{array}$

Use the short method to name the products.

9. $\begin{array}{r} 234 \\ \times\ 3 \\ \hline \end{array}$ **10.** $\begin{array}{r} 458 \\ \times\ 6 \\ \hline \end{array}$ **11.** $\begin{array}{r} 652 \\ \times\ 8 \\ \hline \end{array}$ **12.** $\begin{array}{r} 708 \\ \times\ 5 \\ \hline \end{array}$ **13.** $\begin{array}{r} 498 \\ \times\ 7 \\ \hline \end{array}$ **14.** $\begin{array}{r} 896 \\ \times\ 9 \\ \hline \end{array}$

Name the products.

15. $\begin{array}{r} 4235 \\ \times\ 4 \\ \hline \end{array}$ **16.** $\begin{array}{r} 6027 \\ \times\ 3 \\ \hline \end{array}$ **17.** $\begin{array}{r} 5938 \\ \times\ 6 \\ \hline \end{array}$ **18.** $\begin{array}{r} 4947 \\ \times\ 8 \\ \hline \end{array}$ **19.** $\begin{array}{r} 9619 \\ \times\ 9 \\ \hline \end{array}$ **20.** $\begin{array}{r} 5721 \\ \times\ 7 \\ \hline \end{array}$

21. $3 \times 827 = a$

22. $4 \times 624 = b$

23. $298 \times 6 = c$

24. $507 \times 4 = d$

25. $3 \times 4729 = e$

26. $6 \times 8291 = f$

27. $5714 \times 8 = g$

28. $9273 \times 7 = h$

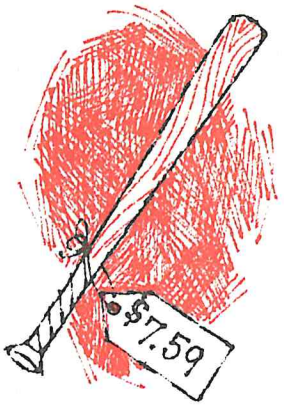
29. $8 \times 345 = i$

We may use these steps to find the cost of 7 baseball bats at \$7.59 each.

Step 1: **Estimate the cost.** If the bats cost \$7 each, the cost would be \$49.

Step 2: **Multiply.** $7 \times 759 = 5313$.

Step 3: **Compare** 5313 with 49 to find where to place the point. We see that the actual cost must be \$53.13, not \$5313.



EXERCISES

Name the total amounts.

1.	\$6.48 × 6	**3.**	\$5.59 × 5	**5.**	\$4.26 × 7	**7.**	\$3.78 × 4	**9.**	\$4.64 × 8	**11.**	\$3.83 × 9
2.	\$9.57 × 2	**4.**	\$8.79 × 3	**6.**	\$5.16 × 7	**8.**	\$3.29 × 9	**10.**	\$6.42 × 6	**12.**	\$3.13 × 5

Solve the problems.

13. What is the cost of 9 bats at \$6.89 each?

14. What is the cost of 6 train tickets at \$4.54 each?

15. What is the cost of 8 purses at \$3.29 each?

16. What is the cost of 7 dictionaries at \$7.38 each?

17. What is the cost of 5 dinners at \$2.75 each?

18. What is the cost of 6 model airplanes at \$1.98 each?

***19.** What is the total cost of 5 train tickets at \$6.58 and 5 meal tickets at \$2.75?

***20.** How much do we save if we buy 6 shirts for \$3.54 each instead of their usual price of \$3.98 each?

***21.** Joanne went shopping for summer clothes. She bought 4 skirts for \$3.78 each and 5 dresses for \$5.37 each. How much did she spend in all?

***22.** Which is cheaper, 8 one-way tickets at \$4.99 each, or 4 round-trip tickets at \$9.72 each?

Tens, hundreds as factors

Compare the equations and the working forms.

$26 \times 30 = 26 \times (3 \times 10)$ $= (26 \times 3) \times 10$ $= 78 \times 10$ $= 780$	$\begin{array}{r} 26 \\ \times\ 30 \\ \hline 780 \end{array}$	$26 \times 300 = 26 \times (3 \times 100)$ $= (26 \times 3) \times 100$ $= 78 \times 100$ $= 7800$	$\begin{array}{r} 26 \\ \times\ 300 \\ \hline 7800 \end{array}$

EXERCISES

Name the products.

1. $48 \times 3 = a$
2. $48 \times 30 = b$
3. $48 \times 300 = c$
4. $128 \times 3 = d$
5. $128 \times 30 = e$
6. $128 \times 300 = f$
7. $476 \times 4 = g$
8. $476 \times 40 = h$
9. $476 \times 400 = i$

Name the products.

10. $\begin{array}{r} 27 \\ \times\ 5 \\ \hline \end{array}$
11. $\begin{array}{r} 27 \\ \times\ 50 \\ \hline \end{array}$
12. $\begin{array}{r} 27 \\ \times\ 500 \\ \hline \end{array}$
13. $\begin{array}{r} 49 \\ \times\ 4 \\ \hline \end{array}$
14. $\begin{array}{r} 49 \\ \times\ 40 \\ \hline \end{array}$
15. $\begin{array}{r} 49 \\ \times\ 400 \\ \hline \end{array}$
16. $\begin{array}{r} 68 \\ \times\ 2 \\ \hline \end{array}$
17. $\begin{array}{r} 68 \\ \times\ 20 \\ \hline \end{array}$
18. $\begin{array}{r} 68 \\ \times\ 200 \\ \hline \end{array}$
19. $\begin{array}{r} 73 \\ \times\ 3 \\ \hline \end{array}$
20. $\begin{array}{r} 73 \\ \times\ 30 \\ \hline \end{array}$
21. $\begin{array}{r} 73 \\ \times\ 300 \\ \hline \end{array}$

Name the products.

22.	$27 \times 6 = a$	$270 \times 6 = b$	$2700 \times 6 = c$
23.	$36 \times 4 = d$	$360 \times 4 = e$	$3600 \times 4 = f$
24.	$56 \times 6 = g$	$560 \times 6 = h$	$5600 \times 6 = i$

Name the products.

25. $\begin{array}{r} 32 \\ \times\ 7 \\ \hline \end{array}$ $\begin{array}{r} 320 \\ \times\ 7 \\ \hline \end{array}$ $\begin{array}{r} 3200 \\ \times\ 7 \\ \hline \end{array}$

26. $\begin{array}{r} 85 \\ \times\ 6 \\ \hline \end{array}$ $\begin{array}{r} 850 \\ \times\ 6 \\ \hline \end{array}$ $\begin{array}{r} 8500 \\ \times\ 6 \\ \hline \end{array}$

27. $\begin{array}{r} 54 \\ \times\ 8 \\ \hline \end{array}$ $\begin{array}{r} 540 \\ \times\ 8 \\ \hline \end{array}$ $\begin{array}{r} 5400 \\ \times\ 8 \\ \hline \end{array}$

28. $\begin{array}{r} 94 \\ \times\ 3 \\ \hline \end{array}$ $\begin{array}{r} 940 \\ \times\ 3 \\ \hline \end{array}$ $\begin{array}{r} 9400 \\ \times\ 3 \\ \hline \end{array}$

29. $\begin{array}{r} 67 \\ \times\ 5 \\ \hline \end{array}$ $\begin{array}{r} 670 \\ \times\ 5 \\ \hline \end{array}$ $\begin{array}{r} 6700 \\ \times\ 5 \\ \hline \end{array}$

30. $\begin{array}{r} 94 \\ \times\ 6 \\ \hline \end{array}$ $\begin{array}{r} 940 \\ \times\ 6 \\ \hline \end{array}$ $\begin{array}{r} 9400 \\ \times\ 6 \\ \hline \end{array}$

Study these methods for naming the product 26×34.

$$\begin{array}{r} 34 \\ \times\ 26 \\ \hline \end{array} = \begin{array}{r} 34 \\ \times\ 20 \\ \hline 680 \end{array} + \begin{array}{r} 34 \\ \times\ 6 \\ \hline 204 \end{array} = 884$$

×	34
20	680
6	204
26	884

$$\begin{array}{r} 20 \times 34 = 680 \\ 6 \times 34 = 204 \\ \hline 26 \times 34 = 884 \end{array}$$

EXERCISES

Solve the equations.

1. $\begin{array}{r} 48 \\ \times\ 73 \\ \hline \end{array} = \begin{array}{r} 48 \\ \times\ 70 \\ \hline a \end{array} + \begin{array}{r} 48 \\ \times\ 3 \\ \hline b \end{array} = c$

2. $\begin{array}{r} 59 \\ \times\ 27 \\ \hline \end{array} = \begin{array}{r} 59 \\ \times\ 20 \\ \hline d \end{array} + \begin{array}{r} 59 \\ \times\ 7 \\ \hline e \end{array} = f$

Complete the number patterns.

3.

×	47
20	a
7	b
27	c

4.

×	28
40	d
8	e
48	f

5.

×	75
30	g
6	h
36	i

6.

×	39
50	j
3	k
53	l

7.

×	86
60	m
9	n
69	p

Solve the equations.

8. $\begin{array}{r} 20 \times 46 = a \\ 7 \times 46 = b \\ \hline 27 \times 46 = c \end{array}$

9. $\begin{array}{r} 30 \times 57 = d \\ 5 \times 57 = e \\ \hline 35 \times 57 = f \end{array}$

10. $\begin{array}{r} 40 \times 64 = g \\ 2 \times 64 = h \\ \hline 42 \times 64 = i \end{array}$

11. $\begin{array}{r} 60 \times 58 = j \\ 7 \times 58 = k \\ \hline 67 \times 58 = l \end{array}$

Use any of the methods on this page to name the products.

12. $36 \times 83 = a$

13. $28 \times 71 = b$

14. $36 \times 73 = c$

15. $89 \times 78 = d$

16. $45 \times 68 = e$

17. $59 \times 34 = f$

Solve the problems.

18. What is the cost of 24 radios at \$59 each?

19. What is the cost of 14 books at 75¢ each?

Both factors greater than 10

Study these methods for naming the product 26×34.

Equations

$$\begin{aligned} 26 \times 34 &= (20 + 6) \times 34 \\ &= (20 \times 34) + (6 \times 34) \\ &= 680 + 204 \\ &= 884 \end{aligned}$$

Three-step

$$\begin{array}{r} 34 \\ \times\ 26 \\ \hline 204 \\ 680 \\ \hline 884 \end{array}$$

EXERCISES

Solve the equations.

1. $\begin{aligned} 48 \times 23 &= (40 + a) \times 23 \\ &= (40 \times 23) + (a \times 23) \\ &= 920 + b \\ &= c \end{aligned}$

2. $\begin{aligned} 67 \times 54 &= (60 + d) \times 54 \\ &= (60 \times 54) + (d \times 54) \\ &= 3240 + e \\ &= f \end{aligned}$

Use equations to name the products.

3. $41 \times 38 = a$

4. $63 \times 35 = b$

5. $48 \times 72 = c$

6. $28 \times 37 = d$

7. $84 \times 26 = e$

8. $35 \times 96 = f$

9–14. Use the 3-step method to name the products in exercises 3-8.

Use the 3-step method to name these products.

15. $\begin{array}{r} 93 \\ \times\ 28 \\ \hline \end{array}$ **16.** $\begin{array}{r} 75 \\ \times\ 46 \\ \hline \end{array}$ **17.** $\begin{array}{r} 27 \\ \times\ 38 \\ \hline \end{array}$ **18.** $\begin{array}{r} 82 \\ \times\ 76 \\ \hline \end{array}$ **19.** $\begin{array}{r} 49 \\ \times\ 53 \\ \hline \end{array}$ **20.** $\begin{array}{r} 51 \\ \times\ 28 \\ \hline \end{array}$

Complete the number charts.

21.

×	63	28	35	42	68	75
87	*r*	*n*	*c*	*s*	*e*	*u*

22.

×	48	26	35	28	93	47
92	*v*	*a*	*i*	*s*	*n*	*x*

Name the number of hours in

23. 35 days **24.** 47 days **25.** 59 days **26.** 76 days

Study these methods for naming the product 48×239.

Equations	Three-step
$8 \times 239 = 1912$ $40 \times 239 = 9560$ $48 \times 239 = 11472$	239×48 $1912 : 8 \times 239$ $9560 : 40 \times 239$ $11472 : 48 \times 239$

EXERCISES

Use equations to name the products.

1. $59 \times 317 = a$
2. $74 \times 684 = b$
3. $92 \times 337 = c$
4. $48 \times 265 = d$
5. $38 \times 579 = e$
6. $48 \times 681 = f$

7–9. Use the 3-step method to name the products in exercises 1, 2, 3.

Use the 3-step method to name the products.

10. 486×27 **11.** 395×47 **12.** 683×39 **13.** 315×96 **14.** 486×27 **15.** 309×84

Solve these problems.

16. Sylvia spends 175 minutes each day in school. How many minutes will she spend in school in 36 days?

17. A bale of wool weighs 475 pounds. What will be the weight of 56 bales of this size?

18. If each bale of wool weighs 575 pounds, what will be the weight of **a.** 29 bales **b.** 64 bales?

19. If it takes Mr. Clark 25 minutes to get to work each day, how many minutes does he spend getting to work in 242 days?

20. If you sleep 495 minutes every day, how many minutes do you sleep in 62 days?

21. Mr. Wilson travels 64 miles each day. How far does he travel in 365 days?

Working with money

To name the cost of 36 books at \$7.59 each, first estimate the total cost. If the books cost \$7 each, the total cost would be \$252. (36 × 7 = 252) Now name the product, 36 × 759 = 27,324. We see that the total cost must be \$273.24, not \$27,324.

EXERCISES

Name the total amounts.

1. \$3.56 × 24	**3.** \$2.97 × 36	**5.** \$6.83 × 29	**7.** \$4.75 × 32	**9.** \$3.64 × 67	**11.** \$7.86 × 23
2. \$6.89 × 45	**4.** \$4.78 × 57	**6.** \$2.99 × 75	**8.** \$5.43 × 64	**10.** \$5.22 × 67	**12.** \$3.49 × 56

Solve the problems.

13. There are 33 children in Shelly's class. They each spent \$2.45 for a trip to the zoo. How much was spent in all?

14. On the street where Danny lives, 27 families pay towards the cost of a bus shelter. If each family pays \$8.75, how much is that all together?

15. Last year the Franklin School bought 37 new desks for \$16.34 each. How much did the desks cost all together?

***16.** The school also bought 44 new chairs for \$10.82 each. How much did the school spend for desks and chairs?

Name the cost of

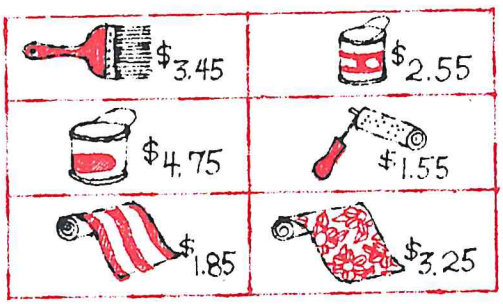
17. 64 brushes at \$3.45 each.

18. 58 cans of paint at \$2.55 each.

19. 83 rolls of wallpaper at \$1.85 each.

20. 23 rolls of wallpaper at \$3.25 each.

21. 55 gallons of paint at \$4.75 a gallon.



Name the products.

1. $\begin{array}{r} 34 \\ \times\ 35 \\ \hline \end{array}$ **2.** $\begin{array}{r} 92 \\ \times\ 28 \\ \hline \end{array}$ **3.** $\begin{array}{r} 49 \\ \times\ 62 \\ \hline \end{array}$ **4.** $\begin{array}{r} 72 \\ \times\ 84 \\ \hline \end{array}$ **5.** $\begin{array}{r} 51 \\ \times\ 37 \\ \hline \end{array}$ **6.** $\begin{array}{r} 65 \\ \times\ 17 \\ \hline \end{array}$ **7.** $\begin{array}{r} 64 \\ \times\ 49 \\ \hline \end{array}$

8. $91 \times 13 = a$

9. $46 \times 52 = b$

10. $83 \times 26 = c$

11. $65 \times 47 = d$

Name the products.

12. $\begin{array}{r} 759 \\ \times\ 82 \\ \hline \end{array}$ **13.** $\begin{array}{r} 658 \\ \times\ 23 \\ \hline \end{array}$ **14.** $\begin{array}{r} 837 \\ \times\ 15 \\ \hline \end{array}$ **15.** $\begin{array}{r} 492 \\ \times\ 43 \\ \hline \end{array}$ **16.** $\begin{array}{r} 307 \\ \times\ 57 \\ \hline \end{array}$ **17.** $\begin{array}{r} 539 \\ \times\ 65 \\ \hline \end{array}$

18. $59 \times 347 = a$

19. $67 \times 562 = b$

20. $49 \times 817 = c$

21. $32 \times 439 = d$

22. $48 \times 864 = e$

23. $25 \times 627 = f$

24. $19 \times 888 = g$

25. $73 \times 582 = h$

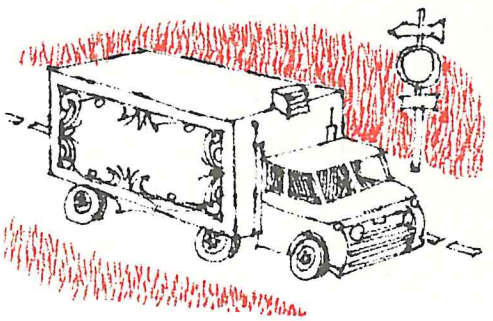
Solve the word problems.

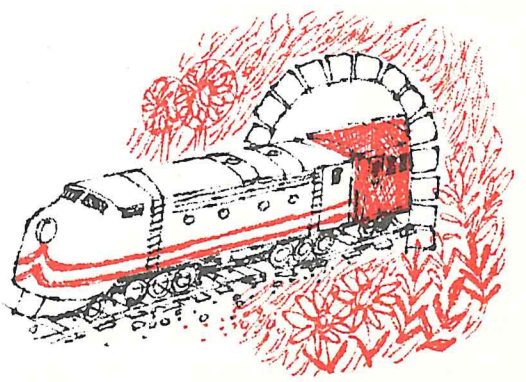
26. There are 437 sacks of peat moss on the truck. If each sack of peat moss weighs 29 pounds, how many pounds of peat moss are on the truck?

27. The large freight truck can hold 842 boxes. If each box weighs 23 pounds, how many pounds can the truck hold?

28. If each car in a passenger train has 118 seats and there are 27 cars, how many people can be seated on the train?

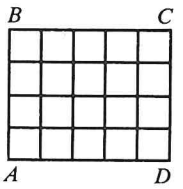
*__29.__ If it costs \$15 a day for a person to stay in a hotel, how much would it cost 5 people to stay for a week?





Naming areas

There are 5 square units per row. There are 4 rows, so there are $4 \times 5 = 20$ square units all together. The **area** of this rectangular region is 20 square units.

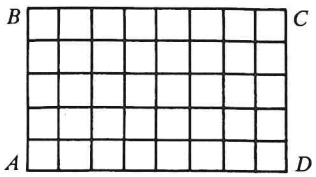


$m(ABCD) = 20$

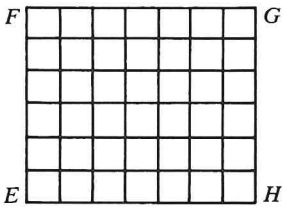
EXERCISES

How many square units are there in each rectangular region?

1.



2.



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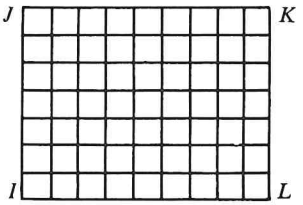
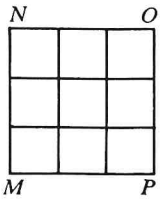


Complete the sentences.

4. For a rectangular region, if l is the number of units of area along the length and w is the number of units of area along the width, then the number of units of ___?___ in the region will be $l \times w$. We can write $A = l \times w$. The area of a rectangular region with sides 5″ and 7″ is $5 \times 7 = 35$ ___?___ inches.

How many square units are there in each square region?

5.



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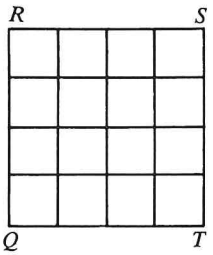

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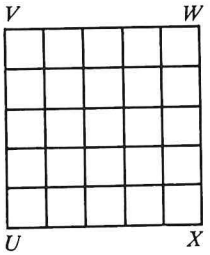


Complete the sentences.

8. For a square region, if s is the number of units of area along each side, the number of units of ___?___ in the region will be $s \times s$ or s^2. The area of a square region with 4″ sides is $4^2 = 16$ ___?___ inches.

Name the area of rectangular regions with sides of

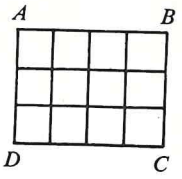
9. 3″ and 8″ **10.** 12′ and 8′ **11.** 11′ and 10′ **12.** 24″ and 9″

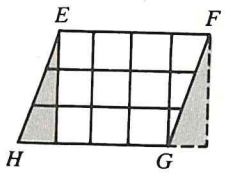
Name the area of square regions with sides of

13. 2′ **14.** 8″ **15.** 9″ **16.** 10″ **17.** 20 yards

Naming areas

The area of rectangular region $ABCD$ is 12 square units. The area of the region formed by the parallelogram $EFGH$ is also 12 square units. One side of the parallelogram is called the **base,** and the perpendicular distance between the base and its opposite side is called the **height.** If b is the number of units along the base and h is the number of units along the height, then the number of units of area is $b \times h$. We write $A = b \times h$.

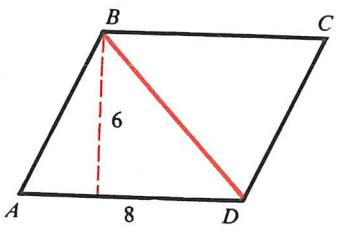




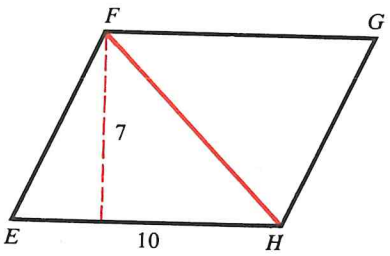
EXERCISES

Name the number of units of area in the regions formed by these parallelograms.

1.



2.



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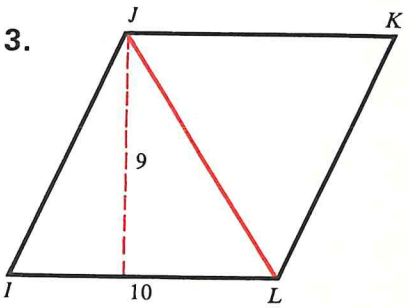
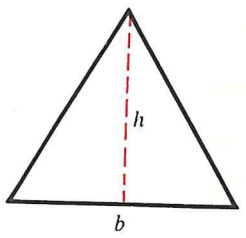


Use the figures above to name the number of units of area in these triangular regions.

4. ABD **5.** BDC **6.** EFH **7.** FGH **8.** IJL **9.** JLK

Complete the sentence.

10. For a triangular region, if b is the number of units of area along the base and h is the number of units of area along the height, then the number of units of ____?____ in the region will be $\frac{1}{2} \times (b \times h)$. We write $A = \frac{1}{2} \times (b \times h)$.



Name the area of the region formed by

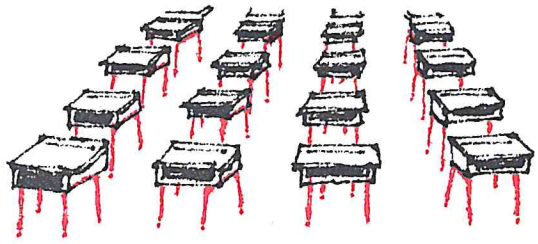
11. a parallelogram with base 12″ and height 6″

12. a triangle with base 12″ and height 10″

13. a parallelogram with base 9″ and height 7″

Matching sets

For each set of 6 cents we may buy 1 candy bar. This is at **the rate of** 6 cents **per** candy bar. So 5 candy bars will cost $5 \times 6 = 30¢$.



There is a set of 4 desks in each row. This is at **the rate of** 4 desks **per** row. So there will be $8 \times 4 = 32$ desks in 8 rows.

EXERCISES

1. Complete this chart of the cost of candy bars.

Cost per bar, in cents	7	8	7	8	9	8	*s*	*x*
Number of bars	6	9	9	12	*v*	*i*	20	8
Total cost, in cents	*a*	*n*	*c*	*r*	90	96	120	160

2. Complete this chart of the number of desks.

Number of desks per row	6	7	8	9	*e*	*n*	20	30
Number of rows	9	12	15	15	7	9	*u*	*c*
Total number of desks	*r*	*i*	*a*	*s*	63	18	180	120

3. Complete this chart of how much may be earned.

Amount earned, in dollars per hour	3	6	8	15	*r*	*v*	4	3
Number of hours	8	40	54	72	20	40	*e*	*x*
Total amount earned, in dollars	*s*	*a*	*n*	*c*	100	320	480	270

4. Complete this chart of the distance that may be traveled.

Speed in miles per hour	8	12	40	45	65	20	9	60
Number of hours	9	3	8	6	4	*n*	*v*	*x*
Total distance, in miles	*e*	*i*	*r*	*s*	*a*	140	360	540

Answer the questions in **A** as quickly as you can. Then see if you can answer the questions in **B** even more quickly.

A

1. The union of a set of 26 apples and a set of 84 apples is a set of ___?___ apples.
2. If 3 apples cost 13 cents, how much would 12 apples cost?
3. Is $1900 + 190 + 19$ greater or less than 2100?
4. Name the area of a rectangular region 16″ long and 8″ wide.
5. Name the difference between 430 and 260.
6. The region formed by a parallelogram has an area of 40 square feet. Name the area of a triangular region with the same base and the same height.
7. $(3 \times 13) + (3 \times 17) = n$
8. $23 \times 6 = 138$, so $230 \times 60 = n$
9. If the product is 800 and one factor is 50, what is the other factor?
10. How many cups match one quart if a cup is half a pint?

B

1. Is 11×90 greater than, less than, or equal to 99×10?
2. Name the area of a triangular region with a base of 20″ and a height of 20″.
3. How many inches in 9 feet? in 3 yards?
4. Name the intersection of
   A = {even whole numbers}
   B = {whole numbers less than 20}.
5. $42 \div 7 = 6$, so $4200 \div 70 = n$
6. Name the sum of the digits in the numeral 3483.
7. Solve the equation:
   $144 = 2 \times 2 \times 2 \times 2 \times 3 \times a$
8. Name the perimeter of a square with sides of 8″.
9. $1234 + 4321 = a$
10. Name the cardinal number of a set of 5 boys and 7 chairs.

Be Careful!

Billy doubles his savings every year. If he has saved nothing when he is 10, how much will he have saved by the time he is 13?

Division with remainders

To name the number of subsets of 4 in a set of 13 we think of the products with 4 as one factor.

Products with 4 as one factor $= \{0, 4, 8, \mathbf{12}, 16, \ldots\}$

$$13 = \mathbf{12} + 1$$
$$= (3 \times 4) + 1$$

There are **3** sets of 4 in a set of 13, with 1 remaining.

$$13 - (3 \times 4) = 1$$

EXERCISES

Complete the sentences.

1. There are a sets of 5 in a set of 33 with b remaining.
2. There are c sets of 7 in a set of 45 with d remaining.
3. There are e sets of 4 in a set of 31 with f remaining.
4. There are g sets of 9 in a set of 68 with h remaining.
5. There are i sets of 6 in a set of 53 with j remaining.
6. There are k sets of 8 in a set of 59 with l remaining.
7. There are m sets of 3 in a set of 26 with n remaining.
8. There are p sets of 7 in a set of 51 with q remaining.
9. There are r sets of 9 in a set of 55 with s remaining.
10. There are t sets of 6 in a set of 39 with u remaining.
11. There are v sets of 8 in a set of 50 with w remaining.

Solve the equations.

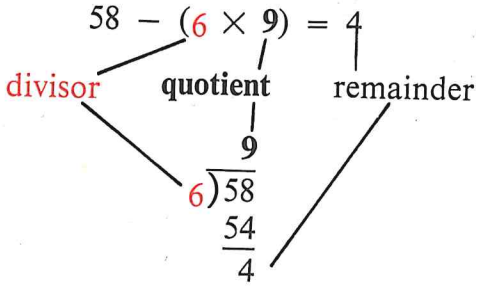
12. $17 = (2 \times 8) + a$
13. $28 = (3 \times 9) + b$
14. $59 = (6 \times 9) + c$
15. $50 = (8 \times 6) + d$
16. $65 = (9 \times 7) + e$
17. $84 = (9 \times 9) + f$
18. $77 = (8 \times g) + 5$
19. $57 = (6 \times h) + 3$
20. $49 = (5 \times i) + 4$
21. $38 = (7 \times j) + 3$
22. $85 = (9 \times k) + 4$
23. $33 = (4 \times l) + 1$

Solve the equations.

24. $57 - (9 \times 6) = a$
25. $53 - (7 \times 7) = b$
26. $38 - (6 \times 6) = c$
27. $66 - (7 \times 9) = d$
28. $29 - (4 \times 7) = e$
29. $45 - (6 \times 7) = f$
30. $59 - (8 \times 7) = g$
31. $36 - (9 \times 4) = h$
32. $69 - (8 \times 8) = i$
33. $25 - (3 \times 7) = j$
34. $20 - (6 \times 3) = k$
35. $32 - (5 \times 6) = l$

Division with remainders

Study the equation and the working form.



When 58 is divided by 6,
the **divisor** is 6,
the **quotient** is 9,
the **remainder** is 4.

The remainder should always be **less** than the divisor. $4 < 6$

EXERCISES

Name the quotients and remainders.

1. $2\overline{)17}$
2. $4\overline{)21}$
3. $7\overline{)31}$
4. $6\overline{)42}$
5. $8\overline{)19}$
6. $3\overline{)25}$
7. $9\overline{)60}$
8. $8\overline{)60}$
9. $7\overline{)60}$
10. $8\overline{)72}$
11. $5\overline{)38}$
12. $9\overline{)87}$

Study the equations, then write the divisions in working form.

13. $26 - (3 \times 8) = 2$
14. $33 - (6 \times 5) = 3$
15. $51 - (7 \times 7) = 2$
16. $48 - (9 \times 5) = 3$
17. $53 - (6 \times 8) = 5$
18. $88 - (9 \times 9) = 7$

Name the quotient and remainder, then write an equation for each of these working forms.

19. $3\overline{)19}$
20. $4\overline{)31}$
21. $8\overline{)38}$
22. $7\overline{)25}$
23. $9\overline{)50}$
24. $6\overline{)59}$
25. $5\overline{)32}$
26. $9\overline{)63}$

Solve the problems.

27. At a garden party there are 46 children. If 5 players are needed for each team for a game, how many teams can be formed? How many children will be left out?
28. Joan says, "When I divide 66 by 7 the quotient is 8 and the remainder is 10." Jean says, "You can subtract another 7. The quotient is 9 and the remainder is 3." Was Jean right?

Division with remainders at first

To name the number of sets of 4 in a set of 56, you may think:

$$\begin{aligned} 56 - (10 \times 4) &= 56 - 40 \\ &= 16 \end{aligned}$$

There are 10 sets of 4 and 16 remaining. But a set of 16 forms 4 sets of 4, so

$$\begin{aligned} 56 &= (10 \times 4) + (4 \times 4) \\ &= 14 \times 4 \end{aligned}$$

There are 14 sets of 4 in a set of 56: $56 - (14 \times 4) = 0$.

EXERCISES

Solve the equations.

1. $52 - (10 \times 3) = 52 - a$
   $= b$
2. $64 - (10 \times 4) = 64 - c$
   $= d$
3. $91 - (10 \times 7) = 91 - e$
   $= f$
4. $88 - (10 \times 6) = 88 - g$
   $= h$
5. $65 - (10 \times 5) = 65 - i$
   $= j$
6. $96 - (10 \times 8) = 96 - k$
   $= l$
7. $56 - (10 \times 5) = 56 - m$
   $= n$
8. $73 - (10 \times 6) = 73 - p$
   $= q$

Solve the equations.

9. $48 = (10 \times 4) + (a \times 4)$
10. $96 = (10 \times 8) + (b \times 8)$
11. $84 = (10 \times 7) + (c \times 7)$
12. $52 = (10 \times 4) + (d \times 4)$
13. $34 = (10 \times e) + (7 \times 2)$
14. $84 = (10 \times f) + (4 \times 6)$
15. $104 = (10 \times 8) + (3 \times g)$
16. $68 = (10 \times 4) + (7 \times h)$
17. $36 = (10 \times 3) + (i \times 3)$
18. $72 = (10 \times 6) + (j \times 6)$
19. $65 = (10 \times 5) + (k \times 5)$
20. $51 = (10 \times 3) + (l \times 3)$
21. $99 = (10 \times m) + (1 \times 9)$
22. $91 = (10 \times n) + (3 \times 7)$
23. $75 = (10 \times 5) + (5 \times p)$
24. $48 = (10 \times 3) + (3 \times q)$
25. $39 = (10 \times 3) + (r \times 3)$
26. $84 = (10 \times 7) + (2 \times s)$
27. $117 = (10 \times t) + (3 \times 9)$
28. $210 = (10 \times 10) + (10 \times u)$

Study these methods for naming the quotient $56 \div 4$.

Equations	Working form
$56 \div 4 = (40 + 16) \div 4$ $= (40 \div 4) + (16 \div 4)$ $= 10 + 4$ $= 14$	$4\overline{)56}$ $40 \mid 10$ 16 $16 \mid 4$ $0 \mid 14$ **Check** $10 \times 4 = 40$ $4 \times 4 = 16$ $14 \times 4 = 56$

EXERCISES

Copy and complete the equations.

1. $42 \div 3 = (30 + 12) \div 3$
$= (a \div 3) + (b \div 3)$
$= c + d$
$= e$

2. $68 \div 4 = (40 + 28) \div 4$
$= (f \div 4) + (g \div 4)$
$= h + i$
$= j$

Use equations to name the quotients.

3. $36 \div 2 = a$
4. $48 \div 3 = b$
5. $51 \div 3 = c$
6. $98 \div 7 = d$
7. $64 \div 4 = e$
8. $96 \div 8 = f$
9. $104 \div 8 = g$
10. $126 \div 9 = h$

Use the working form to name the quotients and check your answers.

11. $4\overline{)52}$
12. $6\overline{)102}$
13. $8\overline{)112}$
14. $3\overline{)57}$
15. $7\overline{)126}$
16. $5\overline{)85}$
17. $9\overline{)153}$
18. $7\overline{)112}$
19. $6\overline{)192}$
20. $4\overline{)180}$
21. $8\overline{)144}$
22. $5\overline{)165}$

Name the quotients.

23. $64 \div 2 = a$
24. $72 \div 2 = b$
25. $118 \div 2 = c$
26. $232 \div 4 = d$
27. $145 \div 5 = e$
28. $175 \div 7 = f$
29. $184 \div 4 = g$
30. $351 \div 9 = h$
31. $424 \div 8 = i$

Use the working form to answer the questions.

32. How many \$8 skirts can be bought for \$112?
33. What is the cost of one skirt if 7 skirts cost \$161, and each of the skirts costs the same?

Division

Study these methods for naming the quotient $56 \div 4$.

Expanded numerals	Shorter working form
$4\overline{)56} = 4\overline{)40 + 16}^{\,10 + 4}$	$4\overline{)56}^{\,14}$ 40 16 16

EXERCISES

Copy and complete the divisions.

1. $6\overline{)60 + 12}^{\,a + b}$ **2.** $5\overline{)50 + 15}^{\,c + d}$ **3.** $8\overline{)80 + 16}^{\,e + f}$ **4.** $9\overline{)90 + 18}^{\,g + h}$

Use expanded numerals to name the quotients.

5. $5\overline{)75}$ **7.** $4\overline{)76}$ **9.** $5\overline{)65}$ **11.** $8\overline{)112}$ **13.** $8\overline{)120}$ **15.** $6\overline{)102}$

6. $7\overline{)119}$ **8.** $6\overline{)84}$ **10.** $9\overline{)108}$ **12.** $3\overline{)51}$ **14.** $9\overline{)117}$ **16.** $4\overline{)76}$

Copy and complete the divisions.

17. $4\overline{)76}^{\,1a}$ (40, 36, 36) **18.** $3\overline{)54}^{\,1b}$ (30, 24, 24) **19.** $5\overline{)65}^{\,1c}$ (50, 15, 15) **20.** $7\overline{)105}^{\,1d}$ (70, 35, 35) **21.** $6\overline{)96}^{\,1e}$ (60, 36, 36) **22.** $9\overline{)126}^{\,1f}$ (90, 36, 36)

Use the shorter working form to name the quotients.

23. $3\overline{)42}$ **25.** $2\overline{)38}$ **27.** $7\overline{)126}$ **29.** $9\overline{)135}$ **31.** $4\overline{)72}$ **33.** $5\overline{)85}$

24. $5\overline{)80}$ **26.** $9\overline{)153}$ **28.** $8\overline{)128}$ **30.** $6\overline{)96}$ **32.** $8\overline{)112}$ **34.** $6\overline{)114}$

Use the shorter working form to answer the questions.

35. How many teams of 9 can be formed from 135 children?

36. How many will there be in each team if 128 children are arranged into 8 teams of the same size?

Study the working forms and the equations.

A	B	Check
$4\overline{)58}$ $40 \mid 10$ 18 $16 \mid 4$ $2 \mid 14$	14 $4\overline{)58}$ 40 18 16 2	$10 \times 4 = 40$ $4 \times 4 = 16$ $14 \times 4 = 56$ $58 - 56 = 2$

When 58 is divided by 4, the quotient is 14 and the remainder is 2. The final remainder should always be less than the divisor. $2 < 4$.

EXERCISES

Copy and complete the divisions.

1. $6\overline{)74}$
$60 \mid 10$
$14 \mid$

2. $7\overline{)86}$
$70 \mid 10$
$16 \mid$

3. $8\overline{)98}$
$80 \mid 10$
$18 \mid$

4. $8\overline{)135}$
$80 \mid 10$
$55 \mid$

5. $9\overline{)140}$
$90 \mid 10$
$50 \mid$

Copy and complete the divisions and the checks.

6.
$$\begin{array}{r} 14 \\ 8\overline{)115} \\ 80 \\ \hline 35 \\ 32 \\ \hline a \end{array}$$

$10 \times 8 = 80$
$4 \times 8 = 32$
$b \times 8 = c$
$115 - c = d$

The quotient is ____?____.
The remainder is ____?____.

7.
$$\begin{array}{r} 14 \\ 6\overline{)87} \\ 60 \\ \hline 27 \\ 24 \\ \hline e \end{array}$$

$10 \times 6 = 60$
$4 \times 6 = 24$
$f \times 6 = g$
$87 - g = h$

The quotient is ____?____.
The remainder is ____?____.

Use method B to name the quotients and remainders.

8. $4\overline{)59}$
9. $6\overline{)105}$
10. $7\overline{)120}$
11. $4\overline{)75}$
12. $9\overline{)165}$
13. $8\overline{)123}$
14. $5\overline{)88}$
15. $7\overline{)95}$
16. $6\overline{)113}$
17. $8\overline{)101}$

Division

Study these methods for dividing 673 by 4.

A	B	Check
$4\overline{)673}$ $400 \mid 100$ 273 $240 \mid 60$ 33 $32 \mid 8$ $1 \mid 168$	168 $4\overline{)673}$ 400 273 240 33 32 1	$100 \times 4 = 400$ $60 \times 4 = 240$ $8 \times 4 = 32$ $168 \times 4 = 672$ $673 - 672 = 1$

EXERCISES

Name the quotients and remainders.

1. $3\overline{)524}$

300	*a*
224	
210	*b*
14	
12	*c*
d	*e*

2. $5\overline{)784}$

500	*f*
284	
250	*g*
34	
30	*h*
i	*j*

3. $6\overline{)892}$

600	*k*
292	
240	*l*
52	
48	*m*
n	*p*

4. $4\overline{)739}$

400	*q*
339	
320	*r*
19	
16	*s*
t	*u*

Complete the divisions.

5. $2\overline{)318}$

a	100
118	
b	50
18	
18	*c*
	d

6. $7\overline{)968}$

e	100
268	
210	*f*
58	
g	8
2	*h*

7. $4\overline{)647}$

i	100
247	
240	*j*
7	
k	1
3	*l*

8. $5\overline{)776}$

m	100
276	
250	*n*
26	
p	5
1	*q*

Use method B to complete the divisions.

9. $4\overline{)573}$

10. $6\overline{)892}$

11. $2\overline{)392}$

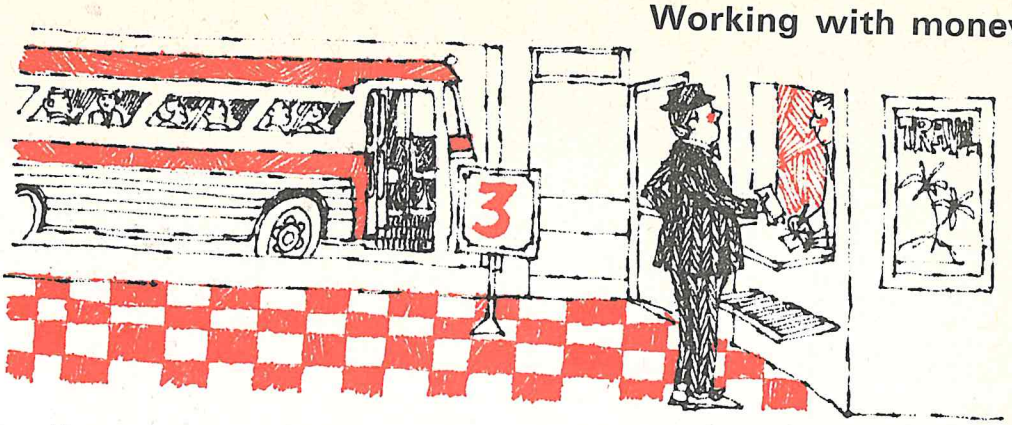
12. $5\overline{)741}$

13. $7\overline{)983}$

14. $3\overline{)482}$

15. $6\overline{)823}$

16. $7\overline{)873}$



To name the cost per passenger if the total cost, \$53.76, is to be shared evenly among 7 passengers, we first estimate like this:

\$53 is between \$49 and \$56.
We choose \$49 because $7 \times 7 = 49$.
We choose \$56 because $7 \times 8 = 56$.
The cost will be between \$7 and \$8.
The cost per passenger is \$7.68.

To name the quotient $5376 \div 7$

$$\begin{array}{r|l} 7)5376 & \\ 4900 & 700 \\ \hline 476 & \\ 420 & 60 \\ \hline 56 & \\ 56 & 8 \\ \hline 0 & 768 \end{array}$$

EXERCISES

Estimate the quotients.

1. \$38.52 to be shared evenly among 4 people.
2. \$65.25 to be shared evenly among 9 people.
3. \$72.64 to be shared evenly among 8 people.
4. \$49.74 to be shared evenly among 6 people.

Name the quotients and remainders.

5. $3854 \div 4$
6. $6523 \div 9$
7. $7214 \div 8$
8. $4972 \div 6$
9. $5785 \div 6$
10. $2961 \div 4$
11. $2675 \div 3$
12. $3728 \div 5$
13. $6529 \div 7$
14. $4682 \div 5$
15. $5947 \div 8$
16. $4573 \div 7$

Solve the problems.

17. If the total cost of \$47.60 is to be shared evenly among 5 people, what is the cost per person?
18. If the total cost of \$68.24 is to be shared evenly among 8 people, what is the cost per person?

Division

To name the number of sets of 23 that may be formed from a set of 575, we may arrange our work like this.

	Check
$23\overline{)575}$ 460 | 20 115 115 | **5** 0 | 25	$20 \times 23 = 460$ $\mathbf{5} \times 23 = 115$ $25 \times 23 = 575$

EXERCISES

Copy and complete.

1. $21\overline{)672}$

630	a
42	
42	b
0	c

$30 \times 21 = d$

$2 \times 21 = e$

$32 \times 21 = f$

2. $53\overline{)742}$

530	g
212	
212	h
0	i

$10 \times 53 = j$

$4 \times 53 = k$

$14 \times 53 = l$

3. $32\overline{)576}$

320	m
256	
256	n
0	a

$10 \times 32 = p$

$8 \times 32 = q$

$18 \times 32 = r$

4. $27\overline{)945}$

810	s
135	
135	t
0	u

$30 \times 27 = v$

$5 \times 27 = w$

$35 \times 27 = x$

Copy and complete the divisions.

5. $24\overline{)592}$

480	a
112	
96	b
c	d

6. $35\overline{)896}$

700	e
196	
175	f
g	h

7. $43\overline{)995}$

860	i
135	
129	j
k	l

8. $67\overline{)887}$

670	m
217	
201	n
p	q

9. $26\overline{)756}$

a	20
236	
b	9
c	d

10. $34\overline{)942}$

e	20
262	
f	7
g	h

11. $53\overline{)875}$

i	10
345	
j	6
k	l

12. $43\overline{)976}$

m	20
116	
n	2
p	q

To name the quotient $575 \div 23$, we may arrange our work like this.

$$\begin{array}{r} 25 \\ 23\overline{)575} \\ 460 \\ \hline 115 \\ 115 \\ \hline 0 \end{array}$$

Step 1: $20 \times 23 = 460$
Step 2: $575 - 460 = 115$
Step 3: $5 \times 23 = 115$
Step 4: $115 - 115 = 0$

EXERCISES

Complete the divisions.

1. $31\overline{)654}$	**5.** $67\overline{)864}$	**9.** $27\overline{)521}$	**13.** $22\overline{)286}$
2. $23\overline{)486}$	**6.** $38\overline{)845}$	**10.** $33\overline{)682}$	**14.** $35\overline{)467}$
3. $34\overline{)763}$	**7.** $46\overline{)606}$	**11.** $36\overline{)550}$	**15.** $45\overline{)498}$
4. $35\overline{)824}$	**8.** $51\overline{)641}$	**12.** $47\overline{)715}$	**16.** $11\overline{)243}$

Complete the divisions.

17. $32\overline{)146}$	**21.** $24\overline{)136}$	**25.** $16\overline{)98}$	**29.** $27\overline{)248}$
18. $46\overline{)186}$	**22.** $36\overline{)218}$	**26.** $18\overline{)124}$	**30.** $35\overline{)229}$
19. $29\overline{)276}$	**23.** $45\overline{)378}$	**27.** $37\overline{)241}$	**31.** $24\overline{)166}$
20. $64\overline{)256}$	**24.** $26\overline{)179}$	**28.** $42\overline{)209}$	**32.** $28\overline{)368}$

Solve the problems.

33. How many 29¢ pizzas can be bought for \$4.24? How much change will there be?

34. How many 49¢ balls can be bought for \$6.50? How much change will there be?

35. How many 59¢ notebooks can you buy with \$5.00? How much change will you receive?

***36.** If a package of 6 candy bars costs 27¢, how many packages can you buy with \$1.00? How much change will you receive?

Estimating the quotient

To estimate the quotient 4478 ÷ 62 we can use the following method.

Step 1: 62 × **10** = **620**

Step 2: 62 × 100 = 6200

Step 3: 4478 is between **620** and 6200. So the quotient must be between **10** and 100.

Step 4: 44 ÷ 6 is about 7.

Step 5: Estimate is 7 tens or 70.

EXERCISES

Complete the sentences.

1. 3982 is between 57 × 10 and 57 × 100 so the quotient 3982 ÷ 57 must be between ***a*** and ***b***.
2. 6743 is between 72 × 10 and 72 × 100 so the quotient 6743 ÷ 72 must be between ***c*** and ***d***.

Complete these steps to estimate the quotients.

3. 3683 ÷ 49

Step 1: 49 × 10 = ***a***

Step 2: 49 × 100 = ***b***

Step 3: 3683 is between 490 and 4900 so the quotient must be between ***c*** and ***d***.

Step 4: 36 ÷ 5 is about ***e***.

Step 5: Estimate is ***f*** or ***g***.

4. 5146 ÷ 73

Step 1: 73 × ***h*** = 730

Step 2: 73 × ***i*** = 7300

Step 3: 5146 is between ***j*** and ***k*** so the quotient must be between 10 and 100.

Step 4: 51 ÷ 7 is about ***l***.

Step 5: Estimate is ***m*** or ***n***.

Use the method in exercises 3 and 4 to estimate these quotients.

5. $51\overline{)4692}$

6. $28\overline{)2576}$

7. $49\overline{)4993}$

8. $31\overline{)2613}$

9. $57\overline{)3590}$

10. $42\overline{)6723}$

11. 6387 ÷ 81

12. 6402 ÷ 92

13. 3716 ÷ 69

14. 5378 ÷ 78

15. 9024 ÷ 23

16. 7123 ÷ 68

Estimating the quotient

To estimate the quotient 4478 ÷ 62 we can use the following method.

Step 1: Hundreds ÷ **tens** is **tens.**
Step 2: 62 is about **6 tens.**
Step 3: Think of 4478 as 44 hundreds.
Step 4: Since 42 ÷ 6 = 7, 44 hundreds ÷ **6 tens** is about **7 tens.**
Step 5: Estimate is 7 tens or 70. **60 × 70** = 4200

EXERCISES

Complete the sentences.

1. 12 hundreds ÷ 3 tens is ***a*** tens.
2. 24 hundreds ÷ 4 tens is ***b*** tens.
3. 48 hundreds ÷ 8 tens is ***c*** tens.
4. 36 hundreds ÷ 9 tens is ***d*** tens.

Complete these steps to estimate the quotients.

5. 4693 ÷ 53

Step 1: Hundreds ÷ tens is ***a***.
Step 2: 53 is about ***b*** tens.
Step 3: 4693 is about ***c*** hundreds.
Step 4: Since 45 ÷ 5 = 9, 46 hundreds ÷ 5 tens is about ***d*** tens.
Step 5: Estimate is ***e*** tens or ***f***.

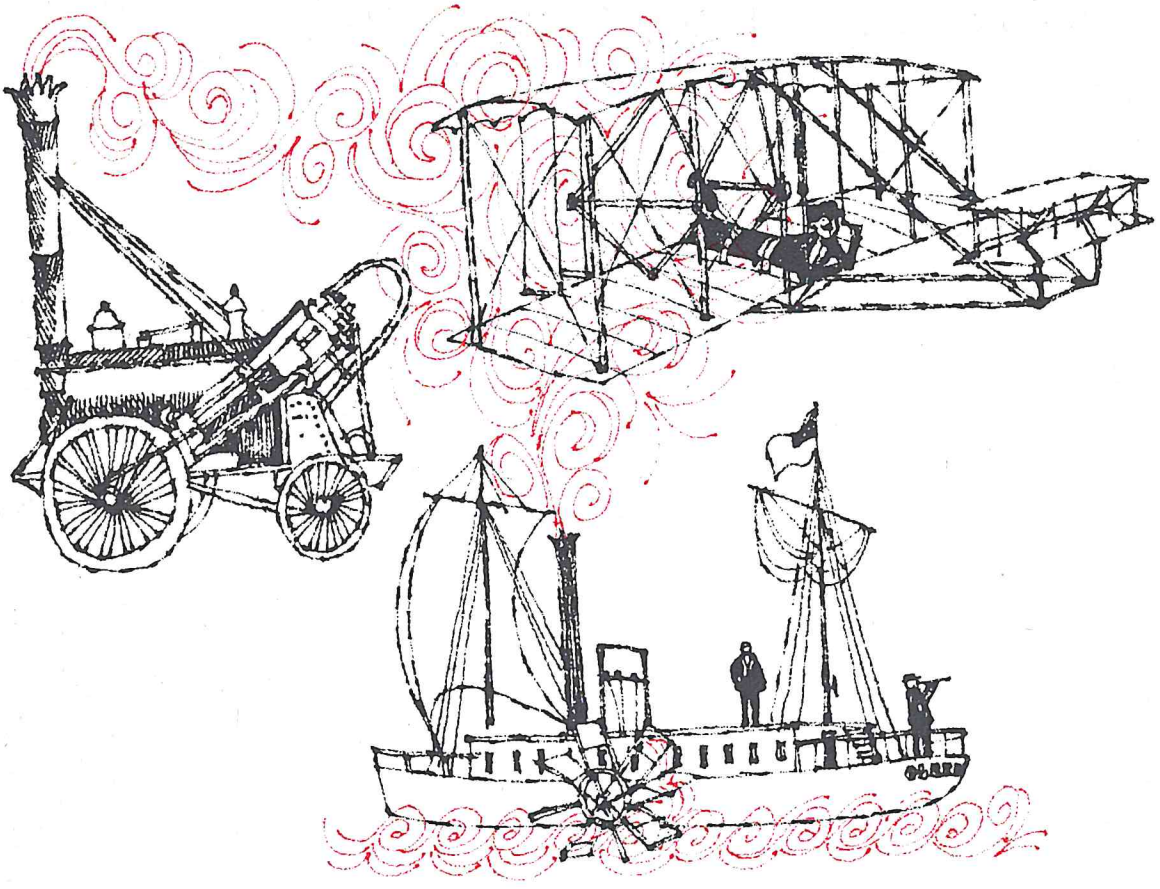
6. 2976 ÷ 38

Step 1: Hundreds ÷ tens is ***g***.
Step 2: 38 is about ***h*** tens.
Step 3: 2976 is about ***i*** hundreds.
Step 4: Since 28 ÷ 4 = 7, 29 hundreds ÷ 4 tens is about ***j*** tens.
Step 5: Estimate is ***k*** tens or ***l***.

Use the method in exercises 5 and 6 to estimate the quotients.

7. $23\overline{)1479}$
8. $32\overline{)2467}$
9. $46\overline{)3598}$
10. $53\overline{)4170}$
11. $89\overline{)6239}$
12. $71\overline{)8046}$
13. 3578 ÷ 61
14. 3029 ÷ 42
15. 2865 ÷ 29
16. 5746 ÷ 83
17. 6891 ÷ 51
18. 5329 ÷ 72

Famous inventions



1. In 1903, Orville Wright made the first successful flight in an airplane. He flew at an air speed of about 31 miles per hour at a height of about 10 feet.
2. George Stephenson built the first steam locomotive in 1825. It could pull about 50 tons and traveled at the rate of 15 miles per hour.
3. Robert Fulton's steamboat the *Clermont* made its first successful trip, from New York to Albany, in 1807.
4. J. L. Baird, a British inventor, made the first color television set in 1944. The first TV show in color was broadcast on June 25, 1951.
5. Eli Whitney invented the cotton gin in 1793.
6. Alexander Graham Bell invented the telephone in 1876.

Study the sets, then solve the equations.

$R = \{x, t, m, y, s\}$ $T = \{0, 2, 4, 6\}$

1. $n(R) = a$ **2.** $n(T) = b$ **3.** $n(R \otimes T) = c$

4. List the set of pairs for $R \otimes T$.

5. How many pairings can be formed between 3 fish and 6 fish-hooks?

Complete the sentences.

6. If 1 is a factor, then the product and the other ____?____ are the same.

7. The order of the factors does not affect the ____?____.

8. The way in which the factors are grouped does not affect the ____?____.

Name the other members of the family of facts for each equation.

9. $9 \times 7 = 63$ **11.** $54 \div 9 = 6$ **13.** $5 \times 8 = 40$ **15.** $21 \div 7 = 3$

10. $6 \times 8 = 48$ **12.** $56 \div 7 = 8$ **14.** $7 \times 7 = 49$ **16.** $36 \div 9 = 4$

Name the products.

17. 18×9 **18.** 54×6 **19.** 37×8 **20.** 86×5 **21.** 831×7 **22.** 694×5

23. 4376×3 **24.** 4852×9 **25.** 3792×9 **26.** 59×27 **27.** 38×43 **28.** 67×82

29. $15 \times 42 = a$ **30.** $97 \times 36 = b$ **31.** $683 \times 29 = c$

32. $903 \times 17 = d$ **33.** $65 \times 428 = e$ **34.** $309 \times 84 = f$

Name the quotients and remainders.

35. $6\overline{)102}$ **36.** $8\overline{)144}$ **37.** $6\overline{)892}$ **38.** $8\overline{)7214}$ **39.** $23\overline{)575}$

40. $9\overline{)20345}$ **41.** $16\overline{)495}$ **42.** $29\overline{)666}$ **43.** $51\overline{)7821}$ **44.** $42\overline{)9876}$

45. $145 \div 5 = a$ **46.** $128 \div 8 = b$ **47.** $983 \div 7 = c$

48. $7625 \div 9 = d$ **49.** $8012 \div 6 = e$ **50.** $4573 \div 7 = f$

LOOKING BACK

> Multiplication is the renaming of two factors as a product.
> Division is the renaming of a product and a factor as a factor.

Complete the chart.

1.

factor	3	8	4	6	7	3	6	*a*	4	*i*	5	*n*
factor	9	7	9	6	5	8	*u*	5	*s*	8	*x*	7
product	*a*	*c*	*i*	*n*	*e*	*s*	54	25	28	64	30	63

There are 36 basic multiplication-division facts to learn. Their factors are greater than 1 and less than 10. There are 5 facts with products 10 or less. Three of them are: $2 \times 2 = 4$, $2 \times 3 = 6$, and $2 \times 4 = 8$.

2. Name the other facts with products 10 or less.

3. Name the 8 facts with products between 10 and 20.

4. Name the 6 facts with products between 20 and 30.

> Multiplication is commutative. This means that the order of the factors does not affect the product.
>
> Multiplication is associative. This means that the way factors are grouped does not affect the product.
>
> The distributive property means, for example, that to name the product 7×54 we multiply (7×50) and (7×4) and add the products. To name the sum $35 + 56$ we may add $35 + 56 = 91$, or we may multiply $7 \times (5 + 8) = 7 \times 13 = 91$.

Use the properties of multiplication to name the products.

5. 3×19 **6.** 3×29 **7.** 3×849 **8.** 23×849

> Some divisions have remainders. For instance, $55 = (9 \times 6) + 1$, so when 55 is divided by 6 the remainder is 1.

Name the quotients and remainders.

9. $6\overline{)46}$ **10.** $23\overline{)72}$ **11.** $36\overline{)481}$ **12.** $52\overline{)681}$

Study this method for naming a sum or product.

$$\begin{array}{r} 2256 \\ 6526 \\ 92 \\ +\quad 8 \\ \hline 8 \\ 7 \\ 16 \\ 22 \\ \hline 8882 \end{array}$$

$$\begin{array}{r} 8256 \\ \times\ 7 \\ \hline 56 \\ 14 \\ 35 \\ 42 \\ \hline 57792 \end{array}$$

Use the method above to name the sums or products.

1. $\begin{array}{r} 8437 \\ 1256 \\ 839 \\ +\ 4156 \\ \hline \end{array}$

2. $\begin{array}{r} 3879 \\ 2594 \\ 4086 \\ +\ 5723 \\ \hline \end{array}$

3. $\begin{array}{r} 6217 \\ 384 \\ 9216 \\ +\ 1375 \\ \hline \end{array}$

4. $\begin{array}{r} 41 \\ 386 \\ 2971 \\ +\ 4865 \\ \hline \end{array}$

5. $\begin{array}{r} 3297 \\ \times\ 6 \\ \hline \end{array}$

6. $\begin{array}{r} 4836 \\ \times\ 5 \\ \hline \end{array}$

7. $\begin{array}{r} 6724 \\ \times\ 4 \\ \hline \end{array}$

8. $\begin{array}{r} 4825 \\ \times\ 9 \\ \hline \end{array}$

To name the product 46×389, the work could be set up like this. The first row shows the product 40×389; the second row shows the product 6×389; and the third row shows the sums along the diagonals. This is called the **lattice method.**

		3	8	9	×
		1 2	3 2	3 6	4
		1 8	4 8	5 4	6
1	7	8	9	4	

Use the lattice method to name products like those on page 113.

Multiplication patterns

9. What is your favorite number less than 10? Multiply 12,345,679 by this number. Then multiply the product by 9. What do you notice?

10. Multiply 142,857 by any number less than 7. What do you notice about the product? What do you notice when you multiply 142,857 by 7?

A visit to New York

Gina is planning a trip to New York City. She lists the days she will be in the city and the sights she wants to see. Her lists look like this.

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday

Empire State Building
Statue of Liberty
United Nations
Bronx Zoo
Rockefeller Center
Central Park

EXERCISES

To show that the Empire State Building is paired with Monday Gina could write (M, E). Her list would be written as (M, E), (T, SL), (W, UN), (Th, BZ), (Fr, RC), (Sat, CP).

1. How many different pairs could Gina form with Monday as the day?
2. Write the pairs for Monday, beginning with (M, E).
3. Write the possible pairs with Tuesday as the day.
4. Write the possible pairs with Wednesday as the day.

How many pairs would there be

5. for Thursday?
6. for Saturday?
7. all together?

How many pairs would there be all together if Gina decided to stay

8. one more day to see the Planetarium?
9. two more days to see the Planetarium and the Museum?

Cross products

We may pair the objects in set A with the objects in set B and form the set of ordered pairs, C. C is called the **cross product** of A and B, written $A \otimes B$. The number of pairs in C is found by multiplying the cardinal numbers of sets A and B.

$A = \{\bullet\ \blacktriangle\}$
$B = \{1, 2, 3\}$
$C = \{(\bullet, 1)(\bullet, 2)(\bullet, 3)(\blacktriangle, 1)$
$(\blacktriangle, 2)(\blacktriangle, 3)\}$
$C = A \otimes B.$
$n(A) = 2$
$n(B) = 3$
$n(A \otimes B) = 2 \times 3 = 6$

EXERCISES

Write cross products for the sets.

$Q = \{t, w\}$ $R = \{\bullet\ \blacktriangle\ C\}$ $S = \{\blacklozenge\ \blacksquare\ \bigstar\}$ $M = \{0, 2, 4, 6, 8\}$

1. $Q \otimes M$ **2.** $R \otimes S$ **3.** $Q \otimes R$ **4.** $M \otimes Q$

Name the cardinal numbers.

5. $n(Q)$ **7.** $n(S)$ **9.** $n(Q \otimes R)$ **11.** $n(R \otimes S)$
6. $n(R)$ **8.** $n(M)$ **10.** $n(Q \otimes M)$ **12.** $n(M \otimes R)$

Write *T* or *F* to show whether these statements are true or false.

13. $n(Q) \times n(R) = n(Q \otimes R)$ **15.** $n(Q) \times n(M) = n(Q \otimes M)$
14. $n(R) \times n(S) = n(R \otimes S)$ **16.** $n(Q) \times n(M) = n(M \otimes Q)$

Complete the chart.

	$n(A)$	$n(B)$	$n(A \otimes B)$
17.	3	5	*a*
18.	6	7	*b*
19.	8	9	*c*
20.	7	*d*	35
21.	*e*	3	27

Solve the problem.

22. If 5 children enter a room with 10 chairs, in how many ways can the children be seated?

Different kinds of pairing

If we pair each member of L with each member of M we get $L \otimes M$.

L = {● ■ ▲}
M = {● ★ ▲}

$L \otimes M$ = {(●, ●), (●, ★), (●, ▲), (■, ●), (■, ★), (■, ▲), (▲, ●), (▲, ★), (▲, ▲)}

If we pair members of the same color, we get N.

N = {(●, ●), (●, ★), (●, ▲)}

If we pair members of the same shape, we get P.

P = {(●, ●), (▲, ▲)}

N and P are **subsets** of $L \otimes M$

EXERCISES

For G = {Tom, Dick, Harry} and H = {Diane, Tina, Sue}, list the sets.

1. $G \otimes H$
2. pairs from $G \otimes H$ of names that begin with the same letter
3. pairs from $G \otimes H$ of names with the same number of letters

For D = {1, 2, 3, 4, 5,} and E = {6, 7, 8}, list the sets.

4. $D \otimes E$
5. pairs from $D \otimes E$ in which both numbers are even
6. pairs from $D \otimes E$ in which both numbers are odd
7. pairs from $D \otimes E$ in which the second number is twice the first
8. pairs from $D \otimes E$ in which the sum of the two numbers is 10
9. pairs from $D \otimes E$ in which the second number is 4 more than the first
10. pairs from $D \otimes E$ in which the second number is 5 more than the first

If we pair every number in A with every number in B, we get:	$A = \{3, 4, 5\}$ $B = \{0, 3, 6\}$
$A \otimes B = \{(3, 0), (3, 3), (3, 6), (4, 0), (4, 3), (4, 6), (5, 0), (5, 3), (5, 6)\}$	
If we pair every number in A with every number in B which is less, we get:	$P = \{(3, 0), (4, 0), (5, 0), (4, 3), (5, 3)\}$
Set P does not include:	$S = \{(3, 3), (3, 6), (4, 6), (5, 6\}$

P and S are **subsets** of $A \otimes B$

EXERCISES

Name the cardinal numbers of the sets.

1. A 2. B 3. $A \otimes B$ 4. P 5. S

For $L = \{6, 8, 12, 14, 15\}$ and $M = \{3, 9, 10, 20\}$, list the sets.

6. $L \otimes M$
7. pairs from $L \otimes M$ in which the second number is greater than the first
8. pairs from $L \otimes M$ in which both numbers are even
9. pairs from $L \otimes M$ in which both numbers are odd

For $T = \{a, r, m, s\}$ and $V = \{l, u, n, g\}$ list the sets.

10. $T \otimes V$
11. pairs from $T \otimes V$ in which the second letter comes later in the alphabet than the first
12. pairs from $T \otimes V$ in which the second letter comes earlier in the alphabet than the first

Look at the sets $Y \otimes Z = \{(a, 2)\ (a, 3)\ (b, 2)\ (b, 3)\ (c, 2)\ (c, 3)\}$ and $L \otimes N = \{(1, g)\ (1, h)\ (1, r)\ (1, s)\ (1, t)\}$.

13. List the members of Y, then of Z.
14. List the members of L, then of N.

Special pairing

If we pair every number in set Q with every number in set R we get:

$Q = \{2, 3, 4\}$
$R = \{5, 6, 7, 8\}$

$Q \otimes R = \{(2, 5), (2, 6), (2, 7), (2, 8), (3, 6), (3, 5), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

If we pair every number in Q with every number in R which is 4 greater, we get S.

$S = \{(2, 6), (3, 7), (4, 8)\}$

S is a set of ordered pairs in which each member from Q (first member) appears in only *one* pair. A set of ordered pairs formed by this kind of pairing is called a **function.**

EXERCISES

For $A = \{1, 3, 5, 7\}$ and $B = \{4, 6, 7, 8\}$, list the sets.

1. $A \otimes B$
2. pairs from $A \otimes B$ in which both numbers are odd
3. pairs from $A \otimes B$ in which the second number is 3 greater than the first
4. pairs from $A \otimes B$ in which the second number is 1 greater than the first
5. pairs from $A \otimes B$ in which the second number is less than the first
6. pairs from $A \otimes B$ in which the second number is greater than the first

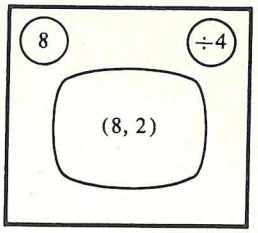
In exercises 3 and 4, the sets are functions.

7. Why are the sets in 1, 2, 5, and 6 *not* functions?

If A = {counting numbers less than 20} and B = {counting numbers less than 20}, list the set of ordered pairs in which

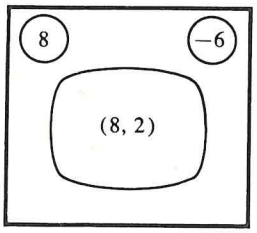
8. the second number is 6 greater than the first number
9. the rule for naming the second number is *add 10*
10. the rule for naming the second number is *subtract 8*
11. the rule for naming the second number is *multiply by 3*

You have often seen our function TV used to show ordered pairs. The number knob shows a counting number, while the rule knob gives a rule for naming the second number in the number pair.

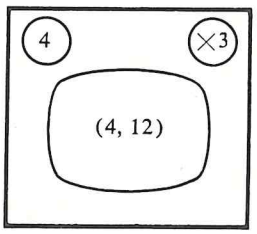


EXERCISES

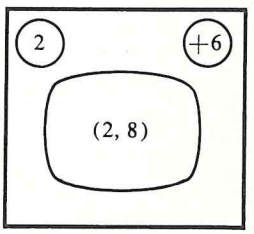
Use the function TV to help you complete each set of ordered pairs.



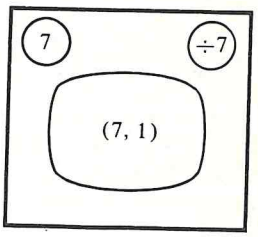
1. $\{(8, 2), (9, a), (6, b), (12, c), (17, d), (32, e), (50, f)\}$



2. $\{(4, 12), (6, g), (9, h), (20, i), (0, j), (32, k), (47, l)\}$



3. $\{(2, 8), (3, m), (0, n), (6, p), (10, q), (18, r), (12, s)\}$



4. $\{(7, 1), (28, t), (14, u), (49, v), (35, w), (21, x), (84, y)\}$

For $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $W = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$, name the rules which would form the functions.

5. $\{(1, 7), (2, 8), (3, 9), (4, 10), (5, 11), (6, 12), (7, 13)\}$

6. $\{(3, 6), (4, 8), (5, 10), (6, 12)\}$

7. $\{(9, 6), (8, 5)\}$

***8.** Form functions by pairing the members of $A = \{2, 4, 6\}$ with members of $B = \{8, 9, 10, 11\}$

Equations can state rules

The equation $m + 4 = n$ gives the rule for a set of ordered pairs. The number that replaces m is the first member of the ordered pair and the number that replaces n is the second member of the pair.

$3 + 4 = n$
$7 = n$
When m is replaced by 3, n is replaced by 7. The ordered pair is (3, 7).

$4 + 4 = n$
$8 = n$
When m is replaced by 4, n is replaced by 8. The ordered pair is (4, 8).

$5 + 4 = n$
$9 = n$
When m is replaced by 5, n is replaced by 9. The ordered pair is (5, 9).

The set of ordered pairs is $\{(3, 7), (4, 8), (5, 9), (6, 10), \ldots\}$.

EXERCISES

Study the rules, then complete the patterns.

Rule: *add 7*

1.

1st number in the pair	2	4	0	9	13	27	35	51	68	26
2nd number in the pair	*a*	*n*	*s*	*v*	*e*	*c*	*r*	*u*	*i*	*x*

Rule: *multiply by 4*

2.

1st number in the pair	2	10	6	0	13	27	16	28	56	79
2nd number in the pair	*v*	*e*	*m*	*n*	*x*	*i*	*u*	*r*	*s*	*a*

$m + 6 = n$

3.

m	2	5	13	0	9	24
n	8	*a*	*s*	*c*	*v*	*e*

$3 \times g = h$

5.

g	4	7	9	11	0	19
h	12	*n*	*i*	*x*	*c*	*r*

$r - 12 = t$

4.

r	15	20	31	42	70	12
t	3	*c*	*i*	30	*s*	*a*

$p + q = 29$

6.

p	2	10	8	4	13	19
q	27	19	*u*	*v*	*w*	*x*

Study the sets of number pairs, then write equations with 2 placeholders to show the functions.

7. $\{(3, 6), (4, 8), (10, 20), \ldots\}$

8. $\{(9, 3), (12, 6), (20, 14), \ldots\}$

Compare these two functions.

A
$3 \times m = n$
$\{(1, 3), (2, 6), (3, 9), \ldots\}$

B
$m \div 3 = n$
$\{(3, 1), (6, 2), (9, 3), \ldots\}$

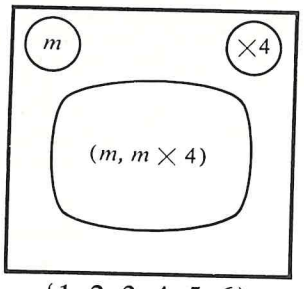
The rule for function B is the opposite of the rule for function A. The order of the numbers in the pairs in B is the opposite of the order of the numbers in the pairs in A.

EXERCISES

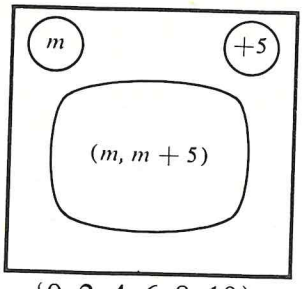
Use the equations to complete the sets of ordered pairs.

1. $x + 8 = y$: $\{(2, 10), (4, a), (7, b), (9, c), (13, d), (26, e)\}$
2. $x - 8 = y$: $\{(10, 2), (12, f), (15, g), (17, h), (21, i), (34, j)\}$
3. $4 \times g = h$: $\{(3, 12), (6, k), (9, l), (13, m), (17, n), (22, p)\}$
4. $g \div 4 = h$: $\{(12, 3), (24, q), (36, r), (52, s), (4, t), (72, u)\}$

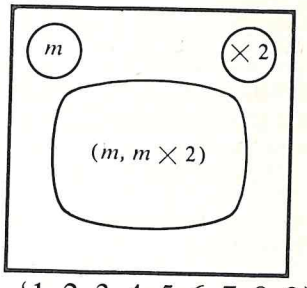
Use the function TV to write ordered pairs, replacing m by the numbers in the given sets.



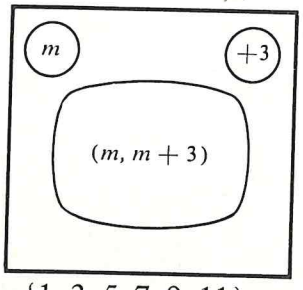
5. $\{1, 2, 3, 4, 5, 6\}$



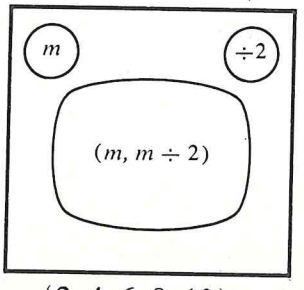
7. $\{0, 2, 4, 6, 8, 10\}$



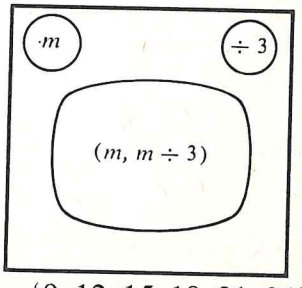
9. $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$



6. $\{1, 3, 5, 7, 9, 11\}$



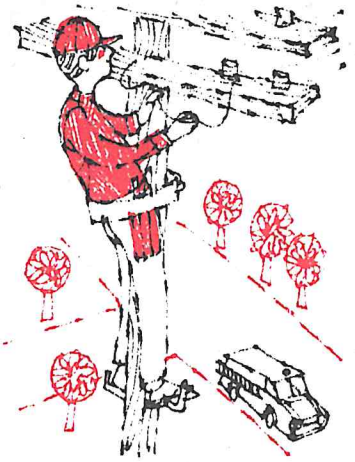
8. $\{2, 4, 6, 8, 10\}$



10. $\{9, 12, 15, 18, 21, 24\}$

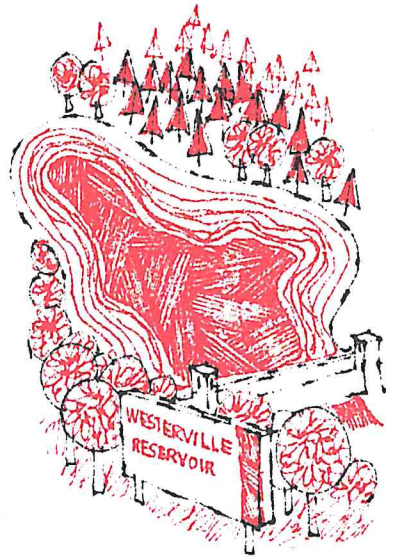
11. Pair the opposite rules in exercises 5–10.

Using sets and numbers



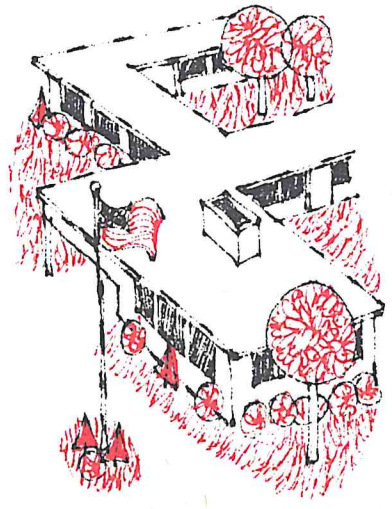
1. There are 715 miles of telephone wire connecting the houses in Westerville. If there are 25 telephone poles for every mile of wire, how many poles are needed for the Westerville system?

2. If it costs $32 per mile for the telephone wire, what is the total cost of the wire used for telephones in Westerville?



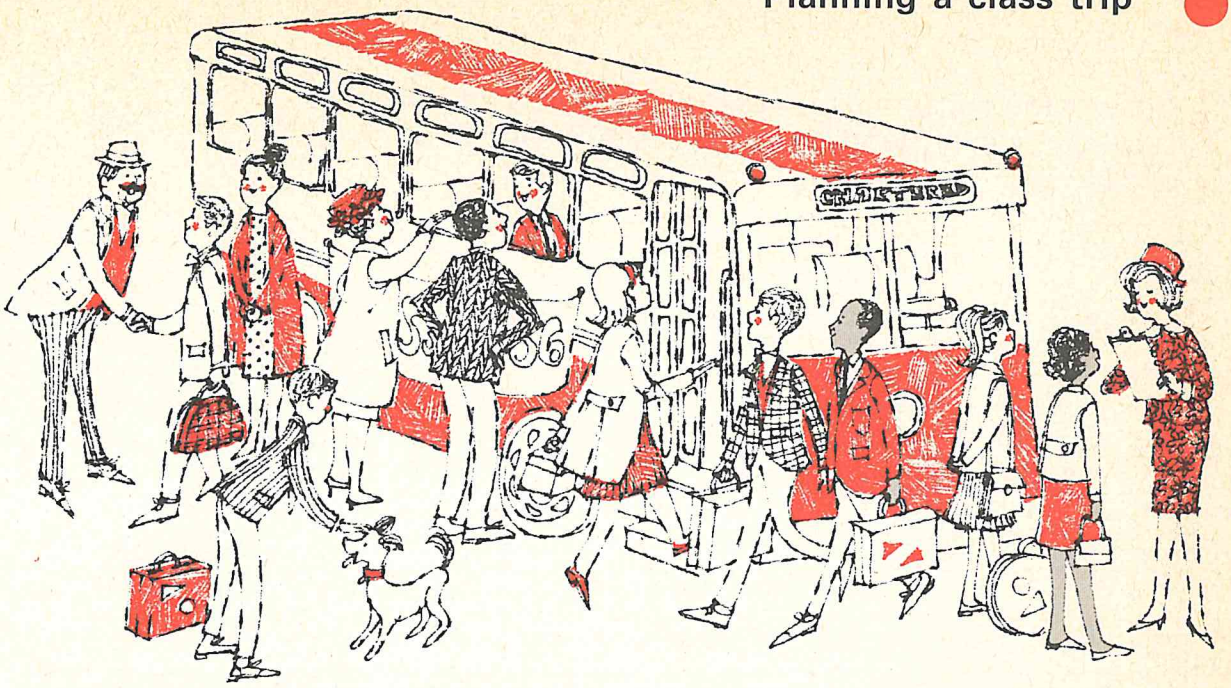
3. To supply water to the houses, the town had to lay 420 miles of pipe. If the cost was $375 per mile, what was the total cost?

4. The reservoir can hold 7,480,000 gallons of water. If this water costs $17 for every 10,000 gallons, what is the total value of the water in the reservoir?



5. The sewage system cost $5,200,000. If there are 52,000 people in Westerville, what is the cost per person of the system?

6. It costs $549 a year for each student in school in Westerville. If there are 11,500 students, what is the total yearly cost for schools?



Tammy's class is planning a bus tour. They are going to travel for 14 days.

1. If the cost of the bus is 17¢ per mile and the trip is planned to cover 1746 miles, what will be the total cost for the bus?

2. If the meals for each person are expected to cost $55 and there are 47 in the group, what will be the total cost for meals?

3. If the cost for motels is expected to be $75 for each person, what will be the total cost for motels?

4. What is the total planned cost for the bus, for meals, and for motels?

5. If meals cost $60 per person, what is the cost? How much more is that than the planned cost?

6. If motels cost $80 per person, what is the total cost? How much more is that than the planned cost?

7. If the distance traveled turns out to be 1925 miles, what is the cost of the bus? How much more than the planned cost is that?

Sharpshooting

Answer the questions in A as quickly as you can. Then see if you can answer the questions in B even more quickly.

A

1. $9 + 9 + 9 + 9 + 9 = a$
2. Name the intersection of $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$ and $B = \{2, 3, 5, 8, 13, 15\}$.
3. $333 \times 4 = n$
4. $65 - (9 \times 7) = n$
5. How many sides has a square with sides of 4″?
6. If $23 \times 9 = 207$, name the product of 230 and 9.
7. $49 \div 7 = 7$ so $4900 \div 70 = a$.
8. Name the perimeter of a triangle with sides of 12″, 13″, and 5″.
9. $100 \times 8 = 800$ so $98 \times 8 = a$
10. Is $3^2 + 5^2$ greater or less than 6^2?

B

1. $(5 \times 7) + (9 \times 7) = a$
2. If $C = \{a, b, c, d\}$ and $D = \{e, f, g\}$ name $n(C)$, $n(D)$ and $n(C \cup D)$.
3. If the perimeter of a square is 28 inches, name the length of each side.
4. $46 \times 5 = 230$ so $49 \times 5 = a$
5. $720 \div 9 = a$
6. Name the difference between 900 and 432.
7. Which is true, $6^2 + 7^2 > 9^2$ or $6^2 + 7^2 < 9^2$?
8. $100 - 64 = 36$ so $1000 - 364 = n$
9. How many sets of 9 can be formed from a set of 99?
10. $123 + 321 = a$

Just For Fun

If the size of an angle is 90°, we call it a right angle. Should we call any other angles wrong angles?

A	B	C
Today is Friday.	Today is not Friday.	Today is Thursday.
$3 \times 4 = 12$	$3 \times 4 \neq 12$	$3 \times 4 < 12$
$6 < 7$	$6 \not< 7$	$6 = 7$
$10 > 9$	$10 \not> 9$	$10 < 9$

The statements in B and C disagree with the statements in A. You remember that $\neq$ means "is not equal to," $\not<$ means "is not less than," $\not>$ means "is not greater than."

EXERCISES

Write *T* or *F* to show whether the statements are true or false.

1. If today is in fact Monday, the first sentences in A and C above are false.
2. If $3 \times a \neq 12$, then $3 \times a > 12$ or $3 \times a < 12$.
3. If $b \not< 7$, then $b = 7$ or $b > 7$.
4. If $c \not> 9$, then $c = 9$ or $c < 9$.

Name the statement in each set which disagrees with the others.

5. $\{n + 4 = 7, 7 - n = 4, 4 + n = 11\}$
6. $\{A$ and B are equivalent sets, $n(A) > n(B)$, $n(A) - n(B) = 0\}$
7. {The product of 3 and n is 18, $n + n + n = 18$, $18 - n = 6\}$

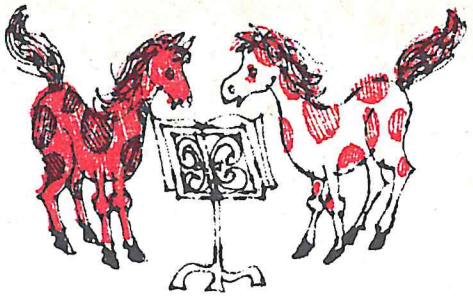
These statements are false. Write a true statement to disagree with each.

8. $15 + 16 \neq 31$
9. $15 + 16 > 31$
10. $15 + 16 < 31$
11. $42 \div 6 \neq 7$
12. $11 + 9 > 20$
13. The size of a right angle is 80°.
14. If 2 line segments are the same length, they are not congruent.
15. The product of 7 and 8 is 55.
16. The difference between 71 and 48 is 39.
17. Seven is an even number.

All — some — no

All trees have roots.

Some birds fly.



No horses sing.

EXERCISES

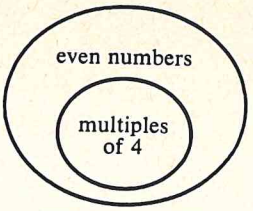
Write *yes* or *no* to answer the questions.

1. If **all** trees have roots, can there be **some** trees **without** roots?
2. If **all** numbers have names, can there be **some** numbers **without** names?
3. If **all** triangles have 3 sides, can there be **some** triangles that have 4 sides?
4. If **some** birds fly, can there be **some** birds that do **not** fly?
5. If **some** numbers are less than 10, can there be **some** numbers greater than 10?
6. If **some** numbers are less than 10, are **all** numbers less than 10?
7. If **some** lines are parallel, are **all** lines parallel?
8. If **no** intersecting lines are parallel, can there be **some** intersecting lines that are parallel?
9. If **no** odd numbers are divisible by 2, are there **some** odd numbers that are divisible by 2?
10. If **no** triangles have more than one right angle, are there **some** triangles with 2 right angles?

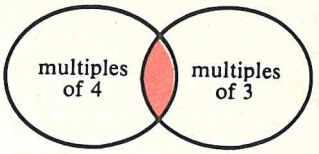
Write *all*, *some*, or *no* to make the statements true.

11. If **all** quadrilaterals have 4 sides, ___?___ quadrilaterals have 5 sides.
12. If **some** numbers are greater than 7, ___?___ numbers can be less than 7.
13. If **no** fish talk, ___?___ fish do not talk.

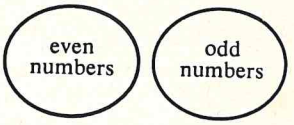
The diagram shows that **all** multiples of 4 are even numbers, and **some,** but **not all,** even numbers are multiples of 4.



The diagram shows that **some** multiples of 4 are multiples of 3.

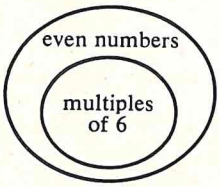


The diagram shows that **no** even numbers are odd numbers.

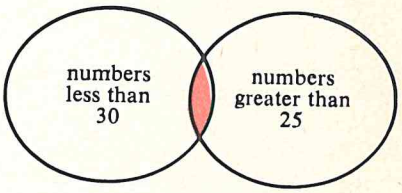


EXERCISES

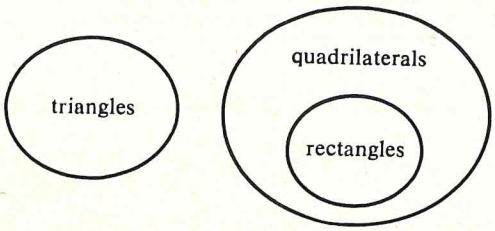
Study the sets, then write *all*, *some*, or *no* to complete each sentence.



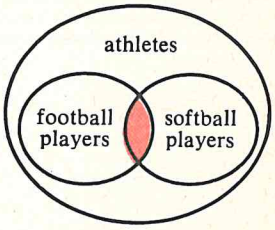
1. ___?___ multiples of 6 are even numbers.
2. ___?___ even numbers are multiples of 6.



3. ___?___ numbers less than 30 are greater than 25.
4. ___?___ numbers greater than 25 are less than 30.



5. ___?___ triangles are quadrilaterals.
6. ___?___ quadrilaterals are triangles.
7. ___?___ rectangles are quadrilaterals.



8. ___?___ football players are athletes.
9. ___?___ football players are softball players.
10. ___?___ athletes are softball players.

Disagreeing with all

All even numbers are divisible by 2.

1. **Some** even numbers are **not** divisible by 2.
2. There is **at least 1** even number that is **not** divisible by 2.

The statements on the right **disagree** with the statement on the left.

EXERCISES

Match each statement on the left with the statement on the right with which it disagrees.

1. **All** numbers divisible by 9 are divisible by 3.
2. **All** triangles have 3 sides.
3. **All** cubes have 6 faces.
4. **All** numbers that are multiples of 6 are multiples of 2.
5. **All** cubes have 12 edges.
6. **All** cars have tires.

a. **Some** cubes do not have 12 edges.

b. **There is at least one** cube that does not have 6 faces.

c. **Some** numbers that are divisible by 9 are not divisible by 3.

d. **Some** numbers that are multiples of 6 are not multiples of 2.

e. **Some** triangles do not have 3 sides.

f. **There is at least 1** car that does not have tires.

Write *yes* or *no* to answer the questions.

7. If all squares are rectangles, can some squares be triangles?
8. If all numbers can be named, is there 1 number that cannot be named?
9. If all whole numbers can be shown as sums, are there some whole numbers that cannot be shown as sums?
10. If all multiples of 10 are even, is there one multiple of 10 that is not even?
11. If all the horses in the stable eat hay, can there be one horse in the stable that does not eat hay?
12. If all the flowers in the vase are pink, can some flowers in the vase be yellow?

Disagreeing with some and no

Some girls like to swim.

No girls like to swim.

No boys like to dance.

1. **Some** boys like to dance.
2. There is **at least 1** boy who likes to dance.

There is only one way to disagree with *some* statements, and that is to use *no* statements. There is more than one way to disagree with *no* statements.

EXERCISES

Pair each statement on the right with the statement on the left with which it disagrees. For example, write (*1*,*c*).

1. No parallel lines intersect.

2. Some numerals name addends.

3. No triangles are squares.

4. Some cars are red.

5. No boys wear hats.

a. Some triangles are squares.

b. There is at least 1 boy who wears a hat.

c. Some parallel lines intersect.

d. No numerals name addends.

e. No cars are red.

Write a statement to disagree with each statement.

6. Some numbers are triangles.

7. Some polygons are pentagons.

8. No girls like music.

9. No boys like football.

10. Some numbers are odd.

11. No numbers are greater than 10.

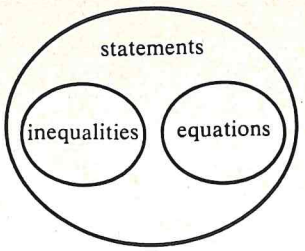
Write *T* or *F* to show whether the statements are true or false.

12. If no rectangles are squares, there is at least 1 rectangle that is a square.

13. If all triangles are 3-sided polygons, then triangle ABC has 3 sides.

***14.** If some angles are right angles, there may be some angles that are not right angles.

Sets — subsets

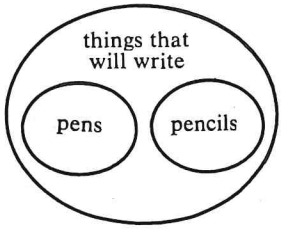


The diagram shows that all inequalities are statements and that all equations are statements.

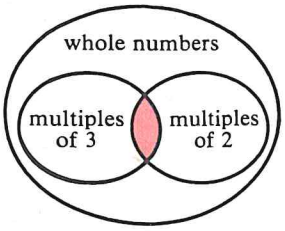
We **can** be sure that if it is an inequality, then it is a statement.
We **cannot** be sure that if it is a statement it is an inequality. It may be an equation or it may be some other kind of statement.

EXERCISES

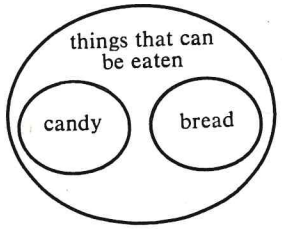
Write *true* or *false* for each statement. The diagram will help you be sure.



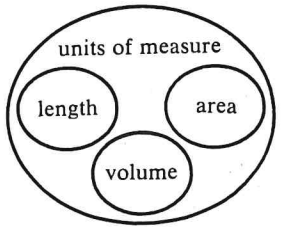
1. If it is a pen, then it will write.
2. If it will write, then it is a pen.
3. If it will write, then it is a pen or pencil.



7. If a number is a multiple of 2, then it is a whole number.
8. If a number is a whole number, then it is a multiple of 3.
9. If a number is a multiple of both 2 and 3, then it is a whole number.



4. If it can be eaten, then it is candy.
5. If it is candy, then it can be eaten.
6. If it can be eaten, then it is bread.



10. If it is a unit of length, then it is a unit of area.
11. If it is a unit of area, then it is a unit of measure.
12. If it is a unit of measure, then it is a unit of volume.

A = {polygons with 3 sides} B = {triangles}

Since $A = B$ the following statements are true:

a. If a polygon has 3 sides, then it is a triangle.
b. If a figure is a triangle, then it is a polygon with 3 sides.

EXERCISES

Write *true* or *false* for each statement.

G = {angles whose size is 90°} H = {right angles} $G = H$

1. If an angle is a right angle, then its size is 90°.
2. If the size of an angle is 90°, then it is a right angle.

E = {even numbers} F = {numbers with 2 as a factor} $E = F$

3. If a number is even, then it has 2 as a factor.
4. If a number has 2 as a factor, then it is even.

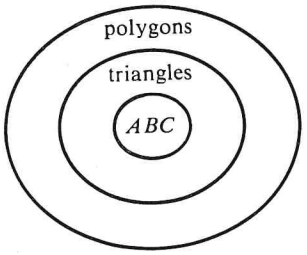
M = {even numbers} N = {numbers with 4 as a factor} $N \subset M$

5. If a number has 4 as a factor, then it is even.
6. If a number is even, then it has 4 as a factor.

For each pair of sets write two *if-then* sentences; write *T* if the sentence is true and *F* if the sentence is false.

7. A = {polygons with 5 sides} and B = {pentagons}
8. M = {multiples of 10} and N = {even numbers}
9. R = {airplanes} and S = {things that fly}
10. V = {parallel lines} and Z = {lines which do not intersect}
11. E = {multiples of 6} and F = {multiples of 3}

If — then

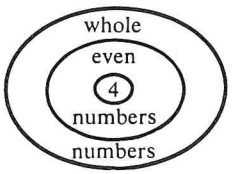


Johnny said, "**If** it is a triangle, **then** it is a polygon."
Mary said, "ABC is a triangle."
Johnny said, "Then I can say that ABC is a polygon."

The diagram shows that Johnny's last statement must be true.

EXERCISES

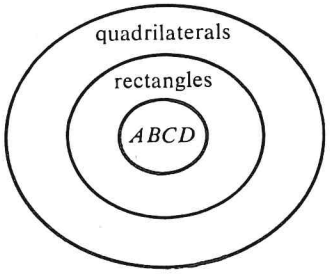
In each exercise study the diagrams and the first 2 statements, then write *true* or *false* for each of the other statements which follow.



1. If a number is even, then it is a whole number.

4 is an even number.

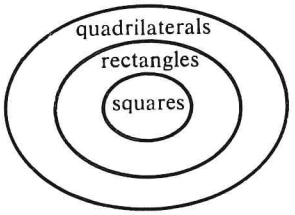
a. Then 4 is a whole number.
b. Then a whole number is an even number.



2. If it is a rectangle, then it is a quadrilateral.

$ABCD$ is a rectangle.

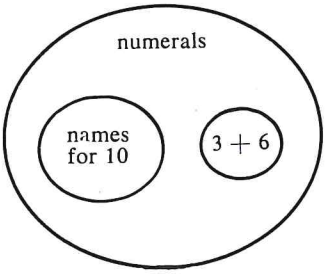
a. Then $ABCD$ is a quadrilateral.
b. Then all quadrilaterals are rectangles.



3. All rectangles are quadrilaterals.

All squares are rectangles.

a. So all squares are quadrilaterals.
b. So all rectangles are squares.



4. All names for 10 are numerals.

3 + 6 is a numeral.

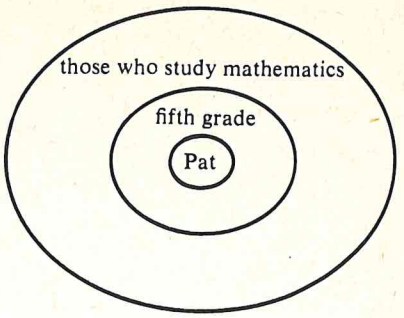
a. So all numerals are names for 10.
b. So 3 + 6 is a name for 10.

A. **If** all the children in fifth grade study mathematics

B. **and** Pat is in fifth grade,

C. **then** Pat studies mathematics.

The diagram shows that Pat must be in the set of those who study mathematics.



EXERCISES

Study the first 2 statements in each exercise then write *yes* or *no* for each of the **then** statements.

1.

If all airplanes fly,
and jets are airplanes,

a. then a jet flies.

b. then all airplanes are jets.

2.

If 2 parallel lines in a plane do not intersect,
and $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel

a. then $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect.

b. then $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ do not intersect.

3.

If all girls like flowers,
and Cindy is a girl,

a. then Cindy likes flowers.

b. then all girls like Cindy.

4.

If all men have hearts,
and cows have hearts

a. then all men are cows.

b. then all cows are men.

Complete the **then** statements.

5. If all girls sing and Phyllis is a girl, then Phyllis ____?____.

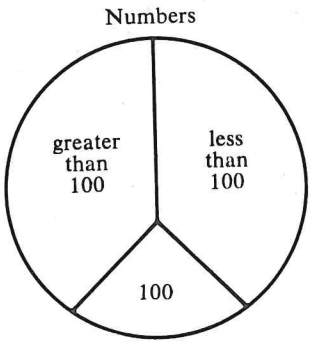
6. If all odd numbers are 1 greater than an even number, and 5 is 1 greater than 4, then 5 is ____?____.

7. If all numerals are names for numbers and $2 + 3$ is a numeral, then $2 + 3$ is ____?____.

8. If all candy is sweet and caramels are candy, then caramels ____?____.

If not — then

The diagram shows that any number is greater than 100, equal to 100, or less than 100. If a number is not equal to or greater than 100, then it is less than 100.



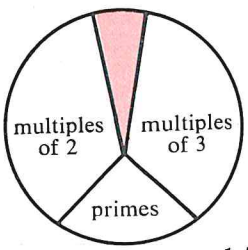
EXERCISES

Complete the sentences.

1. If $n \neq 100$ and $n \not> 100$, then n ___?___ 100.
2. If $n \not< 10$ and $n \not> 10$, then n ___?___ 10.
3. If n may be 2, 3, or 4, but is not 2 or 3, then n is ___?___.
4. If n may be an odd number or an even number, but is not an odd number, then n is ___?___.

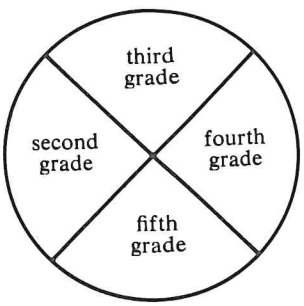
Use the diagrams to help you complete the sentences.

Whole numbers from 2 to 20



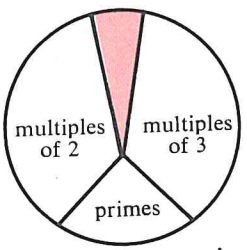
5. Eleven is not a multiple of 2 or 3, so it must be ___?___.

Girls at Maplewood School



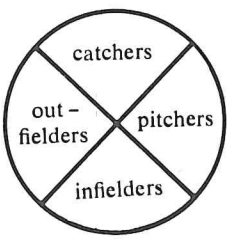
6. If a girl at Maplewood School is not in the second, third, or fourth grade, she must be in the ___?___ grade.

Whole numbers from 2 to 20



7. Sixteen is not a prime and not a multiple of 3, so it must be ___?___.

Softball players



8. If Jim plays softball and is not a pitcher, a catcher, or an infielder, he must be an ___?___.

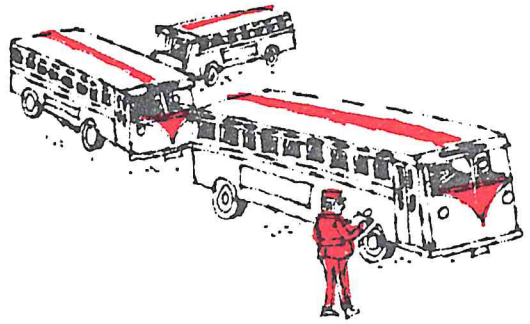
Suppose a friend said, "I am thinking of a whole number between 1 and 10. It is not even and it is not prime." You might find the number by listing the whole numbers between 1 and 10 and striking out the even numbers and the primes. The numbers between 1 and 10 are 2, 3, 4, 5, 6, 7, 8, 9. We strike out 2, 4, 6, and 8 because they are even, and 3, 5, and 7 because they are prime. The only number that is left is 9. So the number your friend is thinking of must be 9.

EXERCISES

Solve the problems.

1. Name the number in the set {3, 4, 5, 6, 7, 8, 9} that is not an odd number, and not a multiple of 4.
2. Name the number in the set {0, 3, 6, 9, 12, 15, 18, 21} that is not even, not less than 10 and not greater than 20.
3. Name the number in the set {7, 11, 17, 21, 27, 31, 37} that is not a prime and not a multiple of 7.
4. If I am thinking of a number that is not greater than 16 and not less than 16, it must be ____?____.
5. The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36. Name the factor of 36 that is not less than 10 or greater than 20 and is not a multiple of 4.
6. If a boy is a member of the set {Jim, Joe, John, Don, David, Dick, Doug} and his name does not begin with D and does not have 3 letters in it, the boy's name is ____?____.
7. If a letter is a member of the set {b, m, p, t, a, e} and is not in the word *apple* or the word *meat*, it is ____?____.
8. If I am thinking of a continent and it does not have A in its name, the continent of which I am thinking is ____?____.

Working with data



The bus company wanted to know how many buses to send out for 8 a.m. on Monday morning. Their data showed that for the previous 6 weeks they had used 36, 34, 39, 33, 30, and 32 buses at this time.

To find the number of buses to send out, they added $36 + 34 + 39 + 33 + 30 + 32 = 204$ and divided by 6. $204 \div 6 = 34$. Thirty-four is the **average.** This is the number of buses they are most likely to need.

EXERCISES

The bus company's data for other times on a Monday could be shown on a table like this.

date	7 a.m.	9 a.m.	4 p.m.	5 p.m.	8 p.m.	10 p.m.
Aug. 1	42	52	32	49	32	17
Aug. 8	32	60	37	50	29	19
Aug. 15	26	51	41	47	36	14
Aug. 22	23	50	29	52	31	12
Aug. 29	29	48	35	53	32	13
Sept. 5	34	51	36	49	36	15

Name the average number of buses used at

1. 7 a.m. **2.** 9 a.m. **3.** 4 p.m. **4.** 5 p.m. **5.** 8 p.m. **6.** 10 p.m.

The table shows the number of students that came to school for one week at Central School.

grade	Mon	Tu	Wed	Thur	Fri
fourth	30	25	27	27	31
fifth	32	30	28	29	31
sixth	31	31	31	31	31

Name the average number of students that came to school that week in

7. fourth grade **8.** fifth grade **9.** sixth grade

Complete the sentences.

10. The average of a fixed sum is the number that each addend would be if they were all the ___?___.

***11.** To find the average, we add, and then ___?___ by the number of addends.

The **average** of these numbers is $280 \div 7 = 40$.

If we order the numbers as in column B, we see that the greatest number is 69 and the least number is 27. The **range** is $69 - 27 = 42$. The number in the center of the data, when it is arranged in order, is 38. It is called the **median.**

Average, range, and median are measures we use to describe a set of data.

A	B
29	69
38	46
46	39
27	38
69	31
32	29
39	27
280	280

EXERCISES

Name the range of the numbers in each set.

1. {29, 81, 61, 84, 97, 63, 28, 17, 62}
2. {33, 38, 49, 57, 63, 58, 29, 37, 41}
3. {48, 46, 81, 37, 62, 51, 58, 49, 36}
4. {16, 28, 39, 45, 67, 46, 18}

Answer the questions.

5. Name the average of the numbers in each set in exercises 1–4.
6. Name the median of the numbers in each set in exercises 1–4.
7. If the highest score on an arithmetic test is 98 and the lowest is 67, what is the range of the scores?
8. In Shipville the average summer temperature is 65°, and the temperatures range from 58° to 71°. In Boattown the average summer temperature is also 65°, and the temperatures range from 48° to 89°. In which of these towns is it easier to tell what the temperature on a particular summer day is likely to be? Explain.

Study this example for naming the median of an even number of data.

{1, 3, 5, 13, 18, 20} has no center number. To name the median of these numbers, find the average of the third and fourth numbers: $\frac{5 + 13}{2} = 9$, so the median is 9.

Name the median of the numbers in each set.

9. {4, 5, 7, 9, 15, 22}
10. {21, 25, 29, 37}
11. {73, 76, 88, 89, 93, 95, 96, 99}

Finding averages

Compare these 2 examples of finding an average.

A To find the average of {37, 48, 67, 32, 21, 16, 58, 81, 54}

Step 1: The sum is 414.

Step 2: The average is 414 ÷ 9 = 46.

B To find the average of {37, 48, 70, 36, 21, 16, 58, 81, 54}

Step 1: The sum is 421.

Step 2: The average is 421 ÷ 9. When we divide 421 by 9, the quotient is 46 and there is a remainder of 7. If we want to state the average as a whole number, since 7 is greater than half of 9, we shall say the average is 47.

EXERCISES

Name the averages.

1. {23, 39, 26, 54, 62, 71, 42, 35}
2. {15, 25, 16, 14, 37, 19}
3. {13, 28, 37, 20, 23, 54, 63}
4. {76, 82, 84, 93, 75}
5. {52, 47, 63, 49, 55, 66, 43, 57}
6. {31, 37, 32, 36, 35, 33}

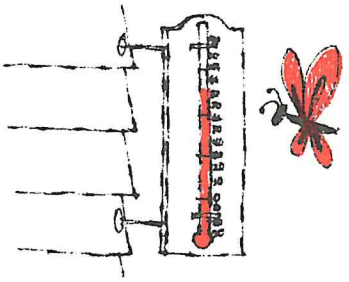
Name the averages as whole numbers.

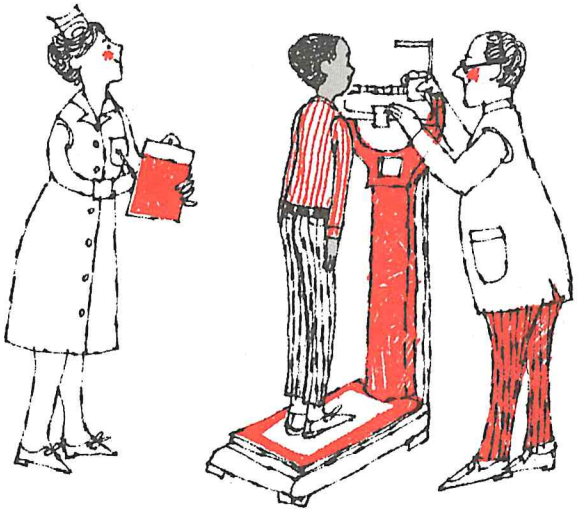
7. {49, 28, 76, 54, 63, 58}
8. {61, 62, 49, 58, 73, 65}
9. {69, 37, 48, 36, 58, 50}
10. {15, 29, 13, 32, 27, 21}
11. {49, 51, 63, 28, 59, 60}
12. {71, 75, 64, 62, 83, 80}

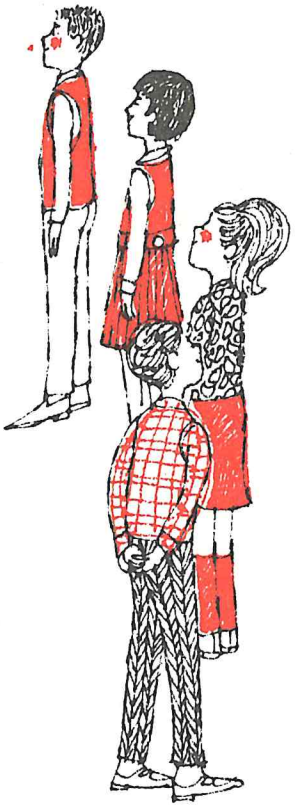
Solve the problems.

13. If you know that the average of five numbers is 60 and four of the numbers are 58, 72, 63 and 48, can you name the fifth number?
14. Is it possible for three hens to lay an average of 24 eggs a month if two of the hens between them lay 73 eggs a month?
15. David got 89 in Math, 54 in Spelling, 76 in Social Studies, and 93 in Science this year. What was his average grade? Do you think that the average tells what grades David is likely to get next year?

1. Mildred's spelling scores for one week were 71, 92, 84, 65, and 70. Find her average and median scores.
2. Tom's arithmetic scores were 73, 94, 87, 94, and 88. Find the range, average, and median of Tom's scores.
3. The high temperatures recorded for one week during the summer were 68, 72, 71, 72, 68, 76, and 84. Find the average and median temperatures.
4. The high temperatures for one week during the winter were 34, 23, 27, 19, 15, 22, and 21. Find the average and median temperatures.







5. The weights in pounds for 8 students in a fifth grade class were 93, 99, 98, 97, 90, 88, 96, and 92. Find the range, average, and median of the weights of the students.
6. The heights in inches of the 9 tallest students in one class at Emerson School were 59, 61, 57, 62, 61, 60, 58, 57 and 56. Find the average and median heights of these students.
7. Record the weights of the students in one row of your class. Find the average and median weights.

CHECKING UP

List the pairs in the cross products.

1. $A \otimes B$ $\quad A = \{1, 2, 3\}$ and $B = \{x, y, z\}$
2. $C \otimes D$ $\quad C = \{\text{Sarah, Billy, Carmen}\}$ and $D = \{\text{apple, pear, plum}\}$

Complete the sets of ordered pairs using the function rule on the left.

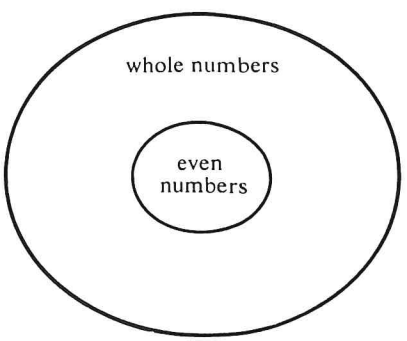
3. $(4 \times m) + 2 = n$ $\quad \{(1, 6), (2, a), (3, b), (4, c), (5, d)\}$
4. $(5 \times m) - 3 = n$ $\quad \{(1, 2), (2, e), (3, f), (4, g), (5, h)\}$
5. $(6 \times m) - 4 = n$ $\quad \{(1, 2), (2, i), (3, j), (4, k), (5, l)\}$

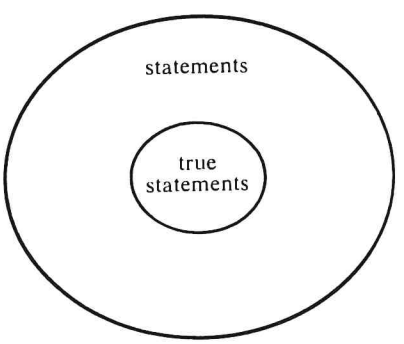
Use the data below to answer the questions.

student	Jim	Anne	Pat	Dave	Tom	Sue	Jan
test scores	69	76	88	75	66	95	77

6. Name the average.
7. Name the median.
8. The scores range from ___?___ to ___?___ .
9. Name the range.

Study the diagrams, then write *all*, *some*, or *no* to complete the sentences.





10. ___?___ whole numbers are even.
11. ___?___ even numbers are whole.
12. ___?___ true statements are statements.
13. ___?___ statements are true statements.

A **function** is a set of ordered pairs with a rule. No element appears as the first number in more than one pair. The rule may be given as an equation, such as $m + 4 = n$, where m holds the place of the first number in any pair, and n holds the place of the second number.

Complete each of the functions for the rule on the left.

1. $m + 17 = n$ $\{(1, 18), (2, a), (7, b), (8, c), (10, d)\}$

2. $(6 \times m) + 1 = n$ $\{(1, 7), (2, e), (3, f), (4, g), (5, h)\}$

3. $(m \div 2) + 1 = n$ $\{(2, 2), (4, i), (6, j), (8, k), (10, l)\}$

Averages, ranges, and medians are single numbers which help us to understand data.

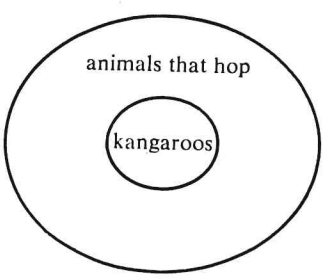
$\{48, 37, 56, 84, 42, 64, 29, 78\}$

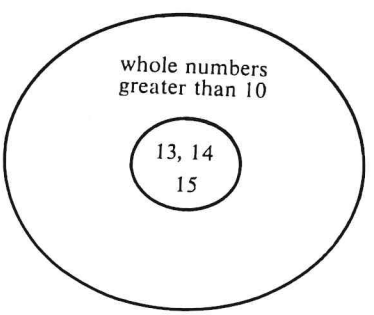
For the data in the set above, name

4. the average **5.** the range **6.** the median

Set diagrams may help us to write "if-then" statements.

Use the diagrams to help you write *T* or *F* for each statement.





7. If it is an animal that hops, then it is a kangaroo.

8. If it is a kangaroo, then it is an animal that hops.

9. If it is a number greater than 10, then it is 13, 14 or 15.

10. If this number is 13, 14 or 15, then it is greater than 10.

LOOKING WAY BACK

Study the sets, then name the cardinal numbers.

$A = \{a, b, c\}$ $\quad B = \{b, e, d, s\}$

$C = \{b, a, n, d\}$ $\quad D = \{g, u, e, s, t\}$

1. $n(A \cup B) = a$
2. $n(A \cap B) = b$
3. $n(A \otimes B) = c$
4. $n(C \cup B) = d$
5. $n(C \cap D) = e$
6. $n(C \otimes D) = f$
7. $n(A \cup C) = g$
8. $n(A \cap C) = h$
9. $n(A \otimes C) = i$
10. $n(B \cup D) = j$
11. $n(B \cap D) = k$
12. $n(B \otimes D) = l$

Write a compact numeral for each.

13. six hundred forty-three million, two hundred seven thousand, nine hundred eighty
14. 764 million, 827 thousand, 348
15. $(2 \times 10^5) + (4 \times 10^4) + (6 \times 10^3) + (5 \times 10^2) + (8 \times 10) + 9$
16. $6{,}000{,}000 + 400{,}000 + 60{,}000 + 3000 + 846$

Solve the equations.

17. $4{,}286{,}941 + 3{,}786{,}927 = a$
18. $29{,}861 - 14{,}867 = b$
19. $3 \times 28{,}697 = c$
20. $37 \times 8954 = d$
21. $369 \times 8471 = e$
22. $62{,}847 \div 9 = f$
23. $9483 \div 37 = g$
24. $2562 \div 61 = h$

Name the quotients and remainders.

25. $7\overline{)698473}$
26. $6\overline{)849603}$
27. $23\overline{)6819}$
28. $38\overline{)6298}$
29. $46\overline{)6847}$
30. $78\overline{)2345}$
31. $27\overline{)2904}$
32. $81\overline{)7593}$

Solve the problem.

33. How far will a car travel in 23 hours if it is traveling at the rate of 48 miles an hour? How long will it take the car to travel 1392 miles at the same rate?

Study this method for naming the average of
$A = \{37, 48, 67, 32, 21, 16, 58, 81, 54\}$.

Step 1: Name the number which seems to be near the middle. In set A it may be 48.

Step 2: Add the differences above and below.

Below	Above
$48 - 37 = 11$	$67 - 48 = 19$
$48 - 32 = 16$	$58 - 48 = 10$
$48 - 21 = 27$	$81 - 48 = 33$
$48 - 16 = 32$	$54 - 48 = 6$
86	68

Step 3: The sum of the differences below is 18 greater than the sum of the differences above. Divide 18 by 9. $18 \div 9 = 2$

Step 4: Subtract 2 from 48. $48 - 2 = 46$. 46 is the average.

1. Use this method to name the average of each set on page 158.

Copy the number patterns and continue them until they each contain 6 equations. Check your patterns by multiplying.

2.

$$(9 \times 9) + 7 = 88$$
$$(9 \times 98) + 6 = 888$$
$$(9 \times 987) + 5 = 8888$$

3.

$$6 \times 7 = 42$$
$$66 \times 67 = 4422$$
$$666 \times 667 = 444{,}222$$

4.

$$2^2 - 1^2 = 3$$
$$3^2 - 2^2 = 5$$
$$4^2 - 3^2 = 7$$
$$5^2 - 4^2 = 9$$

5.

$$20^2 - 10^2 = 300$$
$$30^2 - 20^2 = 500$$
$$40^2 - 30^2 = 700$$
$$50^2 - 40^2 = 900$$

Factors

$2 \times 9 = 18, \quad 3 \times 6 = 18, \quad 1 \times 18 = 18$

The **factors** of 18 are $\{1, 2, 3, 6, 9, 18\}$.

EXERCISES

Complete the sets of factors.

1. factors of 12 = $\{1, 2, 3, a, b, 12\}$
2. factors of 16 = $\{1, 2, c, d, 16\}$
3. factors of 24 = $\{1, 2, 3, e, f, g, h, 24\}$
4. factors of 36 = $\{1, 2, i, j, k, 9, l, m, n\}$
5. factors of 64 = $\{1, 2, 4, p, q, r, 64\}$
6. factors of 70 = $\{1, s, t, u, 10, v, 35, 70\}$

List the set of factors for each number.

7.	20	**9.**	49	**11.**	30	**13.**	51	**15.**	37	**17.**	74
8.	32	**10.**	14	**12.**	72	**14.**	60	**16.**	42	**18.**	91

Answer the questions.

19. If 4 is a factor of a number, will 2 be a factor also?
20. If 6 is a factor of a number, will 3 be a factor also? will 2?
21. If 10 is a factor of a number, name 2 other factors of the number.

List the set of factors for each number.

22.	90	**24.**	100	**26.**	144	**28.**	200	**30.**	256
23.	75	**25.**	126	**27.**	27	**29.**	288	**31.**	150

Complete the chart.

32.

number	10	20	30	12	32	42	14	24	16	36
number of factors	4	6	8	*a*	*n*	*c*	*x*	*e*	*i*	*v*

A = {factors of 18} = {1, 2, 3, 6, 9, 18}
B = {factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}

The **intersection** of A and B is {**common factors** of 18 and 24}.

$$A \cap B = \{1, 2, 3, 6\}$$

EXERCISES

Study these sets, then name the intersections.

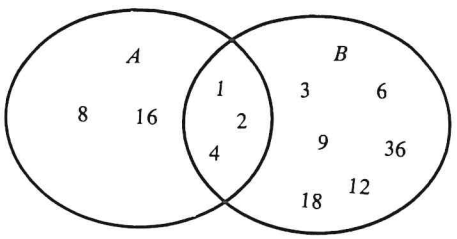
$A = \{1, 2, 3, 4, 6, 12\}$ $B = \{1, 2, 4, 5, 10, 20\}$ $C = \{1, 2, 3, 4, 6, 8, 12, 24\}$

1. $A \cap B$ **2.** $A \cap C$ **3.** $B \cap C$

Name the common factors of

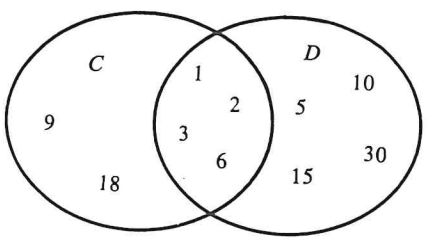
4. 12 and 20 **5.** 20 and 24 **6.** 12 and 24

Study the diagrams, then name the sets.



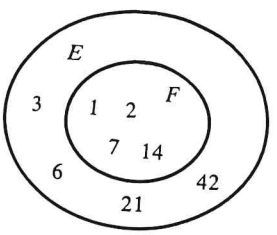
7. A **8.** B **9.** $A \cap B$

10. {common factors of 16 and 36}



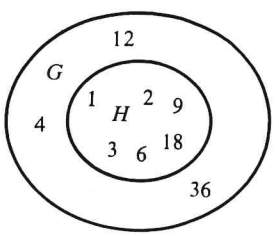
11. C **12.** D **13.** $C \cap D$

14. {common factors of 18 and 30}



15. F **16.** E **17.** $F \cap E$

18. {common factors of 14 and 42}



19. G **20.** H **21.** $G \cap H$

22. {common factors of 18 and 36}

*Use the sets of factors you named on page 164 to list the common factors of

23. 32 and 72 **25.** 90 and 144 **27.** 200 and 256

24. 42 and 60 **26.** 100 and 200 **28.** 144 and 256

Greatest common factor

A = {factors of 20} = {1, 2, 4, 5, 10, 20}
B = {factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}
$A \cap B$ = {common factors of 20 and 24} = {1, 2, 4}

The **greatest common factor (GCF)** of 20 and 24 is 4.

EXERCISES

Study the sets of factors.

$M = \{1, 3, 9, 27\}$ $N = \{1, 3, 5, 15\}$ $O = \{1, 2, 3, 5, 6, 10, 15, 30\}$

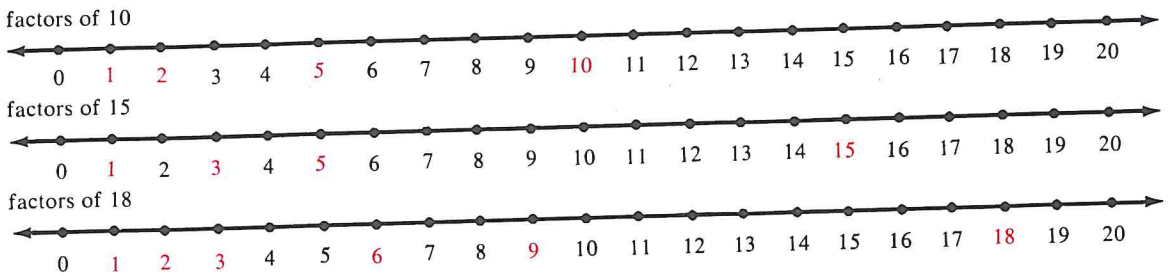
Name the common factors of

1. 27 and 30 **2.** 15 and 27 **3.** 15 and 30

Name the greatest common factor of

4. 27 and 30 **5.** 15 and 27 **6.** 15 and 30

Compare these number lines, which show the factors of 10, 15, and 18.



Name the GCF of

7. 10 and 15 **8.** 10 and 18 **9.** 15 and 18

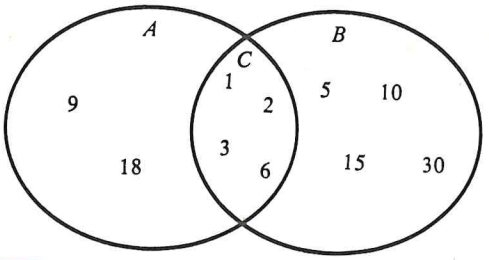
List the factors of each number, then name the GCF of each pair of numbers.

10. 8 and 12 **12.** 14 and 21 **14.** 30 and 45 **16.** 27 and 32
11. 9 and 24 **13.** 20 and 28 **15.** 48 and 64 **17.** 14 and 56

Solve the problem.

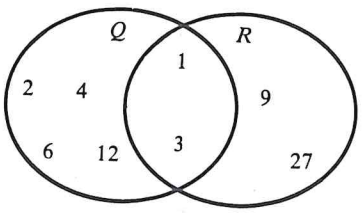
*__18.__ If you have 2 sets of cubes, 12 in one set and 20 in the other, how many cubes would you place in each row if you wanted to separate each pile into rows of the same size?

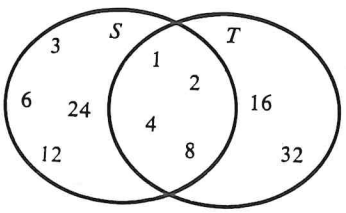
A = {factors of 18}
B = {factors of 30}
$C = A \cap B$
= {common factors of 18 and 30}
= {1, 2, 3, 6}
The *greatest common factor* of 18 and 30 is 6.

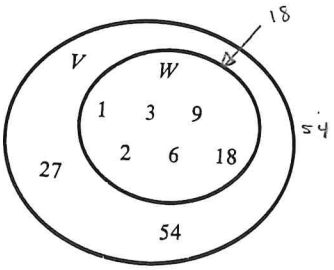


EXERCISES

Study these sets, then name the sets and the GCF.







1. Q **2.** R
3. $Q \cap R$
4. GCF of 12 and 27

5. S **6.** T
7. $S \cap T$
8. GCF of 24 and 32

9. W **10.** V
11. $W \cap V$
12. GCF of 18 and 54

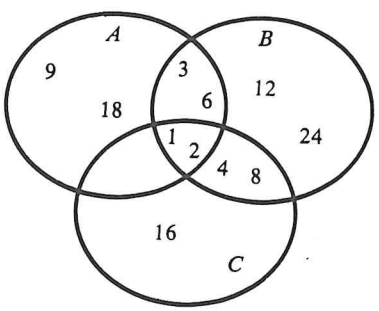
List the factors of each number, then name the GCF of each pair of numbers.

13. 20 and 52
14. 30 and 48
15. 40 and 60
16. 90 and 108
17. 48 and 112
18. 60 and 105
19. 72 and 144
20. 33 and 65

Study this diagram.

A = {1, 2, 3, 6, 9, 18}
B = {1, 2, 3, 4, 6, 8, 12, 24}
C = {1, 2, 4, 8, 16}

The common factors of 18, 24, and 16 are 1 and 2.
The GCF of 18, 24, and 16 is 2.



*Name the GCF of

21. 40, 48, and 64
22. 72, 90, and 144
23. 45, 135, and 210

Multiples

$0 \times 3 = 0$ $1 \times 3 = 3$ $2 \times 3 = 6$ $3 \times 3 = 9$ $4 \times 3 = 12$

0, 3, 6, 9, and 12 are *multiples* of 3.
The set of multiples of 3 is $\{0, 3, 6, 9, 12, \ldots\}$.

> To name multiples of 3, multiply 3 by numbers from the set of whole numbers, $\{0, 1, 2, 3, 4, \ldots\}$.

EXERCISES

Complete the sets of multiples.

1. $\{0, 3, 6, 9, a, b, c, d, e, 27\}$
2. $\{0, 4, 8, 12, f, g, h, i, j, 36\}$
3. $\{0, 5, 10, 15, 20, k, l, m, n, o\}$
4. $\{0, 6, 12, 18, 24, p, q, r, s, t\}$

Complete the number chart.

5.	**multiples of 6**	0	6	12	18	24	*a*	*n*	*o*	*v*	*e*	*i*
6.	**multiples of 7**	0	7	14	21	28	*z*	*r*	*c*	*a*	*x*	*e*
7.	**multiples of 8**	0	8	16	24	32	*s*	*n*	*o*	*u*	*e*	*r*
8.	**multiples of 9**	0	9	18	27	36	*s*	*i*	*u*	*v*	*o*	*x*

Complete the number chart.

9.	**multiples of 4**	56	60	*n*	*r*	*c*	*s*	*v*	*i*	88	92
10.	**multiples of 5**	40	45	*o*	*x*	60	*i*	*g*	*r*	*u*	85
11.	**multiples of 6**	54	60	*a*	*e*	*o*	*x*	90	*v*	*r*	108
12.	**multiples of 7**	*i*	*r*	49	56	63	*u*	*v*	*e*	*x*	*z*

Complete the sentences.

13. If a number is a multiple of 2 then it is called an ___?___ number.
14. If a number is a multiple of 6 then 2 of its factors, other than 1 and 6, are ___?___ and ___?___.
15. The only number which is a multiple of every number is ___?___.
16. The least number which is a multiple of *both* 2 and 3, other than 0, is ___?___.

$36 = 12 \times 3$
$48 = 16 \times 3$
$36 + 48 = (12 + 16) \times 3$
$= 28 \times 3$
$= 84$

36 is a multiple of 3.
48 is a multiple of 3.
$36 + 48 = 84$ is a multiple of 3.

We may use the distributive property to name multiples.

EXERCISES

Complete this table of multiples of 3, 4, 5, and 6.

	×	6	7	13	8	9	17	10	11	21
1.	3	18	21	39	24	27	51	*s*	*a*	*n*
2.	4	24	28	*v*	*e*	*i*	*n*	*r*	*x*	*c*
3.	5	30	*z*	*n*	*x*	*a*	*o*	*c*	*u*	*r*
4.	6	36	*s*	*i*	*u*	*v*	*e*	*x*	*v*	*z*

Complete this table of multiples of 6, 7, 8, and 9.

	×	5	7	12	6	9	15	8	10	18
5.	6	30	42	72	*a*	*n*	*c*	48	60	108
6.	7	35	49	*z*	42	63	*e*	*r*	*i*	*o*
7.	8	*i*	*u*	*a*	*x*	*r*	*n*	*o*	*v*	*s*
8.	9	*r*	*s*	*c*	*u*	*v*	*e*	*x*	*i*	*z*

Solve the equations.

9. $168 = 4 \times 42$ $\quad 126 = 3 \times 42$ $\quad 168 + 126 = a \times 42 = b$
10. $222 = 6 \times 37$ $\quad 148 = 4 \times 37$ $\quad 222 + 148 = c \times 37 = d$

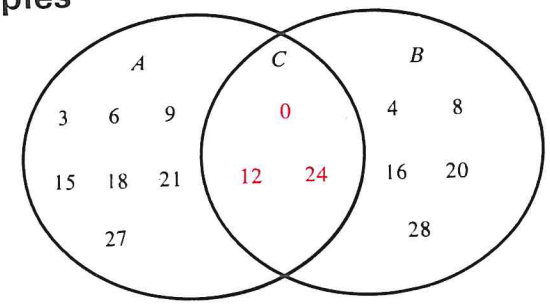
Using the first two equations in each row above, solve the equations.

11. $168 - 126 = a \times 42 = b$
12. $222 - 148 = c \times 37 = d$

Complete the sentence.

13. To name the multiples of 5, we ___?___ 5 by numbers from the set $\{0, 1, 2, 3, 4, 5, 6, \ldots\}$.

Common multiples



$A = \{0, 3, 6, 9, 12, 15, 18, 21, 24, 27\}$
$= \{\text{multiples of 3, less than 30}\}$

$B = \{0, 4, 8, 12, 16, 20, 24, 28\}$
$= \{\text{multiples of 4, less than 30}\}$

$C = A \cap B = \{0, 12, 24\}$
$= \{\textbf{common multiples} \text{ of 3 and 4, less than 30}\}$

EXERCISES

Name the sets of multiples of

1. 2, less than 20
2. 5, less than 30
3. 6, less than 40
4. of 3, less than 20
5. of 4, less than 40
6. of 8, less than 80

Name the sets of multiples of

7. 2 between 20 and 30
8. 3 between 30 and 50
9. 4 between 40 and 70
10. 3 between 20 and 30
11. 5 between 30 and 50
12. 6 between 40 and 70

From the sets in exercises 1–12 name the common multiples of

13. 2 and 3, less than 20
14. 3 and 5, less than 30
15. 4 and 6, less than 40
16. 2 and 3 between 20 and 30
17. 3 and 5 between 30 and 50
18. 4 and 6 between 40 and 70

How many common multiples are there of

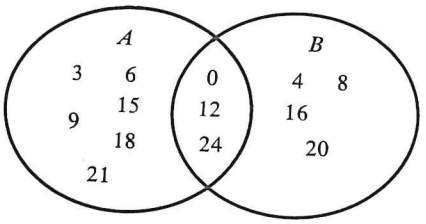
19. 4 and 5, less than 80?
20. 3 and 5, less than 50?
21. 2 and 7, less than 70?
22. 3 and 8, less than 100?
23. 2 and 3, less than 100?
24. 4 and 6, less than 90?
25. 6 and 10, less than 100?
26. 5 and 12, less than 100?

$A = \{0, 3, 6, 9, 12, 15, 18, 21, 24, 27\} = \{\text{multiples of 3, less than 30}\}$
$B = \{0, 4, 8, 12, 16, 20, 24, 28\} = \{\text{multiples of 4, less than 30}\}$
$A \cap B = \{0, 12, 24\} = \{\text{common multiples of 3 and 4, less than 30}\}$

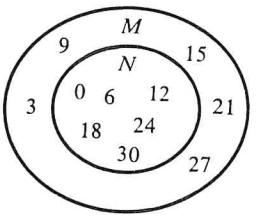
12 is the least number in the set $\{0, 12, 24\}$, other than 0.
12 is the **least common multiple** (LCM) of 3 and 4.
Because 0 is a multiple of every number, it is never called the LCM.

EXERCISES

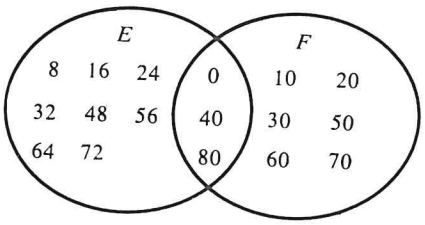
Study each diagram, then name the sets.



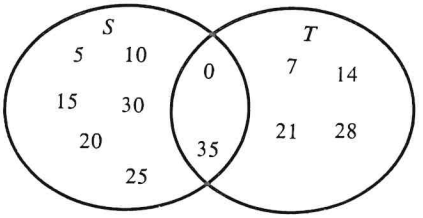
1. A **2.** B **3.** $A \cap B$



7. M **8.** N **9.** $M \cap N$



4. E **5.** F **6.** $E \cap F$



10. S **11.** T **12.** $S \cap T$

From the sets above, name the least common multiple of

13. 3 and 4 **14.** 8 and 10 **15.** 3 and 6 **16.** 5 and 7

Name the common multiples, less than 100, of

17. 6 and 7 **19.** 10 and 15 **21.** 7 and 11 **23.** 12 and 20
18. 8 and 12 **20.** 4 and 14 **22.** 24 and 36 **24.** 20 and 24

25–32. Name the least common multiple of each pair of numbers in exercises 17–24.

*__33.__ The product of 2 numbers will always be a common multiple of the numbers. Will it always be the LCM of the numbers?

Numbers with only 2 factors

Each of the numbers 2, 3, 5, 7, 11, 13 has only 2 factors, itself and 1. They are **prime numbers.**

$2 = 1 \times 2$ $\quad$ $3 = 1 \times 3$
$5 = 1 \times 5$ $\quad$ $7 = 1 \times 7$
$11 = 1 \times 11$ $\quad$ $13 = 1 \times 13$

EXERCISES

Name the factors of each number.

1. 17 **2.** 19 **3.** 21 **4.** 23 **5.** 29 **6.** 31 **7.** 37 **8.** 39

9. Which of the numbers in exercises 1–8 are prime?

List the numbers between 1 and 100 like this.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
91	92	93	94	95	96	97	98	99	100

10. Cross off 1.

11. Cross off all the multiples of: 2, except 2; 3, except 3; 5, except 5; 7, except 7.

12. Is each of the remaining numbers prime?

A pattern such as this is named after a famous Greek mathematician. It is called the Sieve of Eratosthenes. You may use this method with any set of numbers to find out which are prime.

Solve the equations.

13. $7 = (6 \times 1) + a$ **16.** $53 = (6 \times 9) - d$ **19.** $79 = (6 \times g) + 1$

14. $11 = (6 \times 2) - b$ **17.** $97 = (6 \times 16) + e$ **20.** $17 = (6 \times h) - 1$

15. $23 = (6 \times c) - 1$ **18.** $71 = (6 \times f) - 1$ **21.** $91 = (6 \times i) + 1$

22. Can you discover a pattern? Test your pattern with other primes.

23. How many prime numbers are even numbers?

	11		31	41		61	71		
2									
3	13	23		43	53		73	83	
5									
7	17		37	47		67			97
	19	29			59		79	89	

There are 25 primes less than 100.

EXERCISES

How many primes are there

1. less than 20?
2. between 40 and 60?
3. between 80 and 100?
4. between 20 and 40?
5. between 60 and 80?
6. between 0 and 50?

Name

7. 4 digits which do not occur in the ones' place of a prime less than 100.
8. 2 digits that occur only once.
9. 2 digits that occur 5 times.
10. 1 digit that occurs 6 times.
11. 1 digit that occurs 7 times.

Solve these equations. Then see if you can discover a pattern.

12. $7 + 11 = a$
13. $13 + 17 = b$
14. $19 + 23 = c$
15. $29 + 31 = d$
16. $37 + 41 = e$
17. $43 + 47 = f$
18. $59 + 61 = g$
19. $67 + 71 = h$
20. $73 + 79 = i$

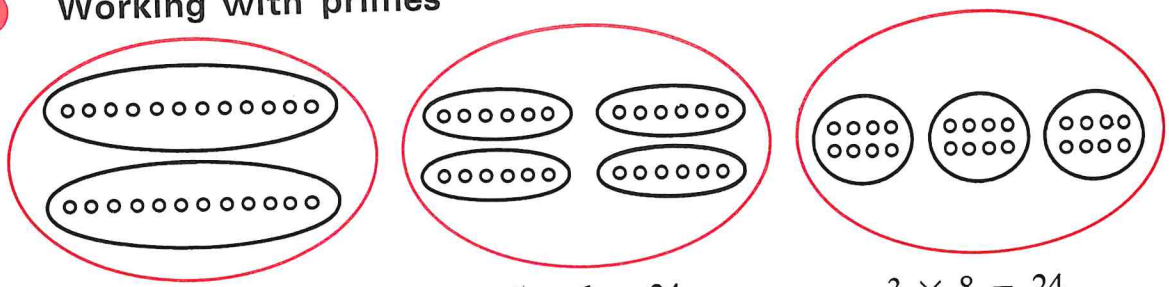
**21. List the 21 primes between 100 and 200.

How many primes are there between

22. 100 and 120?
23. 120 and 140?
24. 140 and 160?
25. 160 and 180?

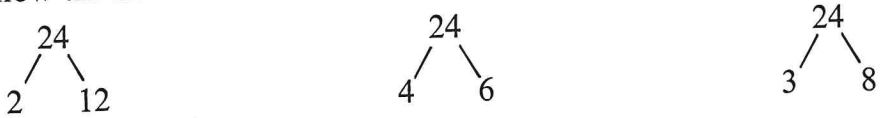
*26. Would you say that the primes between 100 and 200 are evenly spread?

Working with primes



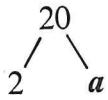
$2 \times 12 = 24$ $\quad$ $4 \times 6 = 24$ $\quad$ $3 \times 8 = 24$

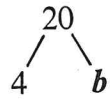
We can show the factors of 24 like this:

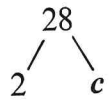


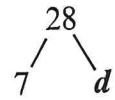
EXERCISES

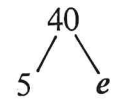
Complete the patterns.

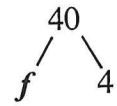
1. 

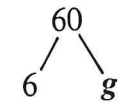
2. 

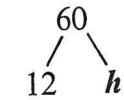
3. 

4. 

5. 

6. 

7. 

8. 

9. Show 2 different patterns like those above for 18; for 30; for 32.

10. Show 3 different patterns like those above for 36; for 48; for 56.

Solve the equations.

11. $20 = 2 \times 10$ and $10 = 2 \times 5$ so $20 = 2 \times 2 \times a$

12. $30 = 2 \times 15$ and $15 = 3 \times 5$ so $30 = 2 \times b \times 5$

13. $63 = 3 \times 21$ and $21 = 3 \times c$ so $63 = 3 \times 3 \times d$

14. $105 = 3 \times 35$ and $35 = e \times 7$ so $105 = 3 \times f \times 7$

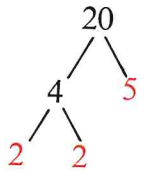
15. Are all the factors in the last equations in exercises 11–14 prime?

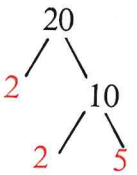
*16. Name the product $2 \times 3 \times 5 \times 7 \times 11$.

*17. Why will this product be the LCM of 2, 3, 5, 7, and 11?

Factoring with primes

Each **factor tree** shows that $20 = 2 \times 2 \times 5$. This shows that 20 can be written as the product of 3 prime numbers.

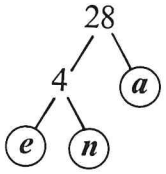




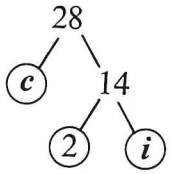
EXERCISES

Complete these factor trees. Then tell whether all the circled numerals name primes.

1.



2.



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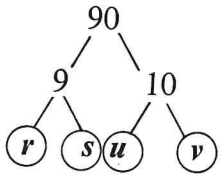

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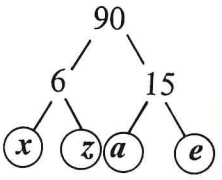


5. Show 2 different factor trees for 18; for 30; for 32; for 45.

6. Show 3 different factor trees for 36; for 48; for 56; for 60.

Study the example, then complete the equations.

$$\begin{aligned} 24 &= 2 \times 12 \\ &= 2 \times \ 2 \times 6 \\ &= 2 \times \ 2 \times 2 \times 3 \end{aligned}$$

7. $\begin{aligned} 66 &= 2 \times 33 \\ &= 2 \times c \times e \end{aligned}$

8. $\begin{aligned} 44 &= 11 \times 4 \\ &= 11 \times a \times r \end{aligned}$

9. $\begin{aligned} 52 &= 2 \times s \\ &= 2 \times u \times v \end{aligned}$

10. $\begin{aligned} 68 &= 2 \times n \\ &= 2 \times 2 \times r \end{aligned}$

11. $\begin{aligned} 56 &= 2 \times x \\ &= 2 \times 2 \times 14 \\ &= 2 \times 2 \times a \times e \end{aligned}$

12. $\begin{aligned} 54 &= 2 \times 27 \\ &= 2 \times 3 \times 9 \\ &= 2 \times 3 \times m \times n \end{aligned}$

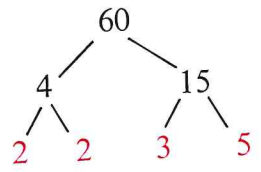
13. $\begin{aligned} 225 &= 3 \times n \\ &= 3 \times 3 \times r \\ &= 3 \times 3 \times s \times u \end{aligned}$

14. $\begin{aligned} 150 &= 2 \times s \\ &= 2 \times 5 \times a \\ &= 2 \times 5 \times n \times c \end{aligned}$

15. Has each number in exercises 7–14 been expressed as a product of prime factors?

Using prime factors

The factor tree shows that 60 is the product of the prime factors 2, 3, and 5. This is called the **prime factorization** of 60.



$60 = 2 \times 2 \times 3 \times 5$

The prime factorization of any number names that number as a **product of prime factors.**

EXERCISES

Use the factor trees on page 175 to help you name the prime factorization of

1. 20 **2.** 28 **3.** 90 **4.** 32 **5.** 48 **6.** 56

Draw a factor tree for each number, then name the prime factorization.

7. 12 **8.** 16 **9.** 58 **10.** 64 **11.** 100 **12.** 144 **13.** 18 **14.** 96 **15.** 72 **16.** 36 **17.** 256 **18.** 150

Complete the equations.

19. $45 = 3 \times 15$
$= 3 \times a \times b$

20. $45 = 5 \times 9$
$= 5 \times c \times d$

21. $42 = 6 \times 7$
$= e \times f \times 7$

22. $42 = 3 \times 14$
$= 3 \times g \times h$

23. $66 = 3 \times 22$
$= 3 \times i \times j$

24. $66 = 33 \times 2$
$= k \times l \times 2$

25. $105 = 3 \times 35$
$= 3 \times m \times n$

26. $105 = 5 \times 21$
$= 5 \times p \times q$

27. $105 = 7 \times 15$
$= 7 \times r \times s$

28. How many prime factorizations are there for 45? for 42? for 66?

Answer the questions.

29. What is the least number that is the product of 2 different primes?

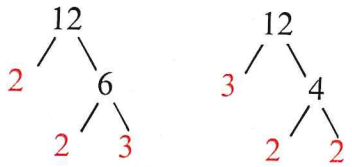
30. What is the least odd number that is the product of 2 different primes?

31. What is the least number that is the product of 3 different primes?

32. What is the least odd number that is the product of 3 different primes?

Prime factorization

These factor trees show that $12 = 2 \times 2 \times 3$. Since the commutative property says that the order of the factors does not affect the product, for any number there is only **one** prime factorization.



$12 = 2 \times 2 \times 3$

EXERCISES

Show a factor tree and prime factorization for

1. 28 **2.** 90 **3.** 120 **4.** 164 **5.** 68 **6.** 102

The prime factorization for 36 is $2 \times 2 \times 3 \times 3$. We can write this as $2^2 \times 3^2$ since each of the factors 2 and 3 is used twice. Remember that the **exponent** tells the number of times the factor is used in a product.

Name each number as a product of prime factors, using exponents.

7. $20 = 2 \times 2 \times 5$
8. $18 = 2 \times 3 \times 3$
9. $16 = 2 \times 2 \times 2 \times 2$
10. $40 = 2 \times 2 \times 2 \times 5$
11. $54 = 2 \times 3 \times 3 \times 3$
12. $75 = 3 \times 5 \times 5$
13. $45 = 3 \times 3 \times 5$
14. $50 = 2 \times 5 \times 5$

Name the prime factorization of each number, using exponents.

15. 48 **16.** 36 **17.** 180 **18.** 28 **19.** 64 **20.** 42 **21.** 100 **22.** 60 **23.** 144 **24.** 90 **25.** 176 **26.** 200

To name a prime factorization, we divide by primes beginning with 2. For example $234 \div 2 = 117$, $117 \div 3 = 39$, $39 \div 3 = 13$. The prime factorization of 234 is $2 \times 3^2 \times 13$.

Use the division method to name the prime factorization of

27. 76 **28.** 104 **29.** 132 **30.** 138 **31.** 153 **32.** 182 **33.** 210 **34.** 225 **35.** 240 **36.** 300 **37.** 969 **38.** 253

Using prime factors

We may use the associative and commutative properties to name all the factors of a number if we know its prime factorization. Study these equations, which use the prime factorization of 30.

$30 = 2 \times 3 \times 5$ $= (2 \times 3) \times 5$ $= 6 \times 5$	$30 = 2 \times 3 \times 5$ $= 2 \times (3 \times 5)$ $= 2 \times 15$	$30 = 2 \times 3 \times 5$ $= 3 \times (2 \times 5)$ $= 3 \times 10$

The factors of 30 are $\{1, 2, 3, 5, 6, 10, 15, 30\}$.

EXERCISES

Study these prime factorizations.

$45 = 3 \times 3 \times 5$ $\qquad$ $36 = 2^2 \times 3^2$

$84 = 2 \times 2 \times 3 \times 7$ $\qquad$ $100 = 2^2 \times 5^2$

1. Use the associative property and the prime factors to name the factors of 45, other than 1 and 45.
2. Use the prime factorizations to name all the factors of
   a. 84 **b.** 36 **c.** 100

Use the prime factorizations to complete the chart.

$42 = 2 \times 3 \times 7$ $\qquad$ $60 = 2 \times 2 \times 3 \times 5$

$105 = 3 \times 5 \times 7$ $\qquad$ $90 = 2 \times 3 \times 3 \times 5$

	numbers	common prime factors	common factors	GCF
3.	42, 105	*a*	*b*	*c*
4.	42, 60	*d*	*e*	*f*
5.	60, 90	*g*	*h*	*i*
6.	90, 105	*j*	*k*	*l*

Complete the sentences.

7. If a number is prime, its only factors are itself and ___?___.
8. If a number greater than 1 is not prime, it can be shown as the ___?___ of only prime factors.

Sharpshooting

Answer the questions in A as quickly as you can. Then see if you can answer the questions in B even more quickly.

A

1. What is the cost of 6 newspapers at 15¢ each?
2. $72 - 28 = n$
3. Name the least common multiple of 6 and 10.
4. How many hours will it take to travel 120 miles at 24 miles per hour?
5. $46 + 28 + 34 = n$
6. How many inches are there in 8 feet 4 inches?
7. How many minutes from 7:45 am to 9:15 am?
8. If 16 pounds of peaches cost $1.92, what is the cost per pound?

B

1. What is the cost of 6 dozen eggs at 47¢ a dozen?
2. $36 + 4 + 8 + 9 + 7 = n$
3. Name the product of 7 and 37.
4. How many ounces are there in 3 pounds 4 ounces?
5. Name the difference between 612 and 216.
6. Name the quotient and remainder when 276 is divided by 12.
7. Name the number which gives a quotient of 4 and remainder of 3 when divided by 9.
8. Write a compact numeral for the number 11 hundreds, 11 tens, 11 ones.

Just For Fun

Jim discovered a quick way to multiply by 9 and by 11.

$$27 \times 9 = 270 - 27 = 243$$
$$27 \times 11 = 270 + 27 = 297$$

Do you understand Jim's methods? If not, study these equations.

$27 \times 9 = 27 \times (10 - 1) = (27 \times 10) - (27 \times 1) = 270 - 27 = 243$
$27 \times 11 = 27 \times (10 + 1) = (27 \times 10) + (27 \times 1) = 270 + 27 = 297$

Use Jim's method to name these products.

9. $46 \times 9 = a$
10. $128 \times 9 = b$
11. $157 \times 11 = c$
12. $95 \times 9 = d$
13. $84 \times 11 = e$
14. $728 \times 11 = f$

Can you extend Jim's method to name these products?

15. $58 \times 99 = a$
16. $93 \times 99 = b$
17. $56 \times 101 = c$

Number patterns

1, 1, 2, 3, 3, 4, 5, 5, 6, . . . 7, 7, 8, 9, 9, 10, 11, 11, . . .

In the pattern on the left every odd number occurs twice, so the pattern would continue as shown on the right.

1, 3, 4, 7, 11, 18, 29, . . . 47, 76, 123, 199, . . .

In the pattern on the left, each number after the second one is the sum of the two numbers that come just before it. It would continue as shown in the pattern on the right.

EXERCISES

Name the next 10 members of each pattern.

1. 1, 2, 3, 6, 5, 4, 7, 8, 9, . . .

2. 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, . . .

3. 0, 1, 2, 3, 5, 8, . . .

***4.** 2, 3, 5, 7, 11, 13, 17, . . .

Name the missing members of each pattern.

5. 1, 4, 7, ***a***, 13, ***b***, ***c***, 22

6. 1, 2, 4, 8, ***d***, ***e***, ***f***, 128

7. 1, 3, 9, ***g***, ***h***, ***i***, 729

8. 2, 6, 18, ***j***, ***k***, ***l***, 1458

9. 17, 15, 13, ***m***, ***n***, ***o***, 5, ***p***, 1

10. 38, 34, 30, ***q***, ***r***, ***s***, 14, 10, ***t***, 2

If m is one member of a pattern and we add 4 to get the next member, n, we may write the rule for the pattern as $m + 4 = n$.

Use the rules on the left to name the next 10 members.

	rule	first member
11.	$(2 \times m) + 1 = n$	1
12.	$(3 \times r) - 1 = s$	2
13.	$(2 \times a) - 5 = b$	6
14.	$(4 \times x) - 5 = y$	2

	rule	first member
15.	$(2 \times s) - 1 = t$	2
16.	$(2 \times m) - 1 = n$	1
17.	$(4 \times r) - 3 = s$	2
18.	$(3 \times m) - 2 = n$	1

Working with number patterns

The numbers in the pattern 2, 4, 6, 8, 10, 12, . . . are ordered. Two is the first **term,** 4 is the second **term** and so on. We can show this in a chart.

number of the term	1	2	3	4	5	6	. . .
term	2	4	6	8	10	12	. . .

Name these terms of the number pattern above.

1. the 7th term
2. the 8th term
3. the 10th term
4. the 13th term
5. the 21st term
6. the 50th term

We can write the number pattern above as a set of ordered pairs, giving the number of the term first and the term of the pattern second.

Complete this set of number pairs, then name the rule.

7. $\{(1, 2), (2, 4), (3, 6), (4, a), (5, b), (6, c), (8, d), (10, e), (13, f), (50, g)\}$

Study this chart.

number of the term	1	2	3	4	5	6
term	4	8	12	16	20	24

8. Name the 7th term; the 10th term; the 15th term; the 25th term.
9. Write the first 10 terms of the pattern as a set of ordered pairs.
10. Name the rule for writing the ordered pairs in the pattern.

Study this chart.

number of the term	1	2	3	4	5	6
term	3	5	7	9	11	13

11. Name the 7th term; the 10th term; the 15th term; the 25th term.
12. Write the first 10 terms of the pattern as a set of ordered pairs.
13. Name the rule for writing the ordered pairs.

Recognizing multiples of 3

$3 \times 5 = 15$ $\quad 3 \times 14 = 42$ $\quad 24 \div 3 = 8$ $\quad 57 \div 3 = 19$

We say that 15, 42, 24, and 57 are **multiples** of 3, or that 15, 42, 24, and 57 are **divisible** by 3.

EXERCISES

Solve the equations.

1. $10 \times 3 = a$, $7 \times 3 = b$, so $17 \times 3 = c$

2. $20 \times 3 = d$, $4 \times 3 = e$, so $24 \times 3 = f$

3. $30 \times 3 = g$, $5 \times 3 = h$, so $35 \times 3 = i$

Complete this chart of multiples of 3.

4.

+	0	3	6	9	12	15	18	21	24	27
30	30	33	36	*a*	*n*	*c*	*i*	*e*	*r*	*s*
60	60	63	*x*	*i*	*a*	*e*	*v*	*m*	*n*	*u*
90	90	*e*	*a*	*r*	*s*	*c*	*u*	*v*	*w*	*x*

Solve these equations.

5. $732 \div 3 = a$ $\quad 7 + 3 + 2 = b = c \times 3$

6. $258 \div 3 = d$ $\quad 2 + 5 + 8 = e = f \times 3$

7. $714 \div 3 = g$ $\quad 7 + 1 + 4 = h = i \times 3$

8. $822 \div 3 = j$ $\quad 8 + 2 + 2 = k = l \times 3$

In the equations on the right above, we have named the sums of the digits in the numerals in the equations on the left. For example, the sum of the digits of 732 is $7 + 3 + 2 = 12$.

9. Name any 10 multiples of 3 from exercise 4, then name the sum of the digits of each multiple.

Complete these sentences.

10. The sum of the digits of a multiple of 3 is a ____?____ of 3.

11. A number is divisible by 3 if the sum of its ____?____ is divisible by 3.

Recognizing multiples of 2 and 4

$A = \{0, 2, 4, 6, 8\} = \{\text{multiples of 2, less than 10}\}$
$B = \{10, 12, 14, 16, 18, 20\} = \{\text{multiples of 2 from 10 to 20}\}$
$C = \{0, 4, 8, 12, 16\} = \{\text{multiples of 4, less than 20}\}$
$D = \{20, 24, 28, 32, 36\} = \{\text{multiples of 4 from 20 to 36}\}$

The ones' digits are 0, 2, 4, 6 or 8.

EXERCISES

List the multiples of 2

1. from 20 to 28
2. from 90 to 98
3. from 50 to 58
4. from 130 to 138

Complete this sentence.

5. If a number is a multiple of 2, the ones' digit in its numeral is ___?___, ___?___, ___?___, ___?___, or ___?___.

Study the chart of multiples of 4, less than 100, then complete the sentences.

6. If a number is a multiple of 4 it is also a multiple of ___?___.
7. A multiple of 4, less than 100, is 0, 4, 8, 12, 16, plus 0, 20, 40, 60, or ___?___.

0	4	8	12	16
20	24	28	32	36
40	44	48	52	56
60	64	68	72	76
80	84	88	92	96

8. Is 100 divisible by 4? 200? 300? 1000? Are all multiples of 100 divisible by 4?

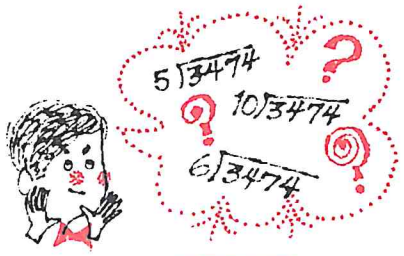
Test these numbers to see if they are divisible by 4.

9. $84 = 80 + 4$
10. $364 = 300 + 64$
11. $98 = 80 + 18$
12. $592 = 500 + 92$
13. $132 = 100 + 32$
14. $758 = 700 + 58$
15. $156 = 100 + 56$
16. $972 = 900 + 72$
17. $634 = 600 + 34$

Complete the sentence.

18. A number greater than 100 is divisible by 4 if its tens and ones are divisible by ___?___.

Recognizing multiples of 5, 10, and 6



1. List the set of multiples of 5, less than 100.
2. List the set of multiples of 5 from 300 to 400.

Complete the sentence.

3. A number is a multiple of 5 if the ones' digit in its numeral is ___?___ or ___?___.

4. List the set of multiples of 10, less than 100.
5. List the set of multiples of 10 between 500 and 600.
6. List the set of multiples of 10 between 800 and 900.

Complete the sentence.

7. A number is a multiple of 10 if the ones' digit in its numeral is ___?___.

Which of these multiples of 3 are divisible by 2? by 6?

8.	96	**10.**	315	**12.**	294	**14.**	441	**16.**	1236	**18.**	558
9.	483	**11.**	945	**13.**	2382	**15.**	5712	**17.**	672	**19.**	12,921

Which of these multiples of 2 are divisible by 3? by 6?

20.	84	**22.**	312	**24.**	972	**26.**	676	**28.**	888	**30.**	4326
21.	550	**23.**	484	**25.**	60	**27.**	782	**29.**	1206	**31.**	92,204

Complete the sentence.

32. If a number is divisible by 2 and by 3 then it will be divisible by ___?___.

Use the rule in exercise 32 to test these numbers for divisibility by 6.

33.	72	**35.**	112	**37.**	576	**39.**	351	**41.**	9786	**43.**	15,624
34.	93	**36.**	216	**38.**	852	**40.**	656	**42.**	1492	**44.**	21,332

1. List the multiples of 9, less than 100.
2. Name the sum of the digits for each multiple of 9 in exercise 1.
3. List the multiples of 9 between 99 and 200.
4. Name the sum of the digits for each multiple of 9 in exercise 3.

Solve these equations.

5. $648 \div 9 = a$
6. $117 \div 9 = d$
7. $999 \div 9 = g$
8. $2340 \div 9 = j$
9. $7875 \div 9 = m$
10. $4122 \div 9 = q$

$6 + 4 + 8 = b = c \times 9$

$1 + 1 + 7 = e = f \times 9$

$9 + 9 + 9 = h = i \times 9$

$2 + 3 + 4 + 0 = k = l \times 9$

$7 + 8 + 7 + 5 = n = p \times 9$

$4 + 1 + 2 + 2 = r = s \times 9$

Complete the sentence.

11. A number is divisible by 9 if the sum of its digits is divisible by ___?___.

555	96	175	396	300	144
6327	6372	6273	6732	7623	2367
1000	1100	1110	3456	9090	1245

From the list above, name the numbers divisible by

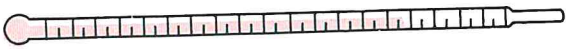
12. 2 **13.** 3 **14.** 4 **15.** 5 **16.** 6 **17.** 9 **18.** 10

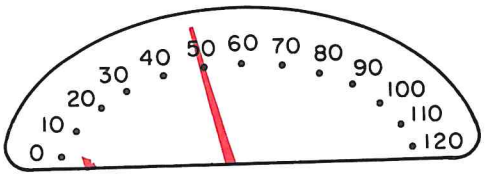
*19. Name the intersection of the set of multiples of 2 and the set of multiples of 3.

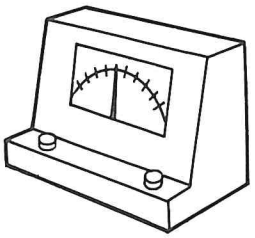
*20. Name the least number which is divisible by 2, 3, 4, 5, 6, and 9.

*21. Name the greatest number between 100 and 1000 divisible by 2, 3, 4, 5, 6, and 9.

Number lines







We use number lines to measure temperature, speed, time, and electric current.

EXERCISES

1. Name other measuring instruments that use number lines to show measures.
2. On a number line like the one at the top of the page the numbers go on without ending. Is the same true for the numbers on a clock? If not, can you say why?
3. On a number line like the one at the top of the page the greater numbers are on the right. Can you give a similar rule for the numbers on a clock?

Answer these questions.

4. What is the time 7 hours after 6 o'clock?
5. What is the time 9 hours after 8 o'clock?
6. What is the time 18 hours after 3 o'clock?
7. What is the time 25 hours after 2 o'clock?

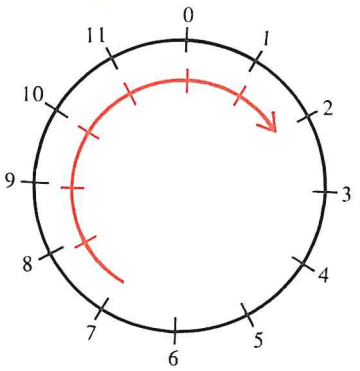
*8. Is the set of whole numbers on a number line finite or infinite?

*9. Is the set of whole numbers on a clock finite or infinite?

On this clock we see the twelve numerals 0–11. We shall use these numerals in a different number system. The arrow shows that 7 hours after 7 o'clock is 2 o'clock. We write

$$7 + 7 \overset{12}{=} 2$$

This is a **12-clock equation.**



EXERCISES

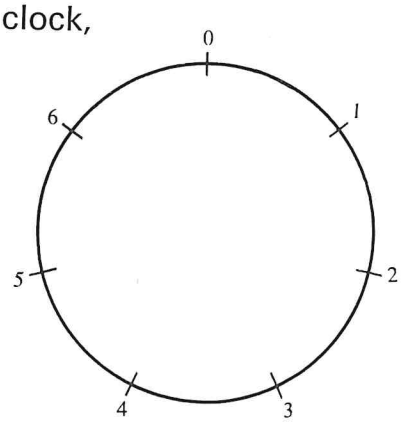
Complete the sentences and solve the 12-clock equations.

1. 5 hours after 2 o'clock is a o'clock. $2 + 5 \overset{12}{=} b$
2. 5 hours after 6 o'clock is c o'clock. $5 + 6 \overset{12}{=} d$
3. 5 hours after 8 o'clock is e o'clock. $5 + 8 \overset{12}{=} f$
4. 5 hours after 10 o'clock is g o'clock. $5 + 10 \overset{12}{=} h$

Solve the 12-clock equations.

5. $3 + 5 \overset{12}{=} a$
6. $6 + 6 \overset{12}{=} b$
7. $8 + 5 \overset{12}{=} c$
8. $9 + 6 \overset{12}{=} d$
9. $10 + 8 \overset{12}{=} e$
10. $8 + 8 \overset{12}{=} f$
11. $11 + 8 \overset{12}{=} g$
12. $11 + 11 \overset{12}{=} h$

Here is a clock with 7 numerals on it. Study the clock, then solve the 7-clock equations.

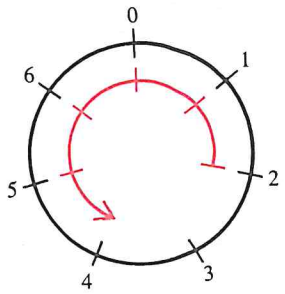


13. $4 + 2 \overset{7}{=} a$
14. $3 + 4 \overset{7}{=} b$
15. $5 + 4 \overset{7}{=} c$
16. $5 + 0 \overset{7}{=} d$
17. $5 + 5 \overset{7}{=} e$
18. $6 + 1 \overset{7}{=} f$
19. $6 + 2 \overset{7}{=} g$
20. $6 + 5 \overset{7}{=} h$
21. $6 + 6 \overset{7}{=} i$
22. $3 + 6 \overset{7}{=} j$

Solve the 7-clock equations.

23. $a + 2 \overset{7}{=} 5$
24. $b + 4 \overset{7}{=} 1$
25. $6 + c \overset{7}{=} 3$
26. $5 + d \overset{7}{=} 2$
27. $6 + e \overset{7}{=} 6$
28. $3 + f \overset{7}{=} 2$
29. $5 + g \overset{7}{=} 1$
30. $h + 4 \overset{7}{=} 3$

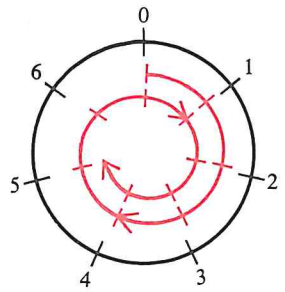
Clock arithmetic



On the 7-clock, 5 hours before 2 o'clock is 4 o'clock, so

$$2 - 5 \stackrel{7}{=} 4$$

We can think of subtraction on the clock as going the opposite way to addition.



On the 7-clock,

$$4 + 4 + 4 \stackrel{7}{=} 5$$
$$3 \times 4 \stackrel{7}{=} 5$$

We can think of multiplication on the clock as repeated addition.

EXERCISES

Use the 7-clock to help you solve the equations.

1. $5 - 3 \stackrel{7}{=} a$
2. $4 - 4 \stackrel{7}{=} b$
3. $4 - 5 \stackrel{7}{=} c$
4. $4 - 6 \stackrel{7}{=} d$
5. $2 - 5 \stackrel{7}{=} e$
6. $3 - 6 \stackrel{7}{=} f$
7. $1 - 4 \stackrel{7}{=} g$
8. $1 - 6 \stackrel{7}{=} h$
9. $2 \times 2 \stackrel{7}{=} i$
10. $4 \times 2 \stackrel{7}{=} j$
11. $3 \times 4 \stackrel{7}{=} k$
12. $4 \times 4 \stackrel{7}{=} l$
13. $2 \times 5 \stackrel{7}{=} m$
14. $2 \times 6 \stackrel{7}{=} n$
15. $4 \times 5 \stackrel{7}{=} p$
16. $5 \times 6 \stackrel{7}{=} q$

Copy and complete the addition and multiplication tables for the 7-clock.

17.

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	*e*	*c*	*a*	*n*
2	2	3	4	5	*e*	*r*	*s*
3	3	4	*i*	*u*	*x*	*a*	*m*
4	4	5	6	*m*	*n*	*v*	*e*
5	5	6	*q*	*r*	*s*	*i*	*u*
6	6	0	*v*	*w*	3	*x*	*z*

18.

×	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	*a*	*n*	*i*	*e*
3	0	3	*e*	*s*	5	*r*	*v*
4	0	4	*i*	5	*c*	*u*	*z*
5	0	5	*m*	*n*	*r*	*a*	*c*
6	0	6	*r*	*s*	*v*	*x*	*i*

Use the addition and multiplication tables on page 188 to help you complete these exercises for the 7-clock.

Solve the clock equations.

1. $3 + 2 \overset{7}{=} a$ **2.** $2 + 3 \overset{7}{=} b$ **7.** $6 + 5 \overset{7}{=} g$ **8.** $5 + 6 \overset{7}{=} h$

3. $4 + 3 \overset{7}{=} c$ **4.** $3 + 4 \overset{7}{=} d$ **9.** $4 + 1 \overset{7}{=} i$ **10.** $1 + 4 \overset{7}{=} j$

5. $3 + 5 \overset{7}{=} e$ **6.** $5 + 3 \overset{7}{=} f$ **11.** $6 + 2 \overset{7}{=} k$ **12.** $2 + 6 \overset{7}{=} l$

13. Do you think addition of 7-clock numbers is commutative?

Solve the equations.

14. $(2 + 1) + 3 \overset{7}{=} a$ **15.** $2 + (1 + 3) \overset{7}{=} b$

16. $(3 + 4) + 5 \overset{7}{=} c$ **17.** $3 + (4 + 5) \overset{7}{=} d$

18. $(4 + 2) + 3 \overset{7}{=} e$ **19.** $4 + (2 + 3) \overset{7}{=} f$

20. Do you think addition of 7-clock numbers is associative?

Solve the equations.

21. $2 \times 3 \overset{7}{=} a$ **22.** $3 \times 2 \overset{7}{=} b$ **27.** $4 \times 3 \overset{7}{=} g$ **28.** $3 \times 4 \overset{7}{=} h$

23. $4 \times 6 \overset{7}{=} c$ **24.** $6 \times 4 \overset{7}{=} d$ **29.** $5 \times 6 \overset{7}{=} i$ **30.** $6 \times 5 \overset{7}{=} j$

25. $2 \times 4 \overset{7}{=} e$ **26.** $4 \times 2 \overset{7}{=} f$ **31.** $5 \times 2 \overset{7}{=} k$ **32.** $2 \times 5 \overset{7}{=} l$

33. Do you think multiplication of 7-clock numbers is commutative?

Solve the equations.

34. $4 \times (3 \times 2) \overset{7}{=} a$ **35.** $(4 \times 3) \times 2 \overset{7}{=} b$

36. $5 \times (2 \times 6) \overset{7}{=} c$ **37.** $(5 \times 2) \times 6 \overset{7}{=} d$

38. $4 \times (6 \times 3) \overset{7}{=} e$ **39.** $(4 \times 6) \times 3 \overset{7}{=} f$

40. Do you think multiplication of 7-clock numbers is associative?

CHECKING UP

List the factors of each number.

1. 24
2. 36
3. 42
4. 75
5. 90
6. 150
7. 120
8. 200

Name the common factors and the GCF of each pair of numbers.

9. 20 and 30
10. 12 and 20
11. 24 and 64
12. 24 and 32
13. 16 and 48
14. 30 and 45

Name the multiples, less than 100, of

15. 8
16. 12
17. 20

Name the common multiples, less than 100, of each pair. Then name the LCM.

18. 10 and 15
19. 16 and 24
20. 9 and 36

Draw a factor tree for

21. 30
22. 54
23. 90

Name the prime factorization of

24. 30 **25.** 54 **26.** 90 **27.** 26 **28.** 84 **29.** 144

Name the next 10 members for each pattern.

30. 5, 6, 10, 11, 15, 16, 20, 21, 25, . . .
31. 7, 6, 5, 10, 9, 8, 13, 12, 11, . . .
32. 1, 2, 3, 4, 8, 7, 6, 5, 9, 10, 11, 12, . . .

Name the numbers in {100, 783, 512, 612, 135, 9846} which are divisible by

33. 2 **34.** 3 **35.** 4 **36.** 5 **37.** 6 **38.** 9

Solve the clock equations.

39. $4 + 8 \overset{12}{=} a$ **40.** $6 + 3 \overset{7}{=} b$ **41.** $2 \times 4 \overset{7}{=} c$ **42.** $2 - 9 \overset{12}{=} d$

A = {factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}
B = {factors of 36} = {1, 2, 3, 4, 6, 9, 12, 18, 36}
$A \cap B$ = {common factors of 24 and 36} = {1, 2, 3, 4, 6, 12}
The **greatest common factor** of 24 and 36 is 12.

List the members of the sets.

1. C = {factors of 48}
2. D = {factors of 60}
3. $C \cap D$
4. {common factors of 48 and 60}

Name the greatest common factor of each pair of numbers.

5. 48 and 60
6. 14 and 21
7. 90 and 75
8. 144 and 120

A = {multiples of 4} = {0, 4, 8, 12, 16, 20, 24, 28, 32, 36 . . .}
B = {multiples of 6} = {0, 6, 12, 18, 24, 30, 36 . . .}
The multiples of 4 (or 6) are named by multiplying 0, 1, 2, 3, 4, 5, 6, . . . by 4 (or 6).
$A \cap B$ = {common multiples of 4 and 6} = {0, 12, 24, 36, . . .}
The **least common multiple** of 4 and 6 is 12.

List the members of the sets.

9. A = {multiples of 8, less than 100}
10. B = {multiples of 12, less than 100}
11. $A \cap B$
12. {common multiples of 8 and 12, less than 100}

Name the least common multiple of each pair of numbers.

13. 8 and 12
14. 9 and 12
15. 7 and 4
16. 16 and 8

A prime number has only 2 factors, itself and 1. The **prime factorization** of 24 is $2 \times 2 \times 2 \times 3 = 2^3 \times 3$.

Name the prime factorization of each number

17. 8
18. 12
19. 20
20. 36
21. 90
22. 200

LOOKING WAY BACK

For the sets $R = \{1, 3, 6, 10, 15, 21, 28\}$ and $S = \{0, 3, 6, 9, 12, 15, 18\}$, name

1. $R \cup S$ **2.** $R \cap S$ **3.** $n(R)$ **4.** $n(S)$

List the members of these sets.

5. {factors of 48} **6.** {prime numbers less than 30}

Name the sums or differences.

7. $512 + 389$ **9.** $706 - 382$ **11.** $4395 + 5827$ **13.** $5321 - 2384$ **15.** $7004 - 4258$

8. $516 + 912$ **10.** $712 - 409$ **12.** $4906 + 2098$ **14.** $3001 - 1624$ **16.** $2783 + 2217$

Name the products.

17. 48×76 **19.** 453×91 **21.** 627×89 **23.** 703×24 **25.** 6734×58

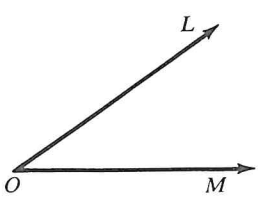
18. 59×23 **20.** 506×48 **22.** 237×54 **24.** 304×79 **26.** 836×90

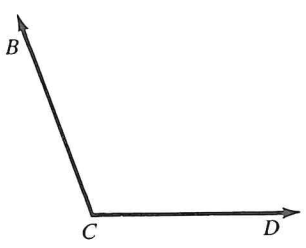
Name the quotients and remainders.

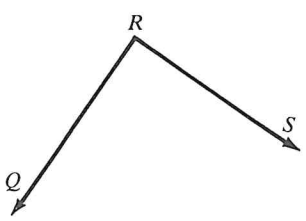
27. $7\overline{)448}$ **29.** $5\overline{)263}$ **31.** $16\overline{)384}$ **33.** $23\overline{)914}$

28. $9\overline{)3004}$ **30.** $6\overline{)2478}$ **32.** $17\overline{)504}$ **34.** $16\overline{)308}$

Name the measures of these angles. The unit is a degree.







35. $m(\angle LOM) = a$ **36.** $m(\angle BCD) = b$ **37.** $m(\angle QRS) = c$

1. Complete these magic squares. Remember: the sum of any 3 numbers in a line must be the same.

36	29	*a*
b	43	*c*
22	*d*	50

e	31	*f*
59	*g*	115
73	143	45

165	*h*	207
i	*j*	102
81	228	123

2. Draw a square and label it as we have done. Then answer the questions, writing your answers in the small squares. If your work is correct, you should finish with a magic square. Check your work by checking your magic square.

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

a. One more than a dozen?
b. Hours in a day?
c. Days in a week?
d. Quarts in a gallon?
e. $84 \div 12 = ?$
f. $51 \div 17 = ?$
g. $(4 \times 7) + 2 = ?$
h. $32 - 24 = ?$
i. $(9 \times 3) - 5 = ?$
j. $701 - 693 = ?$
k. $91 \div 13 = ?$
l. Quarters in \$2.75?
m. Tens' digit in 763?
n. $1000 - 987 = ?$
o. $100 \div 25 = ?$
p. $5 \times 5 = ?$

3. Play the game "Hit the Spot" with your neighbor. One of you names 6 numbers, the other gives rules to reach these numbers as sums, differences, products, or quotients. The first player then checks the second player's rules. For example, if A names 56, 83, 24, 6, 84, 100, B may give the rules:

7×8	$(7 \times 8 = 56)$
$+ 27$	$(56 + 27 = 83)$
$- 59$	$(83 - 59 = 24)$
$\div 4$	$(24 \div 4 = 6)$
$\times 14$	$(6 \times 14 = 84)$
$+ 16$	$(84 + 16 = 100)$

Subsets and supersets

Study the sets.

A = {Monday, Tuesday, Wednesday}
B = {Monday, Tuesday, Wednesday, Thursday, Friday}

A is a **subset** of B
B is a **superset** of A
} Every member of A is a member of B.

$\text{n}(A) = 3$
$\text{n}(B) = 5$

$\frac{\text{n}(A)}{\text{n}(B)} = \frac{3}{5}$

A is three-fifths of B.
A is $\frac{3}{5}$ of B.

> A **fractional number** may be used to compare a subset with a set.

EXERCISES

Name the fractional number which compares the subset with the set.

1. A = {one, two, six}
   B = {one, two, six, ten}

   $\frac{\text{n}(A)}{\text{n}(B)} = a$

2. E = {●●●●}
   F = {●●●●▲▲▲▲}

   $\frac{\text{n}(E)}{\text{n}(F)} = b$

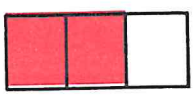
3. C = {to, at, in, on, of}
   D = {to, at, in, on, of, so}

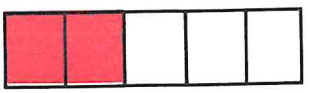
   $\frac{\text{n}(C)}{\text{n}(D)} = c$

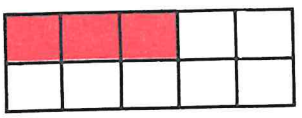
4. G = {do, re, mi, fa, so, la}
   H = {do, re, mi, fa, so, la, ti}

   $\frac{\text{n}(G)}{\text{n}(H)} = d$

Name the fractional number which compares the red region with the total region.

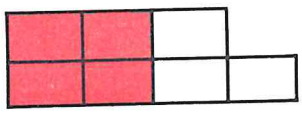
5. 

6. 

7. 

8.

9.

10. 

A = {June, July, May}

A numeral is a name for a number. One of the numerals for the **cardinal number** of A is 3.

A = {March, April, May}
B = {March, April, May, June, July}

One of the names for the **fractional number** which may be used to compare A with B is $\frac{3}{5}$.

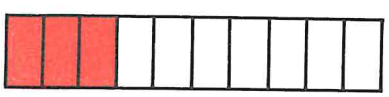
The numeral for a fractional number is a **fraction.**
$\frac{3}{5}$ is a fraction.

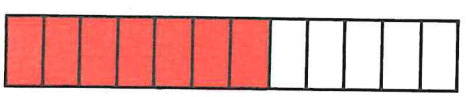
EXERCISES

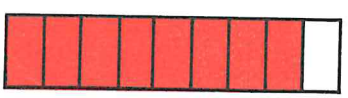
Write the fractions suggested by the sentences.

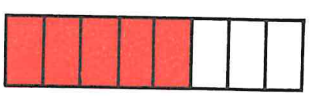
1. Five out of every 6 children at Sarah's school came to school by bus.
2. Three out of every 4 books sold in Mr. Blackstone's book store are paperbacks.
3. Seven out of every 9 puppies sold in Mr. Scott's pet shop are partly black.
4. Six out of every 13 children in the Ridge Street School are boys.
5. John solved 9 out of 10 problems in the test.
6. Kent made 4 out of every 11 shots at the basket.
7. Fred made 2 out of every 3 shots he took.
8. Ellen missed 1 out of every 6 jumps with her jump rope.

Write the fractions for the fractional numbers suggested by the rods.

9. 

10. 

11. 

12. 

The whole set

A = {Monday, Tuesday} $\quad \frac{n(A)}{n(C)} = \frac{2}{5}$

B = {Wednesday, Thursday, Friday} $\quad \frac{n(B)}{n(C)} = \frac{3}{5}$

C = {Monday, Tuesday, Wednesday, Thursday, Friday} $\quad \frac{n(A \cup B)}{n(C)} = \frac{5}{5}$

$$\frac{2+3}{5} = \frac{5}{5} = 1$$

EXERCISES

Study the sets, then solve the equations.

A = {Spelling, Reading}

B = {History, Science, Writing}

C = {Spelling, Reading, Writing, History, Science}

1. $\frac{n(A)}{n(C)} = a$ 2. $\frac{n(B)}{n(C)} = b$ 3. $\frac{n(A \cup B)}{n(C)} = c$

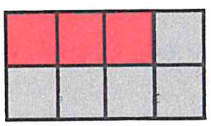
D = {Tom, Mary, Betty}

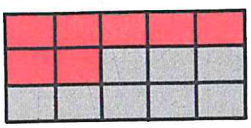
E = {Rita, John, Dick, Jim}

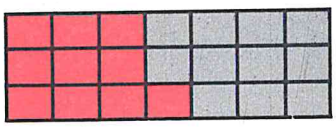
F = {Tom, Mary, Betty, Rita, John, Dick, Jim}

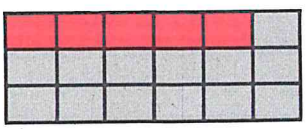
4. $\frac{n(D)}{n(F)} = d$ 5. $\frac{n(E)}{n(F)} = e$ 6. $\frac{n(D \cup E)}{n(F)} = f$

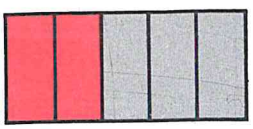
Name the fractional numbers for the red regions, the gray regions, and the red and gray regions together.

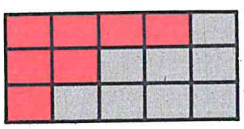
7. 

9. 

11. 

8. 

10. 

12. 

The two numbers in a fractional number

A = {Monday, Tuesday, Wednesday} — The cardinal number of A is 3.

B = {Monday, Tuesday, Wednesday, Thursday} — The cardinal number of B is 4.

$$\frac{n(A)}{n(B)} = \frac{3}{4} = 3 \times \frac{1}{4} = \text{three-fourths}$$

3 is the **numerator** of the fraction, and
4 is the **denominator** of the fraction.

EXERCISES

Name the denominators of the fractional numbers.

1. $\frac{6}{7}$ **2.** $\frac{3}{4}$ **3.** $\frac{2}{5}$ **4.** $\frac{7}{11}$ **5.** $\frac{9}{13}$

Name the numerators of the fractional numbers.

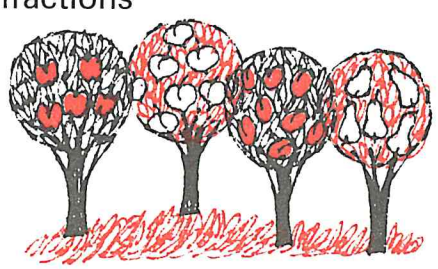
6. $\frac{5}{8}$ **7.** $\frac{2}{9}$ **8.** $\frac{1}{2}$ **9.** $\frac{5}{6}$ **10.** $\frac{11}{13}$

Complete the chart.

11.

numerator	3	5	6	8	9	10	11	12	13
denominator	4	6	7	9	10	11	12	13	14
fractional number	$\frac{3}{4}$	*s*	*n*	*c*	*e*	*a*	*x*	*i*	*v*

Name the numerators and denominators of the fractions suggested by the sentences.



12. Two out of every 7 trees were apple trees.

13. Three out of every 7 trees were peach trees.

14. Five out of every 9 trees were pear trees.

15. Four out of every 9 trees were plum trees.

Name the numerators and denominators of the fractions suggested by the sets.

16. {●●○○} **17.** {●●●●●○○} **18.** {●○○○○○}

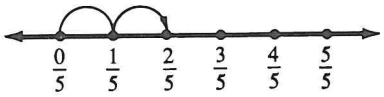
Joining subsets

{● ● ○ ○ ○}

$\frac{1}{5}$ of the circles are red.

$\frac{1}{5}$ of the circles are gray.

$\frac{2}{5}$ of the circles are red or gray.



The arrows show:

$\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

$$\frac{1}{5} + \frac{1}{5} = 2 \times \frac{1}{5} = \frac{2}{5}$$

EXERCISES

Use the sets or regions to help you solve the equations.

{● ● ○}

1. $\frac{1}{3} + \frac{1}{3} = \frac{a}{3}$

2. $2 \times \frac{1}{3} = \frac{n}{3}$

{● ● ○ ○ ○}

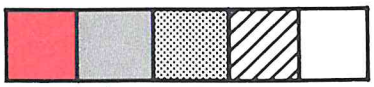
3. $\frac{1}{5} + \frac{1}{5} = \frac{c}{5}$

4. $2 \times \frac{1}{5} = \frac{s}{5}$

{● ● ○ ○}

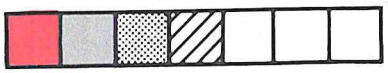
5. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{e}{5}$

6. $3 \times \frac{1}{5} = \frac{r}{5}$



7. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{a}{5}$

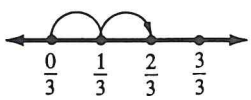
8. $4 \times \frac{1}{5} = \frac{e}{5}$



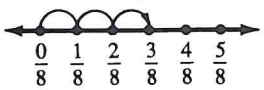
9. $\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{c}{7}$

10. $4 \times \frac{1}{7} = \frac{n}{7}$

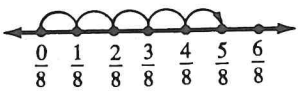
Use the number lines to help you solve the equations.



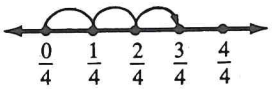
11. $2 \times \frac{1}{3} = \frac{a}{3}$



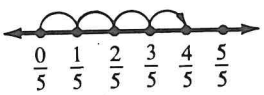
12. $3 \times \frac{1}{8} = \frac{n}{8}$



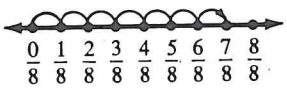
13. $5 \times \frac{1}{8} = \frac{e}{8}$



14. $3 \times \frac{1}{4} = \frac{x}{4}$

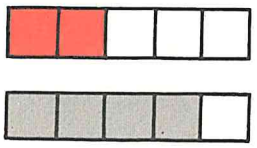


15. $4 \times \frac{1}{5} = \frac{c}{5}$



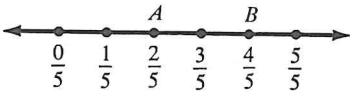
16. $7 \times \frac{1}{8} = \frac{r}{8}$

The fractional number line



$\frac{2}{5}$ of the top rod is red.

$\frac{4}{5}$ of the bottom rod is gray.



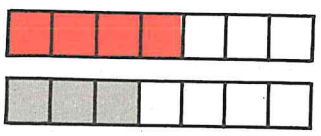
A is to the left of B.

B is to the right of A.

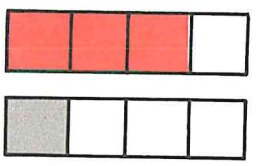
$\frac{2}{5} < \frac{4}{5}$ $\quad$ $\frac{4}{5} > \frac{2}{5}$

EXERCISES

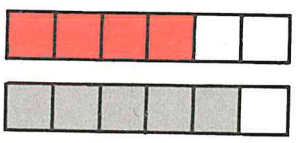
Use the rods to help you. Write $<$, $>$, or $=$ to replace each ◯.



1. $\frac{4}{7}$ ◯ $\frac{3}{7}$ $\quad$ $\frac{3}{7}$ ◯ $\frac{4}{7}$

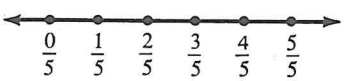


2. $\frac{3}{4}$ ◯ $\frac{1}{4}$ $\quad$ $\frac{1}{4}$ ◯ $\frac{3}{4}$

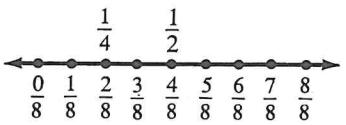


3. $\frac{4}{6}$ ◯ $\frac{5}{6}$ $\quad$ $\frac{2}{3}$ ◯ $\frac{4}{6}$

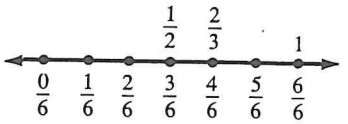
Use the fractional number lines to help you. Write $<$, $>$, or $=$ to replace each ◯.



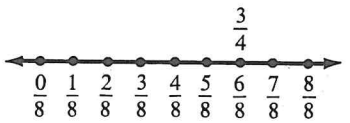
4. $\frac{4}{5}$ ◯ $\frac{3}{5}$ $\quad$ $\frac{3}{5}$ ◯ $\frac{4}{5}$



5. $\frac{3}{8}$ ◯ $\frac{5}{8}$ $\quad$ $\frac{1}{4}$ ◯ $\frac{3}{8}$



6. $\frac{2}{6}$ ◯ $\frac{5}{6}$ $\quad$ $\frac{1}{2}$ ◯ $\frac{2}{3}$

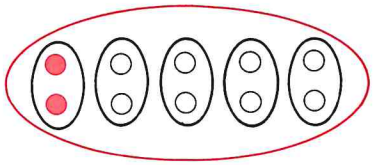


7. $\frac{3}{8}$ ◯ $\frac{1}{2}$ $\quad$ $\frac{7}{8}$ ◯ $\frac{5}{8}$

Complete the sentence.

8. If 2 fractional numbers have the same denominator, the one with the greater numerator is the ____?____ number.

Separating sets

$A =$ 

$\frac{1}{5}$ of the objects in set A are red.

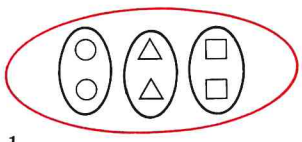
2 of the objects in set A are red.

A set of 10 may be separated to form 5 sets of **2**.

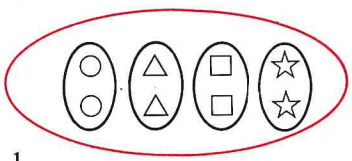
$\frac{1}{5}$ of 10 is **2** $\quad \frac{1}{5} \times 10 = \mathbf{2} \quad 10 \div 5 = \mathbf{2}$

EXERCISES

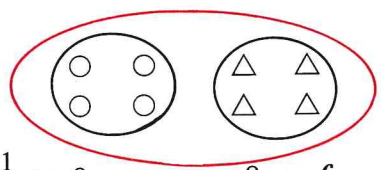
Use the sets to help you solve the equations.



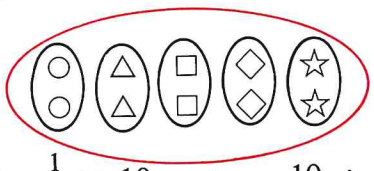
1. $\frac{1}{3} \times 6 = a \quad 6 \div b = 2$



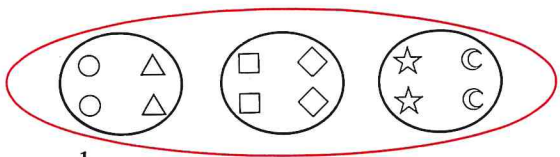
2. $\frac{1}{4} \times 8 = c \quad 8 \div d = 2$



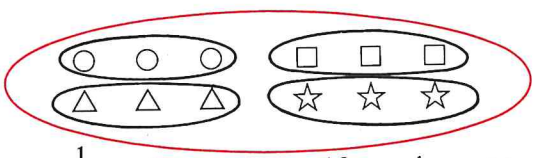
3. $\frac{1}{2} \times 8 = e \quad 8 \div f = 4$



4. $\frac{1}{5} \times 10 = g \quad 10 \div h = 2$

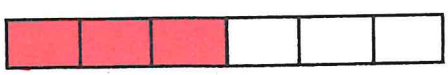


5. $\frac{1}{3} \times 12 = j \quad 12 \div k = 4$

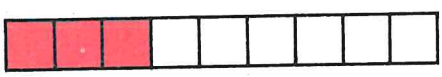


6. $\frac{1}{4} \times 12 = l \quad 12 \div 4 = m$

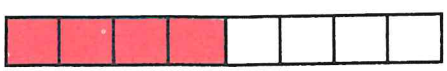
Use the rods to help you solve the equations.



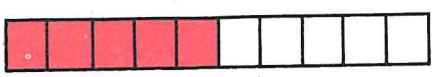
7. $\frac{1}{2} \times 6 = a \quad 6 \div b = 3$



9. $\frac{1}{3} \times 9 = e \quad 9 \div f = 3$



8. $\frac{1}{2} \times 8 = c \quad 8 \div 2 = d$



10. $\frac{1}{2} \times 10 = g \quad 10 \div h = 5$

To name $\frac{1}{5}$ of a set of 80, we divide 80 by 5.

$$\begin{array}{r} 16 \\ 5\overline{)80} \\ \underline{50} \\ 30 \\ \underline{30} \end{array}$$

$\frac{1}{5}$ of 80 is **16**

$\frac{1}{5} \times 80 = \mathbf{16}$

$80 \div 5 = \mathbf{16}$

EXERCISES

Copy and complete the exercises.

1.
$$\begin{array}{r} 5 \\ 4\overline{)20} \\ \underline{20} \end{array}$$

$\frac{1}{4} \times 20 = a$

$20 \div b = 5$

2.
$$\begin{array}{r} 6 \\ 5\overline{)30} \\ \underline{30} \end{array}$$

$\frac{1}{5} \times 30 = c$

$30 \div d = 6$

3.
$$\begin{array}{r} 15 \\ 3\overline{)45} \\ \underline{30} \\ 15 \\ \underline{15} \end{array}$$

$\frac{1}{3}$ of $45 = f$

$\frac{1}{3} \times g = h$

$45 \div i = 15$

4.
$$\begin{array}{r} 12 \\ 4\overline{)48} \\ \underline{40} \\ 8 \\ \underline{8} \end{array}$$

$\frac{1}{4}$ of $48 = j$

$\frac{1}{4} \times k = l$

$48 \div m = 12$

Solve the equations.

5. $60 \div 3 = a$

6. $\frac{1}{3} \times 60 = b$

7. $72 \div 6 = c$

8. $\frac{1}{6} \times 72 = d$

9. $80 \div 4 = e$

10. $\frac{1}{4} \times 80 = f$

11. $93 \div 3 = g$

12. $\frac{1}{3} \times 93 = h$

Solve the equations.

13. $\frac{1}{3} \times 30 = 10$

$\frac{1}{3} \times 6 = a$

$\frac{1}{3} \times 36 = b$

14. $\frac{1}{5} \times 50 = 10$

$\frac{1}{5} \times 5 = c$

$\frac{1}{5} \times 55 = d$

15. $\frac{1}{4} \times 40 = 10$

$\frac{1}{4} \times 8 = e$

$\frac{1}{4} \times 48 = f$

16. $\frac{1}{6} \times 54 = 9$

$\frac{1}{6} \times 12 = g$

$\frac{1}{6} \times 66 = h$

17. $18 \div 3 = i$

$27 \div 3 = j$

$45 \div 3 = k$

18. $16 \div 4 = l$

$28 \div 4 = m$

$44 \div 4 = n$

19. $48 \div 6 = a$

$24 \div 6 = p$

$72 \div 6 = q$

20. $25 \div 5 = r$

$35 \div 5 = s$

$60 \div 5 = t$

21. $\frac{1}{4} \times 60 = a$

22. $\frac{1}{3} \times 60 = b$

23. $\frac{1}{2} \times 30 = c$

24. $\frac{1}{5} \times 65 = d$

Separating subsets

Study the subsets.

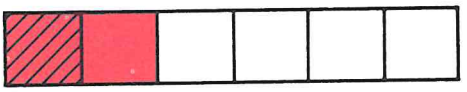
{ ● ○ △ △ △ △}

$\frac{1}{3}$ of the objects are circles.

$\frac{1}{2}$ of the circles are red.

$\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$ of the objects are red circles.

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ $\quad \frac{1}{3} \div 2 = \frac{1}{6}$



$\frac{1}{3}$ of the rod is red.

$\frac{1}{2}$ of the red part is shaded.

$\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$ of the rod is red and shaded.

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ $\quad \frac{1}{3} \div 2 = \frac{1}{6}$

EXERCISES

Study the subsets, then solve the equations.

{ ● ○ △ △}

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{a}$

   $\frac{1}{2} \div b = \frac{1}{4}$

{ ● ○ ○ △ △ / ○ ○ △ △ △}

3. $\frac{1}{5} \times \frac{1}{2} = \frac{1}{e}$

   $\frac{1}{2} \div f = \frac{1}{10}$

{ ● △ □ / ○ △ □ / ○ △ □}

5. $\frac{1}{3} \times \frac{1}{3} = \frac{1}{s}$

   $\frac{1}{3} \div j = \frac{1}{9}$

{ ● ○ △ △ / ○ ○ △ △}

2. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{c}$

   $\frac{1}{2} \div d = \frac{1}{8}$

{ ● ○ ○ △ △ △}

4. $\frac{1}{3} \times \frac{1}{2} = \frac{1}{i}$

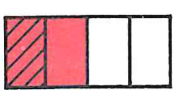
   $\frac{1}{2} \div h = \frac{1}{6}$

{ ● △ □ ☆ / ○ △ □ ☆ / ○ △ □ ☆}

6. $\frac{1}{3} \times \frac{1}{4} = \frac{1}{r}$

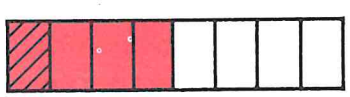
   $\frac{1}{4} \div l = \frac{1}{12}$

Study the rods, then solve the equations.



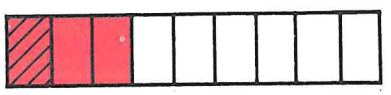
7. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{a}$

   $\frac{1}{2} \div b = \frac{1}{4}$



8. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{c}$

   $\frac{1}{2} \div d = \frac{1}{8}$



9. $\frac{1}{3} \times \frac{1}{3} = \frac{1}{e}$

   $\frac{1}{3} \div f = \frac{1}{9}$

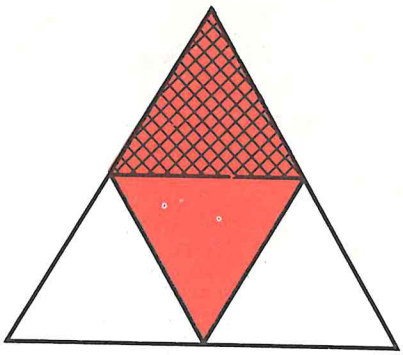
Study the region.

$\frac{1}{2}$ of the region is red.

$\frac{1}{2}$ of the red region is shaded.

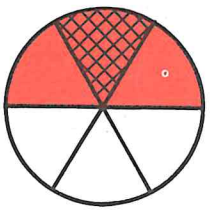
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of the region is red and shaded.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\quad \frac{1}{2} \div 2 = \frac{1}{4}$



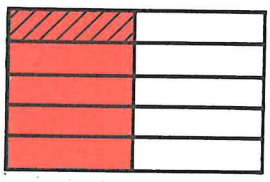
EXERCISES

Study the regions, then solve the equations.



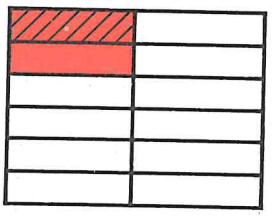
1. $\frac{1}{3} \times \frac{1}{2} = \frac{1}{a}$

   $\frac{1}{2} \div b = \frac{1}{6}$



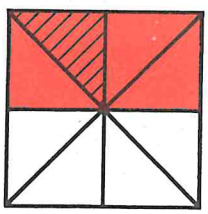
3. $\frac{1}{5} \times \frac{1}{2} = \frac{1}{e}$

   $\frac{1}{2} \div f = \frac{1}{10}$



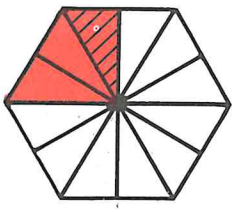
5. $\frac{1}{2} \times \frac{1}{6} = \frac{1}{x}$

   $\frac{1}{6} \div j = \frac{1}{12}$



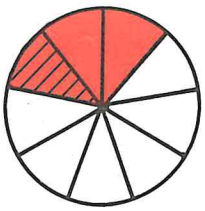
2. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{c}$

   $\frac{1}{2} \div d = \frac{1}{8}$



4. $\frac{1}{3} \times \frac{1}{4} = \frac{1}{i}$

   $\frac{1}{4} \div h = \frac{1}{12}$



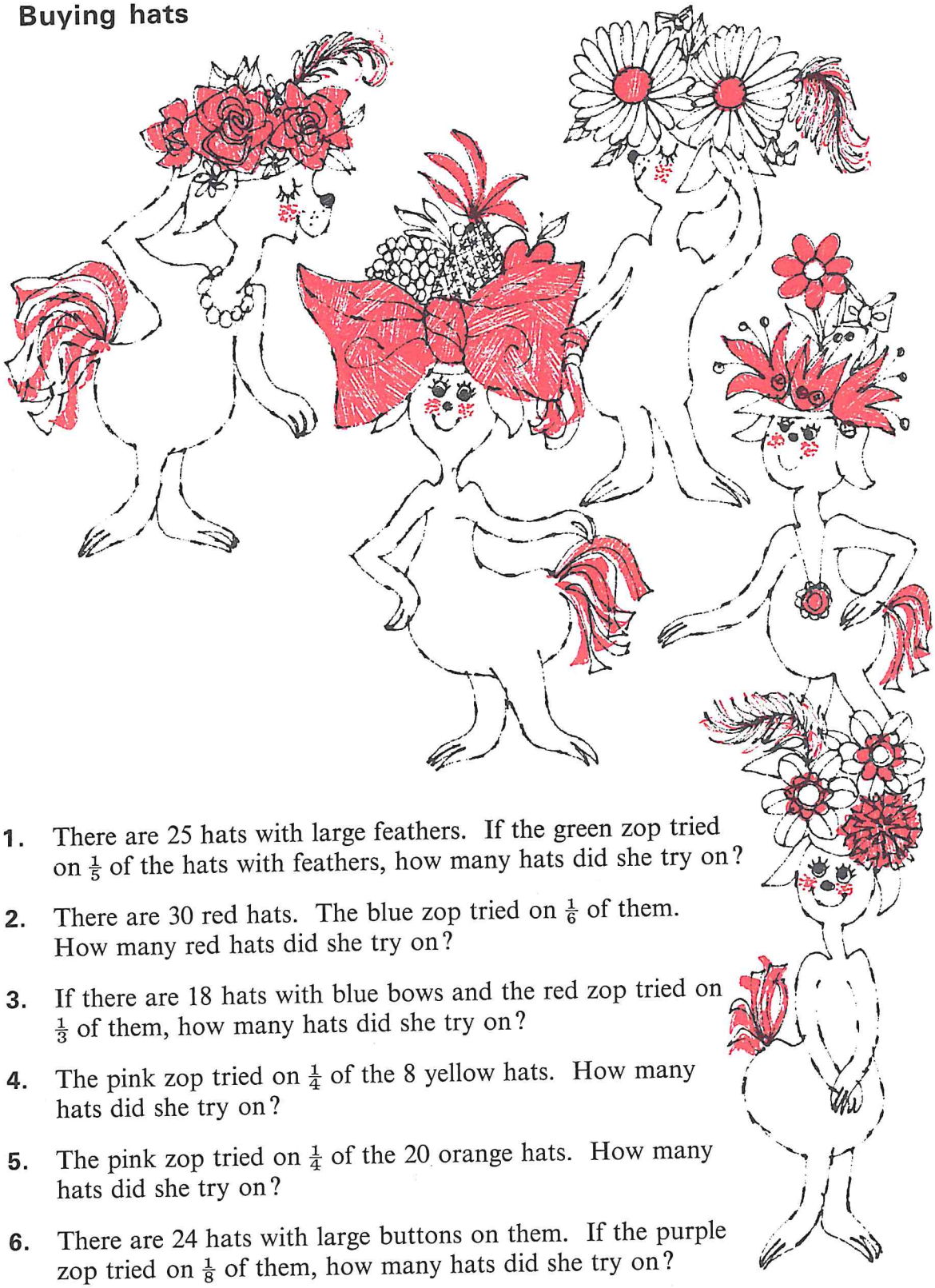
6. $\frac{1}{3} \times \frac{1}{3} = \frac{1}{u}$

   $\frac{1}{3} \div l = \frac{1}{9}$

Solve the problem.

7. One half of a flower bed is planted with roses. One half of the roses are yellow. How much of the flower bed is planted with yellow roses?

Buying hats



1. There are 25 hats with large feathers. If the green zop tried on $\frac{1}{5}$ of the hats with feathers, how many hats did she try on?

2. There are 30 red hats. The blue zop tried on $\frac{1}{6}$ of them. How many red hats did she try on?

3. If there are 18 hats with blue bows and the red zop tried on $\frac{1}{3}$ of them, how many hats did she try on?

4. The pink zop tried on $\frac{1}{4}$ of the 8 yellow hats. How many hats did she try on?

5. The pink zop tried on $\frac{1}{4}$ of the 20 orange hats. How many hats did she try on?

6. There are 24 hats with large buttons on them. If the purple zop tried on $\frac{1}{8}$ of them, how many hats did she try on?

Name the sums to complete the number charts.

1.

+	34	68	29	54	73
28	62	*a*	*i*	*c*	*n*
69	103	*e*	*x*	*r*	*s*
57	91	*i*	*u*	*v*	*z*

2.

+	81	63	54	76	39
41	122	*m*	*n*	*a*	*i*
68	149	*c*	*r*	*s*	*e*
39	120	*u*	*v*	*w*	*x*

Name the products to complete the number charts.

3.

×	38	27	47	29	86
3	114	*x*	*n*	*a*	*c*
7	266	*e*	*r*	*u*	*m*
9	342	*i*	*v*	*s*	*w*

4.

×	54	73	66	85	49
8	432	*m*	*n*	*z*	*a*
6	324	*u*	*v*	*c*	*e*
5	270	*x*	*r*	*s*	*i*

Name the products.

5. $36 \times 476 = a$

6. $29 \times 891 = b$

7. $49 \times 286 = c$

8. $39 \times 205 = d$

9. $48 \times 921 = e$

10. $54 \times 725 = f$

11. $68 \times 739 = g$

12. $47 \times 495 = h$

13. $34 \times 849 = i$

Name the quotients and remainders.

14. $29\overline{)358}$

15. $17\overline{)426}$

16. $23\overline{)498}$

17. $37\overline{)2736}$

18. $28\overline{)7464}$

19. $39\overline{)8292}$

Name each number as the product of its prime factors.

20. 54 **21.** 30 **22.** 60 **23.** 45 **24.** 75

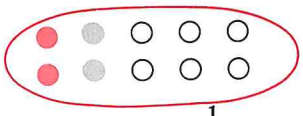
*Use the factors from exercises 20–24 to name the greatest common factor of

25. 54 and 75 **26.** 30 and 54 **27.** 60 and 45

*Use the factors from exercises 20–24 to name the least common multiple of

28. 54 and 60 **29.** 30 and 75 **30.** 45 and 75

Numerators greater than 1



$\frac{1}{5}$ of the circles are red.

$\frac{1}{5}$ of the circles are gray.

$\frac{1}{5} + \frac{1}{5} = 2 \times \frac{1}{5} = \frac{2}{5}$

$\frac{2}{5}$ of the circles are red or gray.

$\frac{1}{5}$ of a set of 10 is 2.

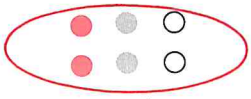
$\frac{1}{5} \times 10 = 2$

$\frac{2}{5}$ of a set of 10 is 4.

$\frac{2}{5} \times 10 = 4$

EXERCISES

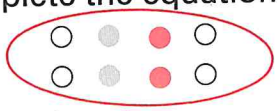
Study the sets, then complete the equations.



1. $\frac{1}{3} + \frac{1}{3} = \frac{a}{3}$

$\frac{2}{3}$ of $6 = b$

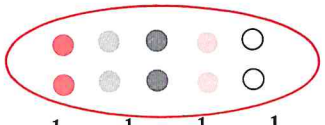
$\frac{2}{3} \times c = 4$



3. $\frac{1}{4} + \frac{1}{4} = \frac{x}{2}$

$\frac{1}{2}$ of $8 = h$

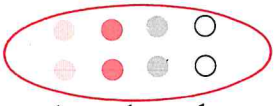
$\frac{1}{2} \times i = 4$



5. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{n}{5}$

$\frac{4}{5}$ of $10 = m$

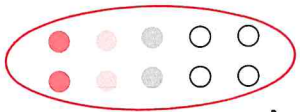
$\frac{4}{5} \times u = 8$



2. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{e}{4}$

$\frac{3}{4}$ of $8 = d$

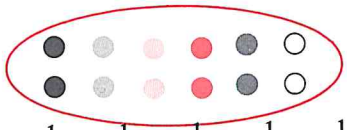
$\frac{3}{4} \times f = 6$



4. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{j}{5}$

$\frac{3}{5}$ of $10 = k$

$\frac{3}{5} \times l = 6$

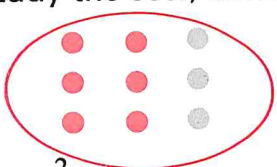


6. $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{v}{6}$

$\frac{5}{6}$ of $12 = q$

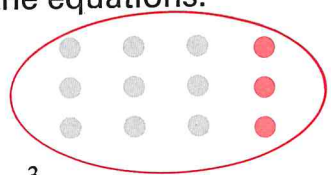
$\frac{5}{6} \times r = 10$

Study the sets, then complete the equations.



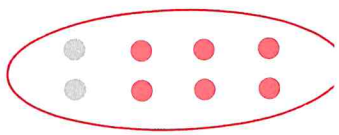
7. $\frac{2}{3}$ of $9 = a$

$\frac{2}{3} \times b = 6$



8. $\frac{3}{4}$ of $12 = c$

$\frac{3}{4} \times d = 9$



9. $\frac{3}{4}$ of $8 = e$

$\frac{3}{4} \times f = 6$

Numerators greater than 1

Study the equations.

$$\begin{aligned}\frac{2}{5} \times 10 &= \left(2 \times \frac{1}{5}\right) \times 10 \\ &= 2 \times \left(\frac{1}{5} \times 10\right) \\ &= 2 \times 2 \\ &= 4\end{aligned}$$

In the first two lines we use the associative property to regroup the factors.

EXERCISES

Copy and complete the equations.

1. $$\begin{aligned}\frac{2}{3} \times 9 &= \left(2 \times \frac{1}{3}\right) \times 9 \\ &= 2 \times \left(\frac{1}{3} \times 9\right) \\ &= 2 \times a \\ &= b\end{aligned}$$

2. $$\begin{aligned}\frac{3}{4} \times 12 &= \left(c \times \frac{1}{4}\right) \times 12 \\ &= 3 \times \left(\frac{1}{4} \times 12\right) \\ &= 3 \times d \\ &= e\end{aligned}$$

3. $$\begin{aligned}\frac{3}{5} \times 10 &= (3 \times f) \times 10 \\ &= 3 \times \left(\frac{1}{5} \times 10\right) \\ &= 3 \times g \\ &= h\end{aligned}$$

4. $$\begin{aligned}\frac{4}{7} \times 14 &= (4 \times i) \times 14 \\ &= 4 \times \left(\frac{1}{7} \times 14\right) \\ &= 4 \times j \\ &= k\end{aligned}$$

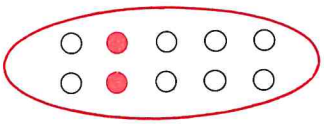
5. $$\begin{aligned}\frac{2}{5} \times 15 &= \left(l \times \frac{1}{5}\right) \times 15 \\ &= l \times \left(\frac{1}{5} \times 15\right) \\ &= l \times m \\ &= n\end{aligned}$$

6. $$\begin{aligned}\frac{2}{3} \times 12 &= (2 \times p) \times 12 \\ &= 2 \times \left(\frac{1}{3} \times 12\right) \\ &= 2 \times q \\ &= r\end{aligned}$$

Solve the equations.

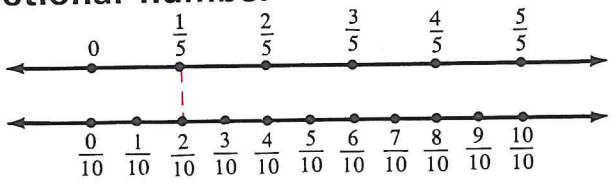
7. $\frac{2}{5} \times 10 = a$
8. $\frac{2}{3} \times 6 = b$
9. $\frac{3}{4} \times 8 = c$
10. $\frac{4}{5} \times 10 = d$
11. $\frac{3}{7} \times 14 = e$
12. $\frac{3}{5} \times 20 = f$

Two names for the same fractional number



$\frac{1}{5}$ of the circles are red.

$\frac{2}{10}$ of the circles are red.

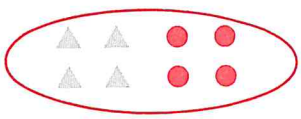


$\frac{1}{5}$ and $\frac{2}{10}$ label the same point.

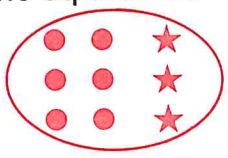
$\frac{1}{5}$ and $\frac{2}{10}$ name the same fractional number.

EXERCISES

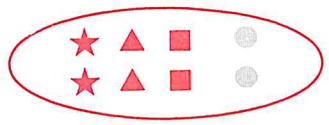
Study the sets, then solve the equations.



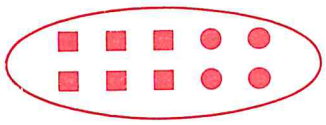
1. $\frac{1}{2} = \frac{a}{8}$



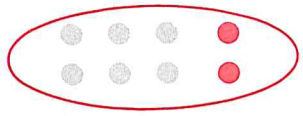
3. $\frac{2}{3} = \frac{c}{9}$



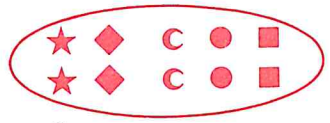
5. $\frac{3}{4} = \frac{e}{8}$



2. $\frac{3}{5} = \frac{n}{10}$

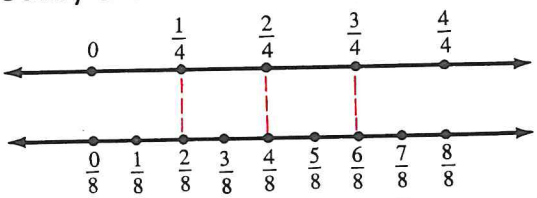


4. $\frac{1}{4} = \frac{x}{8}$

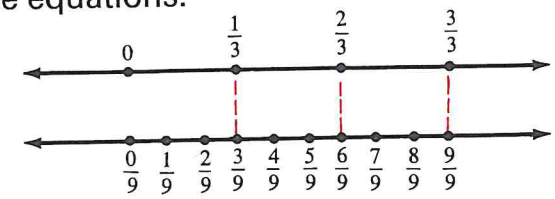


6. $\frac{2}{5} = \frac{v}{10}$

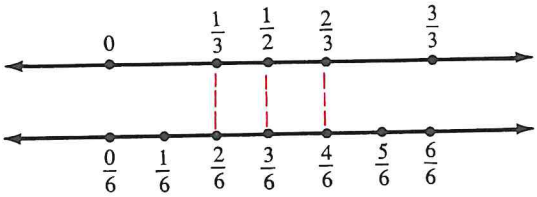
Study the number lines, then solve the equations.



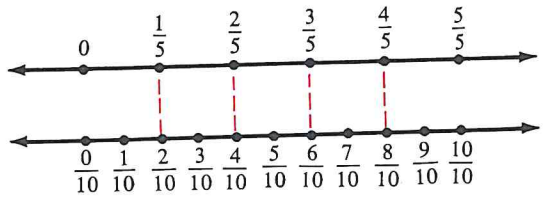
7. $\frac{1}{4} = \frac{a}{8}$ $\frac{1}{2} = \frac{n}{4} = \frac{c}{8}$ $\frac{3}{4} = \frac{s}{8}$



9. $\frac{1}{3} = \frac{u}{9}$ $\frac{2}{3} = \frac{i}{9}$ $1 = \frac{a}{3} = \frac{e}{9}$



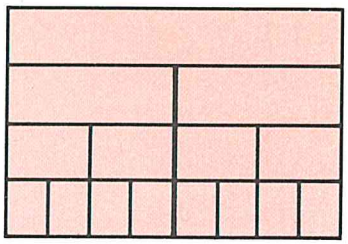
8. $\frac{1}{3} = \frac{e}{6}$ $\frac{1}{2} = \frac{r}{6}$ $\frac{2}{3} = \frac{x}{6}$



10. $\frac{1}{5} = \frac{v}{10}$ $\frac{2}{5} = \frac{z}{10}$ $\frac{3}{5} = \frac{n}{10}$ $\frac{4}{5} = \frac{c}{10}$

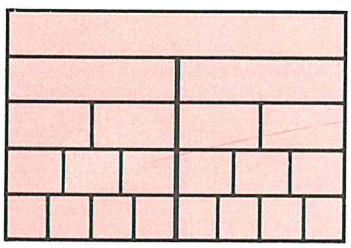
The diagram shows: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

The fractions $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}$ are names for the same fractional number. They are **equivalent fractions.** The simplest name for the number is $\frac{1}{2}$ (one half).

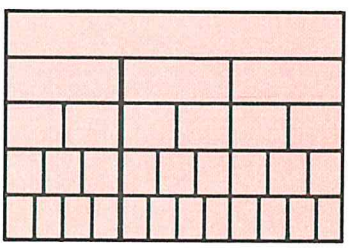


EXERCISES

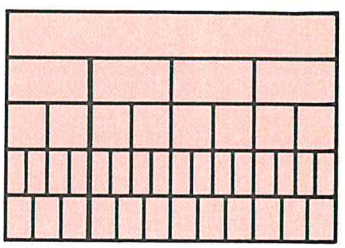
Use the diagrams to help you complete the sets.



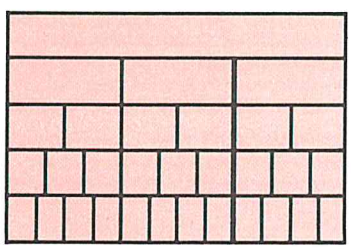
1. $\left\{\frac{1}{2}, \frac{a}{4}, \frac{n}{6}, \frac{c}{8}\right\}$



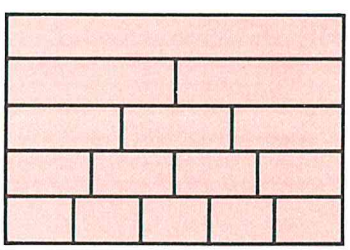
2. $\left\{\frac{1}{3}, \frac{x}{6}, \frac{e}{9}, \frac{a}{12}\right\}$



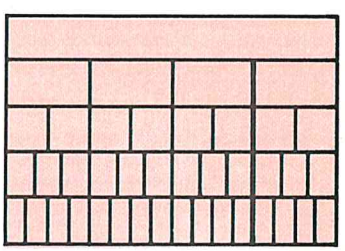
3. $\left\{\frac{1}{4}, \frac{s}{8}, \frac{v}{12}, \frac{i}{16}\right\}$



4. $\left\{\frac{2}{3}, \frac{r}{6}, \frac{z}{9}, \frac{c}{12}\right\}$



5. $\left\{1, \frac{2}{2}, \frac{x}{3}, \frac{n}{4}, \frac{i}{5}\right\}$



6. $\left\{\frac{3}{4}, \frac{u}{8}, \frac{a}{12}, \frac{v}{16}\right\}$

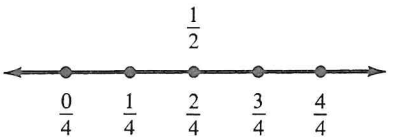
Solve the problem.

7. From their school, Sam walked $\frac{1}{2}$ the distance to the roller skating rink, Frank walked $\frac{2}{4}$ of the distance, and Larry walked $\frac{3}{6}$ of the distance. Did they all walk the same distance?

Renaming fractional numbers

Study the equations and the number line.

$$\frac{1}{2} = 1 \times \frac{1}{2} = \frac{2}{2} \times \frac{1}{2}$$
$$= \frac{2 \times 1}{2 \times 2}$$
$$= \frac{2}{4}$$



$\frac{1}{2}$ and $\frac{2}{4}$ label the same point. $\frac{1}{2} = \frac{2}{4}$

EXERCISES

Copy and complete the equations.

1. $\frac{2 \times 2}{3 \times 2} = \frac{a}{6}$
2. $\frac{1 \times 2}{3 \times 2} = \frac{n}{6}$
3. $\frac{1 \times 3}{4 \times 3} = \frac{c}{12}$
4. $\frac{3 \times 3}{5 \times 3} = \frac{x}{15}$
5. $\frac{1 \times 4}{2 \times 4} = \frac{e}{8}$
6. $\frac{3 \times 2}{4 \times 2} = \frac{s}{8}$
7. $\frac{3 \times 3}{4 \times 3} = \frac{u}{12}$
8. $\frac{2 \times 4}{3 \times 4} = \frac{z}{12}$
9. $\frac{1 \times 2}{5 \times 2} = \frac{i}{10}$
10. $\frac{1 \times 5}{3 \times 5} = \frac{r}{15}$
11. $\frac{2 \times 3}{5 \times 3} = \frac{v}{15}$
12. $\frac{2 \times 2}{7 \times 2} = \frac{e}{14}$

Copy and complete the equations.

13. $\frac{1 \times a}{3 \times a} = \frac{3}{9}$
14. $\frac{1 \times n}{4 \times n} = \frac{2}{8}$
15. $\frac{1 \times u}{2 \times u} = \frac{5}{10}$
16. $\frac{1 \times e}{2 \times e} = \frac{7}{14}$
17. $\frac{3 \times i}{4 \times i} = \frac{6}{8}$
18. $\frac{5 \times c}{6 \times c} = \frac{10}{12}$

Copy and complete the equations.

19. $\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = a$
20. $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = b$
21. $\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = c$
22. $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = d$
23. $\frac{1}{5} = \frac{1 \times 4}{5 \times 4} = e$
24. $\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = f$

Complete the sentence.

25. To rename a fractional number so that it has a greater denominator, we ___?___ both numerator and denominator by the same factor.

Renaming fractional numbers

Study these equations and the reasons for each step.

$$\frac{1}{3} = 1 \times \frac{1}{3}$$ *Step 1:* When 1 is a factor the product is equal to the other factor.

$$= \frac{2}{2} \times \frac{1}{3}$$ *Step 2:* A name for 1 is $\frac{2}{2}$.

$$= \left(2 \times \frac{1}{2}\right) \times \frac{1}{3}$$ *Step 3:* $\frac{2}{2} = 2 \times \frac{1}{2}$

$$= 2 \times \left(\frac{1}{2} \times \frac{1}{3}\right)$$ *Step 4:* The associative property

$$= 2 \times \frac{1}{6}$$ *Step 5:* $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$$= \frac{2}{6}$$ *Step 6:* The simplest name for $2 \times \frac{1}{6}$ is $\frac{2}{6}$.

EXERCISES

Copy and complete the equations.

1. $\frac{1}{2} = 1 \times \frac{1}{2}$
$= \frac{3}{3} \times \frac{1}{2}$
$= a$

2. $\frac{1}{4} = 1 \times \frac{1}{4}$
$= \frac{2}{2} \times \frac{1}{4}$
$= b$

3. $\frac{2}{3} = 1 \times \frac{2}{3}$
$= \frac{3}{3} \times \frac{2}{3}$
$= c$

4. $\frac{2}{5} = 1 \times \frac{2}{5}$
$= \frac{2}{2} \times \frac{2}{5}$
$= d$

5. $\frac{3}{4} = 1 \times \frac{3}{4}$
$= \frac{2}{2} \times \frac{3}{4}$
$= e$

6. $\frac{3}{5} = 1 \times \frac{3}{5}$
$= \frac{4}{4} \times \frac{3}{5}$
$= f$

7. $\frac{5}{6} = 1 \times \frac{5}{6}$
$= \frac{2}{2} \times \frac{5}{6}$
$= g$

8. $\frac{1}{2} = 1 \times \frac{1}{2}$
$= \frac{5}{5} \times \frac{1}{2}$
$= h$

Use equations to show the renaming of

9. $\frac{1}{2}$ as $\frac{5}{10}$

10. $\frac{1}{4}$ as $\frac{2}{8}$

11. $\frac{1}{3}$ as $\frac{3}{9}$

12. $\frac{2}{3}$ as $\frac{4}{6}$

13. $\frac{3}{4}$ as $\frac{6}{8}$

14. $\frac{2}{5}$ as $\frac{4}{10}$

15. $\frac{3}{4}$ as $\frac{9}{12}$

16. $\frac{3}{7}$ as $\frac{6}{14}$

Complete the sets of fractions. Each set contains different names for the same fractional number.

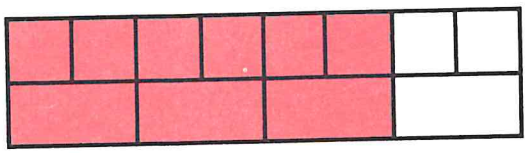
17. $\left\{\frac{1}{2}, \frac{5}{10}, \frac{6}{a}, \frac{9}{n}, \frac{12}{c}, \frac{x}{30}, \frac{e}{40}\right\}$

18. $\left\{\frac{3}{4}, \frac{6}{8}, \frac{15}{v}, \frac{18}{a}, \frac{u}{32}, \frac{n}{40}, \frac{x}{400}\right\}$

Renaming fractional numbers

Study the equations and the colored rods.

$$\frac{6}{8} = \frac{6}{8} \div 1 = \frac{6}{8} \div \frac{2}{2}$$
$$= \frac{6 \div 2}{8 \div 2}$$
$$= \frac{3}{4}$$



$$\frac{6}{8} = \frac{3}{4}$$

EXERCISES

Copy and complete the equations.

1. $\frac{6 \div 2}{8 \div 2} = \frac{n}{4}$
2. $\frac{4 \div 2}{6 \div 2} = \frac{x}{3}$
3. $\frac{2 \div 2}{10 \div 2} = \frac{c}{5}$
4. $\frac{3 \div 3}{12 \div 3} = \frac{z}{4}$
5. $\frac{4 \div 4}{8 \div 4} = \frac{1}{e}$
6. $\frac{6 \div 2}{10 \div 2} = \frac{3}{s}$
7. $\frac{9 \div 3}{12 \div 3} = \frac{3}{i}$
8. $\frac{8 \div 4}{12 \div 4} = \frac{u}{3}$
9. $\frac{8 \div 2}{10 \div 2} = \frac{4}{t}$
10. $\frac{10 \div 5}{15 \div 5} = \frac{a}{3}$
11. $\frac{6 \div 6}{12 \div 6} = \frac{1}{r}$
12. $\frac{20 \div 5}{25 \div 5} = \frac{e}{5}$

Copy and complete the equations.

13. $\frac{3 \div a}{9 \div a} = \frac{1}{3}$
14. $\frac{2 \div v}{8 \div v} = \frac{1}{4}$
15. $\frac{3 \div e}{6 \div e} = \frac{1}{2}$
16. $\frac{14 \div i}{16 \div i} = \frac{7}{8}$
17. $\frac{7 \div r}{14 \div r} = \frac{1}{2}$
18. $\frac{12 \div n}{18 \div n} = \frac{2}{3}$
19. $\frac{24 \div s}{30 \div s} = \frac{4}{5}$
20. $\frac{32 \div x}{40 \div x} = \frac{4}{5}$

Copy and complete the equations.

21. $\frac{6}{10} = \frac{a \times 2}{n \times 2} = \frac{a}{n}$
22. $\frac{10}{16} = \frac{c \times 2}{s \times 2} = \frac{c}{s}$
23. $\frac{12}{20} = \frac{e \times 4}{i \times 4} = \frac{e}{i}$
24. $\frac{15}{20} = \frac{u \times 5}{r \times 5} = \frac{u}{r}$
25. $\frac{4}{10} = \frac{i \times 2}{v \times 2} = \frac{i}{v}$
26. $\frac{12}{16} = \frac{z \times 4}{c \times 4} = \frac{z}{c}$

Complete the sentence.

27. To rename a fractional number so that it has a lesser denominator we ___?___ both numerator and denominator by the same factor.

$$A = \left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \ldots\right\} = \{\text{names for } 1\}$$

$$B = \left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \ldots\right\} = \left\{\text{names for } \frac{1}{2}\right\}$$

The members of B are named by multiplying the matching member of A by $\frac{1}{2}$.

$$\frac{1}{2} = 1 \times \frac{1}{2} = \frac{3}{3} \times \frac{1}{2} = \frac{3}{6}$$

EXERCISES

Use set A above to solve the equations.

1. $\frac{1}{5} = \frac{a}{15}$
2. $\frac{2}{5} = \frac{u}{25}$
3. $\frac{3}{5} = \frac{c}{30}$
4. $\frac{4}{5} = \frac{n}{40}$
5. $\frac{1}{4} = \frac{e}{12}$
6. $\frac{1}{4} = \frac{x}{32}$
7. $\frac{3}{4} = \frac{z}{40}$
8. $\frac{3}{4} = \frac{r}{400}$
9. $\frac{3}{7} = \frac{6}{i}$
10. $\frac{3}{7} = \frac{9}{z}$
11. $\frac{3}{7} = \frac{s}{35}$
12. $\frac{4}{7} = \frac{v}{56}$

To rename $\frac{6}{12}$ as $\frac{1}{2}$ we write 6 as 1×6 and 12 as 2×6.

Then we divide $\frac{6}{12}$ by 1 in the form $\frac{6}{6}$.

$$\frac{6}{12} = \frac{1 \times 6}{2 \times 6} = \frac{(1 \times 6) \div 6}{(2 \times 6) \div 6} = \frac{1}{2}$$

Solve the equations.

13. $\frac{6}{8} = \frac{z}{4}$
14. $\frac{16}{18} = \frac{n}{9}$
15. $\frac{21}{28} = \frac{a}{4}$
16. $\frac{35}{56} = \frac{r}{8}$
17. $\frac{15}{24} = \frac{5}{e}$
18. $\frac{14}{20} = \frac{7}{s}$
19. $\frac{12}{24} = \frac{v}{2}$
20. $\frac{12}{60} = \frac{c}{5}$

> Fractions that name the same fractional number are called **equivalent.**

Use different names for 1 to complete the sets of equivalent fractions.

21. $\left\{\frac{2}{7}, \frac{a}{14}, \frac{6}{u}, \frac{c}{35}, \frac{12}{r}\right\}$
22. $\left\{\frac{3}{8}, \frac{9}{e}, \frac{12}{s}, \frac{x}{40}, \frac{u}{56}\right\}$
23. $\left\{\frac{3}{10}, \frac{9}{i}, \frac{12}{z}, \frac{e}{70}, \frac{n}{400}\right\}$

Fractional numbers greater than 1

$A = \{\bullet\ \bullet\}$ $B = \{\bullet\ \bullet\ \blacksquare\}$

A is a **subset** of B $\quad \frac{n(A)}{n(B)} = \frac{2}{3}$

B is a **superset** of A $\quad \frac{n(B)}{n(A)} = \frac{3}{2}$

$$\frac{3}{2} = 3 \times \frac{1}{2}$$
$$= (2 + 1) \times \frac{1}{2}$$
$$= \left(2 \times \frac{1}{2}\right) + \left(1 \times \frac{1}{2}\right)$$
$$= 1 + \frac{1}{2}$$

EXERCISES

Name the fractional number which compares the superset with the set.

$A = \{\bigcirc\ \bigcirc\ \triangle\ \triangle\ \square\ \square\}$ $\quad B = \{\bigcirc\ \bigcirc\ \triangle\ \triangle\ \square\ \square\ \star\ \star\}$

1. $n(A) = a$ **2.** $n(B) = b$ **3.** $\frac{n(B)}{n(A)} = c$

$C = \{\bigcirc\ \bigcirc\ \bigcirc\ \bigcirc\}$ $\quad D = \{\bigcirc\ \bigcirc\ \bigcirc\ \bigcirc\ \triangle\}$

4. $n(C) = d$ **5.** $n(D) = e$ **6.** $\frac{n(D)}{n(C)} = f$

$E = \{\bigcirc\ \bigcirc\ \triangle\ \triangle\ \square\}$ $\quad F = \{\bigcirc\ \bigcirc\ \triangle\ \triangle\ \square\ \square\ \star\ \star\ \diamond\}$

7. $n(E) = g$ **8.** $n(F) = h$ **9.** $\frac{n(F)}{n(E)} = i$

$G = \{\bigcirc\ \bigcirc\ \square\ \square\ \star\ \star\}$ $\quad H = \{\bigcirc\ \bigcirc\ \square\ \square\ \star\ \star\ \diamond\}$

10. $n(G) = j$ **11.** $n(H) = k$ **12.** $\frac{n(H)}{n(G)} = l$

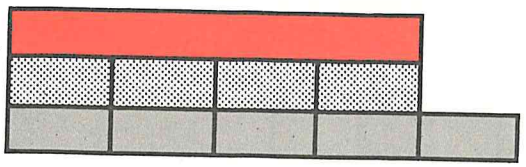
Solve the equations.

13. $\frac{4}{3} = 1 + a$ **16.** $\frac{7}{4} = 1 + d$ **19.** $\frac{8}{5} = 1 + g$ **22.** $\frac{9}{7} = 1 + j$

14. $\frac{5}{3} = 1 + b$ **17.** $\frac{6}{5} = 1 + e$ **20.** $\frac{9}{5} = 1 + h$ **23.** $\frac{10}{7} = 1 + k$

15. $\frac{5}{4} = 1 + c$ **18.** $\frac{7}{5} = 1 + f$ **21.** $\frac{8}{7} = 1 + i$ **24.** $\frac{13}{7} = 1 + l$

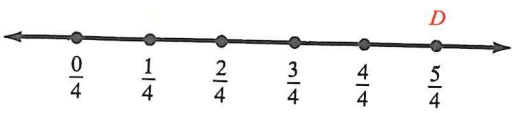
Fractional numbers greater than 1



The red rod shows 1 and the dotted rod, $\frac{4}{4} = 1$. The gray rod shows $\frac{5}{4} = \frac{4}{4} + \frac{1}{4} = 1 + \frac{1}{4}$.

$1 + \frac{1}{4}$ is called an **expanded numeral.**

We may write $1 + \frac{1}{4}$ in short form as $1\frac{1}{4}$.



The number line is divided into fourths. Point D labels $1 + \frac{1}{4}$ and also $\frac{5}{4}$.

We shall call $1\frac{1}{4}$ a **mixed numeral** because part of the numeral names a whole number and part names a fractional number.

EXERCISES

Write mixed numerals to replace the expanded numerals.

1. $1 + \frac{1}{2}$ **2.** $2 + \frac{1}{4}$ **3.** $1 + \frac{1}{3}$ **4.** $3 + \frac{1}{4}$

Solve the equations.

5. $1 + \frac{1}{4} = \frac{a}{4}$ **6.** $1 + \frac{3}{4} = \frac{i}{4}$ **7.** $2 + \frac{1}{2} = \frac{e}{2}$ **8.** $4 + \frac{1}{2} = \frac{s}{2}$

Solve the equations.

9. $1\frac{1}{4} = \frac{e}{4}$ **10.** $1\frac{3}{4} = \frac{c}{4}$ **11.** $2\frac{1}{2} = \frac{s}{2}$ **12.** $4\frac{1}{2} = \frac{r}{2}$

Name the number of fifths in

13. 1 **14.** $1\frac{1}{5}$ **15.** $1\frac{2}{5}$ **16.** $1\frac{3}{5}$ **17.** 2 **18.** $2\frac{3}{5}$

Name the number of fourths in

19. 1 **21.** $1\frac{1}{4}$ **23.** $2\frac{3}{4}$ **25.** $4\frac{1}{4}$ **27.** $6\frac{3}{4}$

20. 2 **22.** $1\frac{3}{4}$ **24.** $3\frac{1}{4}$ **26.** $5\frac{1}{4}$ **28.** $9\frac{1}{4}$

Fractional numbers and division

Here are 4 ways to show a division.

$12 \div 3 = 4$	$\frac{1}{3} \times 12 = 4$	$3\overline{)12}\ 4$	$\frac{12}{3} = 4$

We see that $12 \div 3 = \frac{12}{3}$

EXERCISES

Name the quotients.

1. $\frac{30}{6} = a$
2. $\frac{20}{4} = b$
3. $\frac{18}{6} = c$
4. $\frac{15}{5} = d$
5. $\frac{21}{7} = e$
6. $\frac{28}{4} = f$
7. $\frac{42}{7} = g$
8. $\frac{36}{4} = h$
9. $\frac{27}{9} = i$
10. $\frac{32}{8} = j$
11. $\frac{54}{9} = k$
12. $\frac{36}{6} = l$
13. $\frac{35}{5} = m$
14. $\frac{24}{6} = n$
15. $\frac{48}{8} = p$
16. $\frac{72}{9} = q$

Name the products.

17. $\frac{1}{3} \times 18 = a$
18. $\frac{1}{2} \times 10 = b$
19. $\frac{1}{4} \times 12 = c$
20. $\frac{1}{5} \times 25 = d$
21. $\frac{1}{7} \times 14 = e$
22. $\frac{1}{4} \times 20 = f$
23. $\frac{1}{3} \times 27 = g$
24. $\frac{1}{6} \times 30 = h$

Complete the number patterns.

25.
$48 \div 4 = a$
$24 \div 4 = b$
$12 \div 4 = c$
$6 \div 4 = d$
$3 \div 4 = e$

26.
$64 \div 8 = f$
$32 \div 8 = g$
$16 \div 8 = h$
$8 \div 8 = i$
$4 \div 8 = j$

27.
$625 \div 5 = k$
$125 \div 5 = l$
$25 \div 5 = m$
$5 \div 5 = n$
$1 \div 5 = p$

Solve the problem.

28. Jo Ann had 30 boxes of Girl Scout cookies to sell. If she sold $\frac{1}{6}$ of them, how many boxes of cookies did she sell?

Here are two ways of showing the division $\frac{21}{4}$.

The quotient is 5 and the remainder is 1.

A

$$4\overline{)21}\quad 5,\ \frac{20}{1}$$

B

$$\frac{21}{4} = \frac{20}{4} + \frac{1}{4}$$
$$= 5 + \frac{1}{4}$$
$$= 5\frac{1}{4}$$

EXERCISES

Name the quotients and remainders. For example, for the first exercise, write *Q1, R2*.

1. $3\overline{)5}$ **2.** $2\overline{)5}$ **3.** $2\overline{)9}$ **4.** $4\overline{)9}$ **5.** $5\overline{)8}$ **6.** $5\overline{)11}$ **7.** $3\overline{)14}$ **8.** $5\overline{)12}$ **9.** $8\overline{)13}$ **10.** $6\overline{)23}$ **11.** $5\overline{)19}$ **12.** $8\overline{)19}$ **13.** $8\overline{)23}$ **14.** $6\overline{)25}$ **15.** $10\overline{)33}$

Use the answers in exercises 1–15 to write the fractions as mixed numerals.

16. $\frac{5}{3}$ **17.** $\frac{5}{2}$ **18.** $\frac{9}{2}$ **19.** $\frac{9}{4}$ **20.** $\frac{8}{5}$ **21.** $\frac{11}{5}$ **22.** $\frac{14}{3}$ **23.** $\frac{12}{5}$ **24.** $\frac{13}{8}$ **25.** $\frac{23}{6}$ **26.** $\frac{19}{5}$ **27.** $\frac{19}{8}$ **28.** $\frac{23}{8}$ **29.** $\frac{25}{6}$ **30.** $\frac{33}{10}$

Copy and complete the equations. The solutions will be shown as mixed numerals.

31. $\frac{17}{5} = \frac{a}{5} + \frac{2}{5}$
$= b\frac{2}{5}$

32. $\frac{16}{7} = \frac{c}{7} + \frac{2}{7}$
$= d\frac{2}{7}$

33. $\frac{14}{3} = \frac{n}{3} + \frac{2}{3}$
$= h\frac{2}{3}$

34. $\frac{13}{6} = \frac{i}{6} + \frac{1}{6}$
$= j\frac{1}{6}$

35. $\frac{19}{3} = \frac{18}{3} + \frac{v}{3}$
$= n$

36. $\frac{27}{8} = \frac{24}{8} + \frac{x}{8}$
$= p$

Comparing two sets

A = {Betty's records}
B = {Barbara's records}

$$\frac{n(A)}{n(B)} = \frac{3}{5}$$

Set A can match a subset of set B. A fractional number which is used to compare 2 disjoint sets is called a **ratio.** We say, "The ratio of Betty's records to Barbara's records is three to five."

EXERCISES

Name the fractional number that compares the red circles with the gray circles.

1. {●●}
   {○○○○○}

2. {●●●}
   {○○○○○○○}

3. {●●●●}
   {○○○○○}

4. {●●●●●}
   {○○○○○○○}

5. {●●●●●●}
   {○○○○○○○}

6. {●●}
   {○○○○○○○}

Name the ratio of set A to set B.

7. A = {○○○}
   B = {●●●●●●}

8. A = {★★}
   B = {★★★★★}

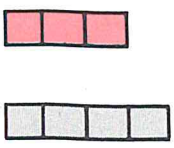
9. A = {▲▲▲}
   B = {△△△△}

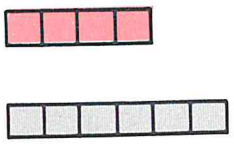
10. A = {○○○○}
    B = {●●●●●●●}

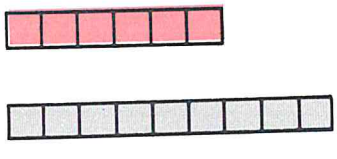
11. A = {■■■}
    B = {□□□□□}

12. A = {CCCC}
    B = {CCCCCCC}

Name the ratio of the red rod to the gray rod.

13. 

14. 

15. 

John has 4 airplanes, Jim has 5.

John has $\frac{4}{5}$ as many airplanes as Jim.

The ratio of John's airplanes to Jim's is 4 to 5.

EXERCISES

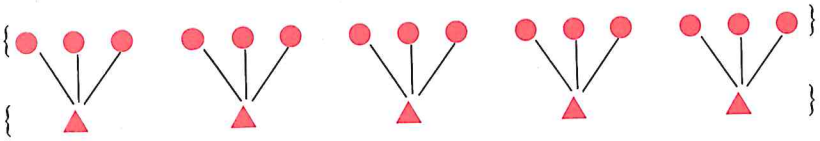
Complete the sentences.

1. Larry has 4 books and John has 7 books. The ratio of Larry's books to John's is 4 to 7 or ___?___.
2. Rita has 8 dresses and Sally has 5. The ratio of Rita's dresses to Sally's is 8 to 5 or ___?___.
3. Fred has 3 suits and Mike has 4. The ratio of Fred's suits to Mike's is 3 to 4 or ___?___.
4. There are 3 boys and 2 girls in the Johnson family. The ratio of boys to girls is ___?___.
5. Nancy has 7 records and Edith has 11. The ratio of Nancy's records to Edith's is ___?___.
6. Jerry has 8 trading cards and Tim has 15. The ratio of Jerry's trading cards to Tim's is ___?___.
7. Martin's Pet Shop has 3 parrots and 14 parakeets for sale. The ratio of parrots to parakeets is ___?___.

Name the ratios.

8. 5 to 7 $= \frac{a}{7}$
9. 3 to 11 $= \frac{n}{11}$
10. 4 to 9 $= c$
11. 2 to 7 $= d$
12. 6 to 13 $= e$
13. 4 to 8 $= f$
14. 9 to 15 $= g$
15. 11 to 16 $= h$
16. 3 to 17 $= i$
17. 2 to 10 $= j$
18. 3 to 9 $= k$
19. 13 to 19 $= l$
20. 12 to 7 $= m$
21. 15 to 5 $= n$
22. 7 to 18 $= p$

A different kind of matching

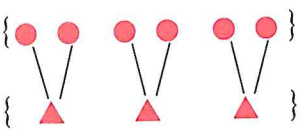
$A = \{$  $\}$ $\quad n(A) = 15$

$B = \{$ $\}$ $\quad n(B) = 5$

If we match the objects in set A with the objects in set B, we see that the 15 circles match the 5 triangles at the **rate** of 3 circles **per** triangle. $15 \div 5 = 3$.

EXERCISES

Use the sets to complete the sentences.

$A = \{$  $\}$

$B = \{$ $\}$

1. a circles per triangle
   $6 \div 3 = b$

$C = \{$ $\}$

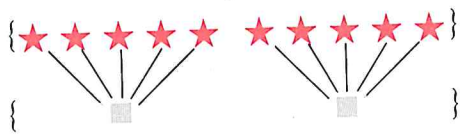
$D = \{$ $\}$

2. c circles per triangle
   $9 \div 3 = d$

$E = \{$ $\}$

$F = \{$ $\}$

3. e triangles per square
   $12 \div 3 = f$

$G = \{$  $\}$

$H = \{$ $\}$

4. g stars per square
   $10 \div 2 = h$

Complete this chart. In each row the first set is being matched with the second set.

	first set	second set	rate
5.	35 cents	5 apples	a cents per apple
6.	40 cents	8 oranges	b cents per orange
7.	\$16	8 hours	\$$c$ per hour
8.	\$12	4 hours	\$$d$ per hour
9.	40 miles	5 hours	e miles per hour
10.	600 miles	3 hours	f miles per hour
11.	24 books	6 girls	g books per girl
12.	30 books	5 boys	h books per boy

A different kind of matching

Phil's hens laid 35 eggs in a week.
Phil thought, "35 ÷ 7 = 5. This is at the *rate* of 5 eggs *per* day."

EXERCISES

Complete the sentences using either *rate* or *per*.

1. Bret's hens laid 42 eggs in a week; 42 ÷ 7 = 6. This is at the ___?___ of 6 eggs ___?___ day.
2. Ted's cow gives 49 gallons of milk each week; 49 ÷ 7 = 7. This is at the ___?___ of 7 gallons ___?___ day.
3. Jane's cow gives 54 gallons every 6 days; 54 ÷ 6 = 9. This is at the ___?___ of 9 gallons ___?___ day.

Complete the sentences.

4. Bonnie read 30 pages in 5 days. This is at the *rate* of ___?___ pages ___?___ day.
5. Howard read 40 pages in 5 days. This is at the *rate* of ___?___ pages per day.
6. Mark did 24 arithmetic problems in 3 minutes. This is at the ___?___ of ___?___ problems ___?___ minute.
7. Theresa did 36 arithmetic problems in 4 minutes. This is at the ___?___ of ___?___ problems ___?___ minute.
8. 27 every 3 days would be at the rate of ___?___ per day.
9. 64 every 8 days would be at the rate of ___?___ per day.
10. 56 every 7 weeks would be at the rate of ___?___ per week.
11. 36 every 3 minutes would be at the rate of ___?___ per minute.
12. 50 every 10 months would be at the rate of ___?___ per month.

Rates

Each boy is carrying 6 books. For every boy there is a set of 6 books or 6 books **per** boy. There are 4 boys, so there must be $4 \times 6 = 24$ books.

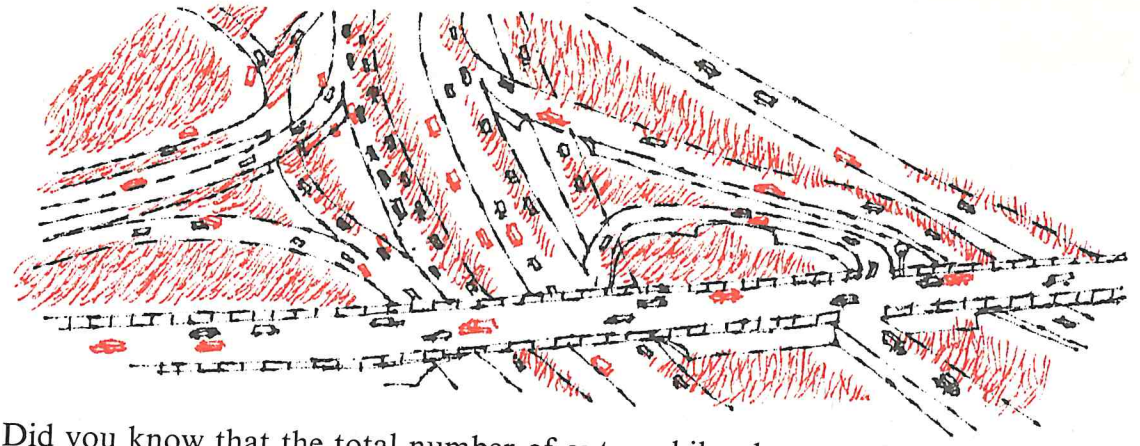
EXERCISES

Complete the sentences.

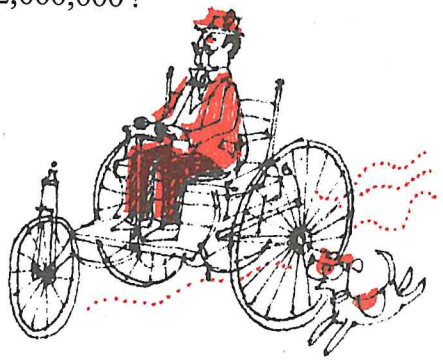
1. If each boy is carrying 5 books and there are 9 boys, there must be $5 \times 9 = a$ books.
2. If each girl is carrying 3 dolls and there are 6 girls, there must be $6 \times 3 = b$ dolls.
3. If each tricycle has 3 wheels and there are 8 tricycles there must be c wheels.
4. There are 4 wheels per car and 7 cars. There must be d wheels.
5. There are 8 spokes per wheel and 9 wheels. There are e spokes.
6. There are 12 eggs per box and 6 boxes. There are f eggs.

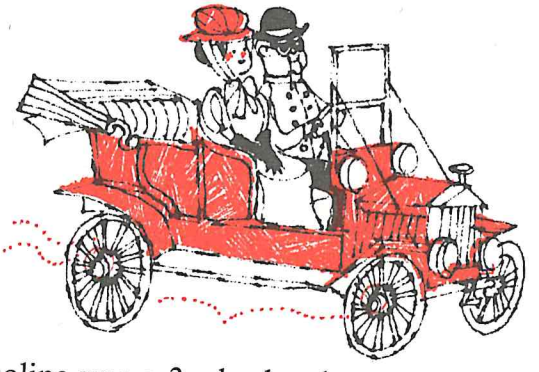
Complete the sentences.

7. If there are 5 marbles per box, in 8 boxes there are a marbles.
8. If there are 7 oranges per box, in 7 boxes there are b oranges.
9. If there are 4 pencils per box, in 12 boxes there are c pencils.
10. If there are 8 crayons per box, in 15 boxes there are d crayons.
11. If there are 14 apples per box, in 8 boxes there are e apples.
12. If there are 12 cookies per box, in 9 boxes there are f cookies.



Did you know that the total number of automobiles, buses and trucks on the roads of the United States in 1963 was about 82,000,000?





1. One of the first cars to run on gasoline was a 3-wheeler that was made in Germany about 1885. It went about 8 miles per hour, about as fast as a man can run.

2. The Model T Ford was probably the cheapest car ever made. Some could be bought for less than $300. It could travel at the rate of 30 miles per hour.

3. One of the longest bicycles ever built could seat 10 people at one time. It was about 24 feet long and could go at the rate of 10 miles per hour.

4. One of the longest trailer trucks in the world is about $\frac{1}{10}$ of a mile long. It takes 6 men to drive it and travels at a rate of about 15 miles per hour.

5. One of the largest buses in the world is in Moscow. It is about 60 feet long and can hold over 190 people. If it goes at the rate of 15 miles per hour it travels 60 miles in 4 hours.

CHECKING UP

Name the numbers.

A = {★ ★ ● ●}

B = {★ ★ ● ● ▲ ▲ ■}

C = {Lee, Sue, Tom}

D = {Dee, Jon, Ray, Lee, Sue, Tom}

1. $\text{n}(A) = a$, $\text{n}(B) = b$ $\quad \dfrac{\text{n}(A)}{\text{n}(B)} = c$

2. $\text{n}(C) = d$, $\text{n}(D) = e$ $\quad \dfrac{\text{n}(C)}{\text{n}(D)} = f = g$

Complete the sets of equivalent fractions.

3. $\left\{\frac{1}{2}, \frac{a}{4}, \frac{v}{6}, \frac{4}{c}, \frac{5}{x}, \frac{e}{12}, \frac{7}{s}\right\}$

4. $\left\{\frac{1}{3}, \frac{c}{6}, \frac{3}{u}, \frac{i}{12}, \frac{5}{a}, \frac{n}{18}\right\}$

5. $\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{z}, \frac{n}{16}, \frac{5}{r}\right\}$

6. $\left\{\frac{2}{3}, \frac{i}{6}, \frac{6}{a}, \frac{r}{12}, \frac{10}{s}, \frac{c}{18}, \frac{14}{u}\right\}$

Solve the equations.

7. $\frac{1}{2} \times 12 = a$

8. $\frac{1}{4} \times 8 = b$

9. $\frac{2}{3} \times 6 = c$

10. $\frac{1}{5} \times 10 = d$

11. $\frac{3}{4} \times 20 = e$

12. $\frac{1}{5} \times 15 = f$

13. $\frac{1}{3} \times 9 = g$

14. $\frac{1}{6} \times 18 = h$

15. $\frac{3}{5} \times 20 = i$

Name the ratios.

16. 7 books compared with 11 books.

17. 3 apples compared with 4 apples.

18. 2 boys compared with 5 girls.

19. 9 inches compared with 5 inches.

20. 8 marbles compared with 13 marbles.

21. 6 feet compared with 7 feet.

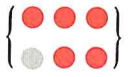
Complete the chart.

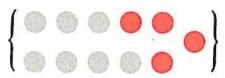
22.

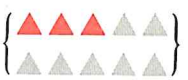
articles per box	5	9	10	11	13	20	25	50
number of boxes	8	9	5	7	6	5	4	3
total articles	40	e	x	n	i	s	a	v

A **fractional number** may be used to compare the cardinal numbers of a subset and a set. A **fraction** is a numeral for a fractional number.

Write fractions to compare the number of red objects with the total number of objects.

1. 

2. 

3. 

In the fractional number $\frac{3}{5}$, the cardinal number of the subset, 3, is the **numerator;** the cardinal number of the set, 5, is the **denominator.**

Fractional numbers greater than 1 may be represented by **mixed numerals.** For example, $2 + \frac{1}{3} = 2\frac{1}{3}$.

Name the sums as mixed numerals.

4. $4 + \frac{1}{2}$ **5.** $3 + \frac{1}{4}$ **6.** $2 + \frac{3}{4}$ **7.** $5 + \frac{7}{8}$

Fractional numbers may be associated with division. For example, $\frac{1}{3} \times 12 = 12 \div 3 = 4$.

Complete the equations.

8. $\frac{1}{5} \times 10 = 10 \div a = b$ **9.** $\frac{1}{6} \times 24 = 24 \div c = d$

A **ratio** is a fractional number used to compare the cardinal numbers of 2 disjoint sets.

Name the ratios to compare

10. 2 feet with 7 feet **11.** 3 apples with 8 peaches

LOOKING WAY BACK

Name the sums or differences.

1. $\begin{array}{r} 346 \\ +\ 598 \\ \hline \end{array}$ **2.** $\begin{array}{r} 4758 \\ +\ 2395 \\ \hline \end{array}$ **3.** $\begin{array}{r} 682 \\ -\ 359 \\ \hline \end{array}$ **4.** $\begin{array}{r} 4293 \\ -\ 1586 \\ \hline \end{array}$ **5.** $\begin{array}{r} 7053 \\ -\ 3759 \\ \hline \end{array}$

Study each equation, then complete the tables.

$x + 12 = y$

6.

x	3	7	9	24	30	48
y	15	a	b	c	d	e

$7 \times a = b$

8.

a	3	7	12	0	19	24
b	21	k	l	m	n	x

$x - 28 = y$

7.

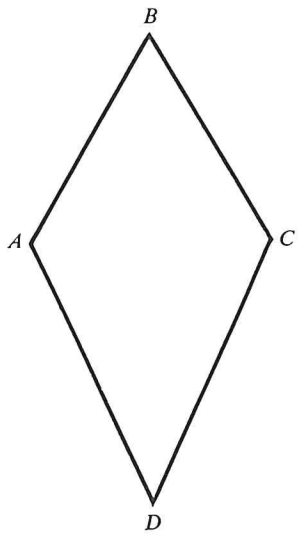
x	40	52	64	73	89	91
y	12	f	g	h	i	j

$a + b = 36$

9.

a	20	15	q	r	s	23
b	16	p	8	9	18	t

Use the figure at the right to name the measures of the sides and angles in inches or degrees.



10. $m(\overline{AB})$

11. $m(\overline{BC})$

12. $m(\overline{CD})$

13. $m(\overline{DA})$

14. $m(\angle ADC)$

15. $m(\angle DAB)$

16. $m(\angle ABC)$

17. $m(\angle BCD)$

Use your answers in exercises 10–17 to

18. name the congruent sides

19. name the congruent angles

Name the products.

20. $\begin{array}{r} 296 \\ \times\ 3 \\ \hline \end{array}$ **22.** $\begin{array}{r} 784 \\ \times\ 9 \\ \hline \end{array}$ **24.** $\begin{array}{r} 59 \\ \times\ 28 \\ \hline \end{array}$ **26.** $\begin{array}{r} 34 \\ \times\ 63 \\ \hline \end{array}$ **28.** $\begin{array}{r} 345 \\ \times\ 82 \\ \hline \end{array}$

21. $\begin{array}{r} 438 \\ \times\ 7 \\ \hline \end{array}$ **23.** $\begin{array}{r} 47 \\ \times\ 32 \\ \hline \end{array}$ **25.** $\begin{array}{r} 78 \\ \times\ 94 \\ \hline \end{array}$ **27.** $\begin{array}{r} 47 \\ \times\ 76 \\ \hline \end{array}$ **29.** $\begin{array}{r} 842 \\ \times\ 65 \\ \hline \end{array}$

Name the quotients and remainders.

30. $4\overline{)116}$ **31.** $9\overline{)416}$ **32.** $17\overline{)476}$ **33.** $24\overline{)983}$ **34.** $58\overline{)846}$

EXTRA FOR EXPERTS

1. Bob looked at these squares: $5^2 = 25$, $6^2 = 36$. He said, "The square of 5 ends in 5, the square of 6 ends in 6. The square of any number always ends with the number as the ones' digit." Was he right?

Continue each number pattern until there are 10 equations.

2.
$$1 = 1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$1 + 2 + 3 + 4 = 10$$
$$1 + 2 + 3 + 4 + 5 = 15$$

3.
$$1^3 = 1$$
$$1^3 + 2^3 = 3^2$$
$$1^3 + 2^3 + 3^3 = 6^2$$
$$1^3 + 2^3 + 3^3 + 4^3 = 10^2$$
$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 15^2$$

In 1745 a famous mathematician named Goldbach said that every even number greater than 2 can be written as the sum of 2 prime numbers. For instance,

$4 = 2 + 2$ $\quad 6 = 3 + 3$ $\quad 8 = 5 + 3$ $\quad 10 = 7 + 3$

Although this has been tested for many thousands of numbers and found to be true in every case, no one has ever been able to prove that it is true for *all* even numbers.

4. See if you can write the first 15 even numbers greater than 2 as sums of 2 primes. Remember that 1 is not a prime.

Just For Fun

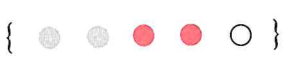
Solve the problems and make up others like them.

5. Bob said, "This glass is half full." Sharon said, "You're wrong. It is half empty." Who was right?

6. Milly said, "You can go only halfway into a tunnel." What did she mean?

7. Bill said, "I had 8 marbles. I have lost one fourth." Garry said, "That means you have $7\frac{3}{4}$ marbles." Bill said, "That's not what I meant." What did Bill mean to say?

Joining and separating sets



$\frac{2}{5}$ of the circles are gray.

$\frac{2}{5}$ of the circles are red.

$\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$ of the circles are red or gray.

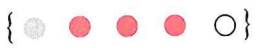
$\frac{3}{5}$ of the objects are red.

$\frac{2}{5}$ of the objects are red circles.

$\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$ of the objects are red triangles.

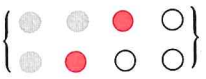
EXERCISES

Study the sets, then name the sums.

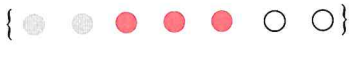


1. $\frac{1}{5} + \frac{3}{5} = a$

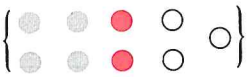
3. $\frac{2}{4} + \frac{1}{4} = c$



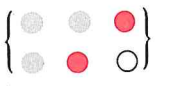
5. $\frac{3}{8} + \frac{2}{8} = e$



2. $\frac{2}{7} + \frac{3}{7} = b$

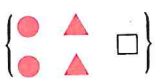


4. $\frac{4}{9} + \frac{2}{9} = d$

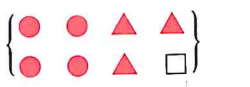


6. $\frac{3}{6} + \frac{2}{6} = f$

Study the sets, then name the differences.

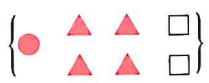


7. $\frac{4}{5} - \frac{2}{5} = a$

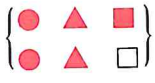


9. $\frac{7}{8} - \frac{3}{8} = c$

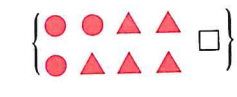
11. $\frac{3}{4} - \frac{1}{4} = e$



8. $\frac{5}{7} - \frac{4}{7} = b$



10. $\frac{5}{6} - \frac{3}{6} = d$



12. $\frac{8}{9} - \frac{5}{9} = f$

Using the distributive property

Study these equations.

$$(5 \times 10) + (4 \times 10) = (5 + 4) \times 10$$
$$= 9 \times 10$$
$$= 90$$

$$(9 \times 10) - (4 \times 10) = (9 - 4) \times 10$$
$$= 5 \times 10$$
$$= 50$$

$$\left(5 \times \frac{1}{12}\right) + \left(4 \times \frac{1}{12}\right) = (5 + 4) \times \frac{1}{12}$$
$$= 9 \times \frac{1}{12}$$
$$= \frac{9}{12}$$
$$= \frac{3}{4}$$

$$\left(9 \times \frac{1}{12}\right) - \left(4 \times \frac{1}{12}\right) = (9 - 4) \times \frac{1}{12}$$
$$= 5 \times \frac{1}{12}$$
$$= \frac{5}{12}$$

EXERCISES

Use the distributive property to solve the equations.

1. $(3 \times 7) + (2 \times 7) = (3 + 2) \times a$
$= b \times a$
$= c$

2. $(8 \times 11) - (3 \times 11) = (8 - 3) \times d$
$= e \times d$
$= f$

3. $\left(2 \times \frac{1}{10}\right) + \left(7 \times \frac{1}{10}\right) = (2 + 7) \times g$
$= h \times g$
$= i$

4. $\left(6 \times \frac{1}{11}\right) + \left(4 \times \frac{1}{11}\right) = (6 + 4) \times j$
$= k \times j$
$= l$

5. $\left(7 \times \frac{1}{9}\right) - \left(4 \times \frac{1}{9}\right) = (7 - 4) \times m$
$= n \times m$
$= p$

6. $\left(6 \times \frac{1}{8}\right) - \left(3 \times \frac{1}{8}\right) = (6 - 3) \times q$
$= r \times q$
$= s$

Addition-subtraction, fractional numbers

Study these equations.

$$\frac{5}{9} + \frac{2}{9} = \frac{5+2}{9} = \frac{7}{9} \qquad \frac{7}{9} - \frac{4}{9} = \frac{7-4}{9} = \frac{3}{9} = \frac{1}{3}$$

EXERCISES

Complete the number charts.

1.

fractional numbers	$\frac{1}{4} + \frac{2}{4}$	$\frac{2}{5} + \frac{2}{5}$	$\frac{2}{6} + \frac{3}{6}$	$\frac{2}{7} + \frac{3}{7}$	$\frac{3}{8} + \frac{4}{8}$	$\frac{4}{9} + \frac{2}{9}$
sum of numerators	$1 + 2$	$2 + 2$	$2 + 3$	m	n	v
denominator	4	a	c	7	s	x
sum	$\frac{3}{4}$	e	i	r	u	z

2.

fractional numbers	$\frac{4}{5} - \frac{2}{5}$	$\frac{8}{9} - \frac{5}{9}$	$\frac{7}{8} - \frac{3}{8}$	$\frac{9}{10} - \frac{4}{10}$	$\frac{5}{7} - \frac{4}{7}$	$\frac{5}{6} - \frac{1}{6}$
difference of numerators	$4 - 2$	$8 - 5$	$7 - 3$	m	r	v
denominator	5	a	e	10	s	x
difference	$\frac{2}{5}$	c	i	n	u	z

Complete the sentences.

3. To name the sum of 2 fractional numbers with the same denominator, name the ___?___ of the numerators and keep the same denominator.

4. To name the difference between 2 fractional numbers with the same denominator, name the difference between the ___?___ and keep the same denominator.

Name the sums and differences.

5. $\frac{3}{7} + \frac{2}{7}$ **6.** $\frac{3}{5} + \frac{1}{5}$ **7.** $\frac{9}{10} - \frac{4}{10}$ **8.** $\frac{5}{8} - \frac{3}{8}$ **9.** $\frac{1}{4} + \frac{1}{4}$ **10.** $\frac{5}{6} - \frac{3}{6}$ **11.** $\frac{2}{3} + \frac{1}{3}$

1. Brenda painted $\frac{3}{8}$ of the tulips red. Gail painted another $\frac{1}{8}$ of the tulips red. What part of the tulips did the girls paint red?

2. If Marjorie painted $\frac{2}{5}$ of the tulips yellow and Hope also painted $\frac{2}{5}$ of them yellow, what part of the tulips did the girls paint yellow?

3. Sue and Julie together painted $\frac{3}{4}$ of the daisies. If Sue painted $\frac{1}{4}$ of them, what part of the daisies did Julie paint?

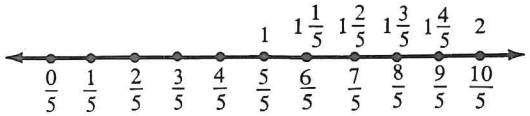
4. If Lucille painted $\frac{3}{10}$ of the lilacs and Doreen painted another $\frac{4}{10}$, what part of the lilacs did the girls paint?

5. Marian and Jane together painted $\frac{4}{5}$ of the violets. If Jane painted $\frac{2}{5}$ of them, what part of the violets did Marian paint?

6. Carrie painted $\frac{2}{7}$ of the roses and Patricia painted $\frac{3}{7}$ of the roses. What part of the roses did the girls paint?

7. Peggy and Anna together painted $\frac{9}{11}$ of the daffodils. If Anna painted $\frac{5}{11}$ of them, what part of the daffodils did Peggy paint?

Fractional numbers greater than 1

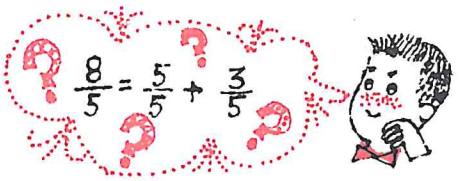


The number line shows that:

$\frac{5}{5} = 1$ $\quad \frac{8}{5} = 1\frac{3}{5}$

$\frac{6}{5} = 1\frac{1}{5}$ $\quad \frac{9}{5} = 1\frac{4}{5}$

$\frac{7}{5} = 1\frac{2}{5}$ $\quad \frac{10}{5} = 2$

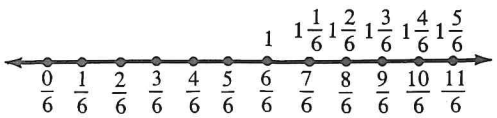


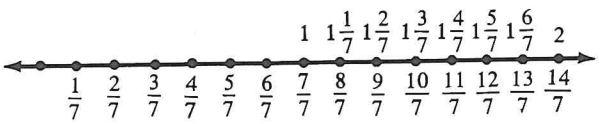
Arnold renamed $\frac{8}{5}$ as $1\frac{3}{5}$.

$$\frac{8}{5} = \frac{5}{5} + \frac{3}{5}$$
$$= 1 + \frac{3}{5}$$
$$= 1\frac{3}{5}$$

EXERCISES

Use the number lines to help you solve the equations.


1 $1\frac{1}{6}$ $1\frac{2}{6}$ $1\frac{3}{6}$ $1\frac{4}{6}$ $1\frac{5}{6}$
$\frac{0}{6}$ $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$ $\frac{7}{6}$ $\frac{8}{6}$ $\frac{9}{6}$ $\frac{10}{6}$ $\frac{11}{6}$


1 $1\frac{1}{7}$ $1\frac{2}{7}$ $1\frac{3}{7}$ $1\frac{4}{7}$ $1\frac{5}{7}$ $1\frac{6}{7}$ 2
$\frac{1}{7}$ $\frac{2}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{5}{7}$ $\frac{6}{7}$ $\frac{7}{7}$ $\frac{8}{7}$ $\frac{9}{7}$ $\frac{10}{7}$ $\frac{11}{7}$ $\frac{12}{7}$ $\frac{13}{7}$ $\frac{14}{7}$

1. $\frac{6}{6} = a$ **3.** $\frac{10}{6} = c$ **5.** $\frac{7}{7} = e$ **7.** $\frac{11}{7} = g$

2. $\frac{7}{6} = b$ **4.** $\frac{11}{6} = d$ **6.** $\frac{8}{7} = f$ **8.** $\frac{13}{7} = h$

Use Arnold's method to solve the equations.

9. $\frac{5}{3} = \frac{3}{3} + \frac{a}{3}$
$= b + \frac{a}{3}$
$= c$

10. $\frac{9}{5} = \frac{e}{5} + \frac{4}{5}$
$= f + \frac{4}{5}$
$= g$

11. $\frac{13}{8} = h + j$
$= 1 + \frac{i}{8}$
$= k$

Complete the sentence.

12. A numeral like $1\frac{2}{5}$ is called a mixed numeral because it names the sum of a whole number and a ____?____ number.

Rename the fractional numbers as mixed numerals.

13. $\frac{16}{7}$ **14.** $\frac{9}{6}$ **15.** $\frac{11}{7}$ **16.** $\frac{8}{3}$ **17.** $\frac{9}{4}$ **18.** $\frac{13}{5}$

Study these methods of naming the sum $\frac{5}{7} + \frac{4}{7}$.

Equations	Working form
$\frac{5}{7} + \frac{4}{7} = \frac{5+4}{7}$ $= \frac{9}{7}$ $= 1\frac{2}{7}$	$\frac{5}{7}$ $+\ \frac{4}{7}$ $\frac{9}{7} = 1\frac{2}{7}$

Study these methods of naming the difference $\frac{9}{7} - \frac{4}{7}$.

Equations	Working form
$\frac{9}{7} - \frac{4}{7} = \frac{9-4}{7}$ $= \frac{5}{7}$	$\frac{9}{7}$ $-\ \frac{4}{7}$ $\frac{5}{7}$

EXERCISES

Solve the equations.

1. $\frac{5}{4} + \frac{2}{4} = \frac{a}{4}$
2. $\frac{9}{7} - \frac{3}{7} = \frac{c}{7}$
3. $\frac{8}{9} - \frac{5}{9} = \frac{e}{9}$
4. $\frac{7}{4} - \frac{3}{4} = \frac{i}{4}$
5. $\frac{4}{6} + \frac{7}{6} = \frac{n}{6}$
6. $\frac{8}{5} - \frac{4}{5} = \frac{r}{5}$
7. $\frac{10}{8} - \frac{5}{8} = \frac{s}{8}$
8. $\frac{6}{7} + \frac{4}{7} = \frac{u}{7}$

Solve the equations.

9. $\frac{5}{9} + \frac{6}{9} = a$
10. $\frac{13}{9} - \frac{3}{9} = b$
11. $\frac{15}{8} - \frac{6}{8} = c$
12. $\frac{5}{4} + \frac{1}{4} = d$
13. $\frac{3}{5} + \frac{4}{5} = e$
14. $\frac{7}{10} + \frac{6}{10} = f$
15. $\frac{12}{7} - \frac{3}{7} = g$
16. $\frac{11}{6} - \frac{5}{6} = h$

Name the sums as mixed numerals.

17. $\frac{2}{7} + \frac{6}{7}$
18. $\frac{3}{5} + \frac{4}{5}$
19. $\frac{5}{8} + \frac{4}{8}$
20. $\frac{5}{6} + \frac{4}{6}$
21. $\frac{8}{9} + \frac{6}{9}$
22. $\frac{6}{11} + \frac{8}{11}$
23. $\frac{3}{4} + \frac{2}{4}$

Name the differences.

24. $\frac{8}{5} - \frac{2}{5}$
25. $\frac{11}{7} - \frac{3}{7}$
26. $\frac{4}{3} - \frac{2}{3}$
27. $\frac{10}{6} - \frac{3}{6}$
28. $\frac{5}{4} - \frac{2}{4}$
29. $\frac{13}{8} - \frac{6}{8}$
30. $\frac{14}{9} - \frac{7}{9}$

Equivalent fractions

The set of equivalent fractions for one-half is $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\right\}$. The simplest name for the members of this set is $\frac{1}{2}$. We may use the multiplication property of 1 to rename fractions.

Study these examples.

$$\frac{1}{2} = \frac{1}{2} \times 1 = \frac{1}{2} \times \frac{4}{4} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

$$\frac{8}{16} = \frac{1 \times 8}{2 \times 8} = \frac{1}{2} \times \frac{8}{8} = \frac{1}{2} \times 1 = \frac{1}{2}$$

EXERCISES

Solve the equations.

1. $\frac{2}{3} = \frac{a}{12}$
2. $\frac{3}{4} = \frac{e}{20}$
3. $\frac{4}{5} = \frac{u}{20}$
4. $\frac{4}{7} = \frac{v}{21}$
5. $\frac{9}{8} = \frac{x}{24}$
6. $\frac{8}{12} = \frac{z}{3}$
7. $\frac{6}{18} = \frac{i}{3}$
8. $\frac{15}{24} = \frac{c}{8}$
9. $\frac{16}{18} = \frac{r}{9}$
10. $\frac{35}{20} = \frac{s}{4}$

To rename $\frac{12}{18}$ in its simplest form we look for the greatest common factor of 12 and 18. Since the greatest common factor is 6 we write: $\frac{12}{18} = \frac{2 \times 6}{3 \times 6} = \frac{2}{3} \times \frac{6}{6} = \frac{2}{3} \times 1 = \frac{2}{3}$.

Name the greatest common factor of each numerator and denominator, then rename the fraction in simplest form.

11. $\frac{8}{12}$
12. $\frac{6}{9}$
13. $\frac{4}{8}$
14. $\frac{10}{25}$
15. $\frac{14}{21}$
16. $\frac{16}{40}$
17. $\frac{24}{36}$

Complete the sentences.

18. To rename a fractional number with a greater denominator we ___?___ the numerator and denominator by a common factor.
19. To rename a fractional number in its simplest form we ___?___ the numerator and denominator by their greatest common factor.
20. A fraction is in its simplest form when the only common factor of the numerator and denominator is ___?___.

Fractions with the same denominators

We cannot use the distributive property to name the sum $(2 \times 3) + (4 \times 5)$ because the addends do not have a common factor. We cannot use the distributive property to name the sum $\frac{1}{2} + \frac{1}{6}$ until the addends have the same denominator.

We know that $\frac{3}{6}$ is another name for $\frac{1}{2}$, so:

$$\begin{aligned}\frac{1}{2} + \frac{1}{6} &= \frac{3}{6} + \frac{1}{6} \\ &= \frac{4}{6} \\ &= \frac{2}{3}\end{aligned}$$

EXERCISES

Use the sets to help you name the sums and differences.

$$\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \ldots\right\}$$

1. $\frac{1}{2} + \frac{1}{4} = \frac{a}{4} + \frac{1}{4} = \frac{e}{4}$
2. $\frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{c}{6} = \frac{r}{6}$
3. $\frac{7}{10} - \frac{1}{2} = \frac{7}{10} - \frac{e}{10} = \frac{v}{10}$
4. $\frac{1}{8} + \frac{2}{4} = \frac{1}{8} + \frac{s}{8} = \frac{x}{8}$
5. $\frac{3}{6} - \frac{5}{12} = \frac{i}{12} - \frac{5}{12} = \frac{u}{12}$
6. $\frac{11}{16} - \frac{4}{8} = \frac{11}{16} - \frac{n}{16} = \frac{z}{16}$

$$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \ldots\right\}$$

7. $\frac{5}{9} - \frac{1}{3} = \frac{5}{9} - \frac{a}{9} = \frac{r}{9}$
8. $\frac{4}{6} + \frac{1}{3} = \frac{4}{6} + \frac{c}{6} = \frac{s}{6}$
9. $\frac{7}{12} - \frac{2}{6} = \frac{7}{12} - \frac{e}{12} = f$
10. $\frac{1}{3} + \frac{5}{12} = \frac{v}{12} + \frac{5}{12} = h$
11. $\frac{3}{9} - \frac{4}{18} = \frac{i}{18} - \frac{4}{18} = j$
12. $\frac{9}{15} + \frac{1}{3} = \frac{9}{15} + \frac{m}{15} = l$

$$\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \ldots\right\}$$

13. $\frac{5}{8} + \frac{1}{4} = \frac{5}{8} + a = b$
14. $\frac{5}{12} - \frac{1}{4} = \frac{5}{12} - c = d$
15. $\frac{7}{16} - \frac{2}{8} = \frac{7}{16} - e = f$
16. $\frac{3}{20} + \frac{1}{4} = \frac{3}{20} + g = h$

Renaming fractional sums and addends

The set of equivalent fractions for $\frac{1}{3}$ is $\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \ldots\right\}$.

To name the sum $\frac{1}{3} + \frac{2}{9}$, first rename $\frac{1}{3}$ as $\frac{3}{9}$.

$$\frac{1}{3} + \frac{2}{9} = \frac{3}{9} + \frac{2}{9}$$
$$= \frac{3 + 2}{9}$$
$$= \frac{5}{9}$$

To name the difference $\frac{7}{12} - \frac{1}{3}$, first rename $\frac{1}{3}$ as $\frac{4}{12}$.

$$\frac{7}{12} - \frac{1}{3} = \frac{7}{12} - \frac{4}{12}$$
$$= \frac{7 - 4}{12}$$
$$= \frac{3}{12} = \frac{1}{4}$$

EXERCISES

Rename $\frac{2}{3}$ as an equivalent fraction with a denominator of

1. 6 **2.** 12 **3.** 15 **4.** 18 **5.** 30 **6.** 24

Name the sums in simplest form.

7. $\frac{1}{2} + \frac{1}{4} = a$

8. $\frac{2}{3} + \frac{1}{6} = b$

9. $\frac{1}{2} + \frac{1}{8} = c$

10. $\frac{3}{5} + \frac{3}{10} = d$

11. $\frac{5}{6} + \frac{5}{18} = e$

12. $\frac{7}{10} + \frac{1}{20} = f$

13. $\frac{1}{3} + \frac{5}{6} = g$

14. $\frac{5}{6} + \frac{2}{3} = h$

15. $\frac{9}{10} + \frac{4}{5} = i$

16. $\frac{5}{8} + \frac{17}{40} = j$

17. $\frac{5}{12} + \frac{19}{24} = k$

18. $\frac{9}{8} + \frac{3}{4} = l$

Name the differences in simplest form.

19. $\frac{4}{5} - \frac{3}{10} = a$

20. $\frac{1}{2} - \frac{1}{6} = b$

21. $\frac{11}{12} - \frac{1}{3} = c$

22. $\frac{5}{9} - \frac{1}{3} = d$

23. $\frac{5}{8} - \frac{1}{4} = e$

24. $\frac{13}{18} - \frac{15}{36} = f$

25. $\frac{3}{4} - \frac{9}{24} = g$

26. $\frac{9}{8} - \frac{3}{4} = h$

27. $\frac{6}{10} - \frac{1}{5} = i$

Fractions with different denominators

To name the sum and difference for $\frac{1}{6}$ and $\frac{3}{4}$ we must rename both of the addends. Since the least common multiple of 6 and 4 is 12, we can rename each fraction as an equivalent fraction which has a denominator of 12.

$$\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12} = \frac{9 + 2}{12} = \frac{11}{12}$$

$$\frac{3}{4} - \frac{1}{6} = \frac{9}{12} - \frac{2}{12} = \frac{9 - 2}{12} = \frac{7}{12}$$

EXERCISES

Name the least common multiple of each pair of numbers.

1. 3 and 4 **2.** 8 and 6 **3.** 10 and 4 **4.** 2 and 7 **5.** 8 and 10

Use the least common multiples you named in exercises 1–5 to help you name these sums or differences.

6. $\frac{2}{3} + \frac{1}{4} = a$

7. $\frac{3}{4} - \frac{1}{3} = b$

8. $\frac{3}{8} + \frac{1}{6} = c$

9. $\frac{5}{6} - \frac{5}{8} = d$

10. $\frac{7}{10} + \frac{1}{4} = e$

11. $\frac{9}{10} - \frac{3}{4} = f$

12. $\frac{4}{7} + \frac{1}{2} = g$

13. $\frac{1}{2} - \frac{3}{7} = h$

14. $\frac{7}{10} + \frac{1}{8} = i$

15. $\frac{9}{10} + \frac{5}{8} = j$

16. $\frac{7}{8} - \frac{3}{10} = k$

17. $\frac{9}{10} - \frac{7}{8} = l$

18. Tom and Sharon found the sum of $\frac{1}{6}$ and $\frac{1}{4}$ in different ways. Can you tell why Sharon's method is easier?

Tom's method

$$\frac{1}{6} + \frac{1}{4} = \frac{4}{24} + \frac{6}{24}$$

$$= \frac{10}{24}$$

$$= \frac{5}{12}$$

Sharon's method

$$\frac{1}{6} + \frac{1}{4} = \frac{2}{12} + \frac{3}{12}$$

$$= \frac{5}{12}$$

Renaming first

Study these methods for naming the sum $\frac{1}{3}+\frac{1}{4}$.

Equations	Working form
$\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}$	$\frac{1}{3}=\frac{4}{12}$
$=\frac{7}{12}$	$+\frac{1}{4}=+\frac{3}{12}$
	$\frac{7}{12}$

Study these methods for naming the difference $\frac{1}{3}-\frac{1}{4}$.

Equations	Working form
$\frac{1}{3}-\frac{1}{4}=\frac{4}{12}-\frac{3}{12}$	$\frac{1}{3}=\frac{4}{12}$
$=\frac{1}{12}$	$-\frac{1}{4}=-\frac{3}{12}$
	$\frac{1}{12}$

EXERCISES

Solve the equations.

1. $\frac{1}{2}+\frac{1}{3}=\frac{a}{6}+\frac{e}{6}=\frac{c}{6}$
2. $\frac{3}{4}+\frac{1}{6}=\frac{u}{12}+\frac{e}{12}=\frac{v}{12}$
3. $\frac{2}{3}+\frac{1}{5}=\frac{n}{15}+\frac{r}{15}=\frac{i}{15}$
4. $\frac{1}{2}+\frac{3}{7}=\frac{e}{14}+\frac{a}{14}=\frac{x}{14}$
5. $\frac{1}{4}+\frac{2}{5}=\frac{s}{20}+\frac{n}{20}=\frac{x}{20}$
6. $\frac{5}{6}+\frac{1}{4}=\frac{v}{12}+\frac{r}{12}=\frac{s}{12}=t$

Name the sums or differences.

7. $\frac{1}{2}+\frac{1}{3}$
8. $\frac{2}{3}+\frac{1}{6}$
9. $\frac{3}{4}-\frac{1}{5}$
10. $\frac{2}{3}+\frac{3}{4}$
11. $\frac{3}{4}-\frac{1}{2}$
12. $\frac{2}{3}-\frac{2}{5}$
13. $\frac{1}{4}+\frac{1}{6}$
14. $\frac{2}{3}-\frac{1}{3}$
15. $\frac{1}{6}+\frac{3}{4}$
16. $\frac{2}{5}-\frac{1}{6}$
17. $\frac{2}{3}-\frac{1}{5}$
18. $\frac{1}{4}+\frac{2}{3}$
19. $\frac{2}{5}+\frac{1}{4}$
20. $\frac{1}{4}-\frac{1}{6}$

Complete the sentences.

21. To add or subtract two fractional numbers which have different denominators, first rename one or both fractions as equivalent fractions so that the denominators are ___?___.

Adding and subtracting fractional numbers

Study these methods for naming fractional sums and differences.

$$\frac{3}{4}+\frac{2}{3}=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}=1\frac{5}{12}$$

$$\begin{array}{r} \frac{3}{4}=\frac{9}{12} \\ +\frac{2}{3}=+\frac{8}{12} \\ \hline \frac{17}{12}=1\frac{5}{12} \end{array}$$

$$\frac{8}{6}-\frac{3}{4}=\frac{16}{12}-\frac{9}{12}=\frac{7}{12}$$

$$\begin{array}{r} \frac{8}{6}=\frac{16}{12} \\ -\frac{3}{4}=-\frac{9}{12} \\ \hline \frac{7}{12} \end{array}$$

EXERCISES

Name the sums.

1. $\frac{1}{2}+\frac{2}{3}=a$
2. $\frac{1}{6}+\frac{3}{4}=b$
3. $\frac{5}{6}+\frac{1}{2}=c$
4. $\frac{5}{8}+\frac{1}{3}=d$
5. $\frac{4}{7}+\frac{1}{2}=e$
6. $\frac{5}{9}+\frac{5}{6}=f$
7. $\frac{4}{8}+\frac{5}{10}=g$
8. $\frac{3}{5}+\frac{3}{2}=h$

Name the differences.

9. $\frac{3}{4}-\frac{1}{3}=a$
10. $\frac{7}{10}-\frac{1}{4}=b$
11. $\frac{7}{4}-\frac{5}{5}=c$
12. $\frac{19}{12}-\frac{3}{8}=d$
13. $\frac{6}{7}-\frac{1}{3}=e$
14. $\frac{1}{2}-\frac{1}{5}=f$
15. $\frac{1}{3}-\frac{1}{8}=g$
16. $\frac{11}{8}-\frac{5}{6}=h$

Name the sums and differences.

17. $\begin{array}{r} \frac{5}{6} \\ +\frac{2}{3} \\ \hline \end{array}$
18. $\begin{array}{r} \frac{4}{9} \\ +\frac{3}{5} \\ \hline \end{array}$
19. $\begin{array}{r} \frac{7}{8} \\ +\frac{4}{5} \\ \hline \end{array}$
20. $\begin{array}{r} \frac{6}{7} \\ -\frac{1}{4} \\ \hline \end{array}$
21. $\begin{array}{r} \frac{9}{8} \\ -\frac{5}{6} \\ \hline \end{array}$
22. $\begin{array}{r} \frac{13}{12} \\ -\frac{7}{8} \\ \hline \end{array}$

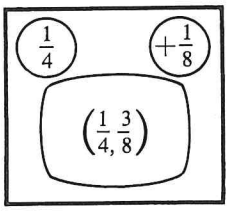
Solve the problem.

23. Leon gave $\frac{1}{2}$ of his trading cards to Mike and $\frac{2}{5}$ of them to Jay. What part of his cards did he give away?

Fractional number patterns, functions

+	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1

The number pattern shows additions with one addend $\frac{1}{8}$.



The function TV shows the tuned-in number $\frac{1}{4}$, the tuned-in rule *add* $\frac{1}{8}$.

The set of number pairs is $\left\{\left(\frac{1}{4}, \frac{3}{8}\right), \left(\frac{3}{8}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{8}\right), \ldots\right\}$

EXERCISES

Complete the number patterns.

1.

+	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$	1
$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{a}{2}$	$\frac{c}{5}$	$\frac{e}{10}$	$\frac{i}{5}$	$\frac{n}{10}$	f	$\frac{r}{10}$

2.

−	$\frac{11}{6}$	$\frac{5}{3}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{7}{6}$	1	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{2}$
$\frac{1}{6}$	$\frac{s}{3}$	$\frac{l}{2}$	$\frac{u}{3}$	$\frac{v}{6}$	p	$\frac{x}{6}$	$\frac{z}{3}$	$\frac{a}{2}$	$\frac{c}{3}$

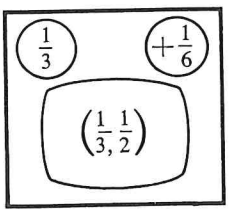
3.

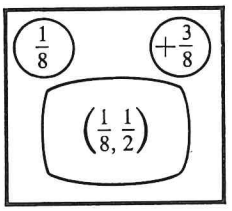
+	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$
$\frac{1}{4}$	$\frac{3}{4}$	h	$\frac{e}{4}$	$\frac{i}{2}$

4.

−	$\frac{5}{3}$	$\frac{4}{3}$	1	$\frac{2}{3}$
$\frac{1}{3}$	$\frac{4}{3}$	t	$\frac{n}{3}$	$\frac{r}{3}$

Use the function TV to help you complete each set.





5. $\left\{\left(\frac{1}{3}, \frac{1}{2}\right), \left(\frac{2}{3}, \frac{a}{6}\right), \left(\frac{5}{6}, b\right), \left(1, \frac{c}{6}\right), \left(\frac{7}{6}, \frac{e}{3}\right), \left(\frac{4}{3}, \frac{i}{2}\right)\right\}$

6. $\left\{\left(\frac{1}{8}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{n}{8}\right), \left(\frac{5}{8}, g\right), \left(1, \frac{r}{8}\right), \left(\frac{11}{8}, \frac{s}{4}\right)\right\}$

Sharpshooting

Answer the questions in set A as quickly as you can. Then see if you can answer the questions in set B even more quickly.

A

1. What is the difference between 298 and 402?
2. What is the perimeter of a square with sides of 27″?
3. Write a fraction with a denominator of 45 that is equivalent to $\frac{7}{9}$.
4. Name the greatest common factor of 30 and 48.
5. How many inches are in 14 feet?
6. Name the prime factorization of 72.
7. $6^2 + 8^2 = a$
8. Name the greatest number whose compact numeral has 4 digits.
9. $8257 + 13975 = a$
10. Name the number whose square is 49.

B

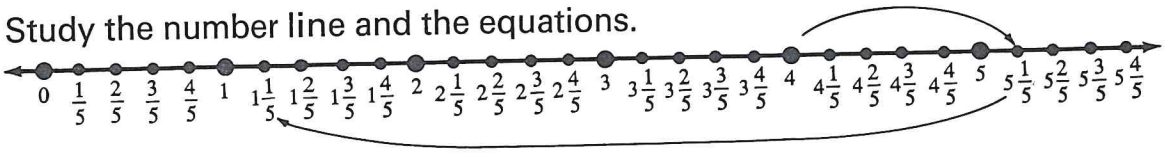
1. $123 + 321 = a$
2. If you travel at 30 miles per hour for 7 hours, how far will you go?
3. Is $1900 + 190 + 19$ greater or less than 2100?
4. Name the greatest prime number less than 100.
5. $1600 \div 40 = a$
6. Name the least number which has 3, 5, and 7 as prime factors.
7. List the factors of 56.
8. What is the name we give to 1000×1000?
9. If the perimeter of a rectangle is 24″ and the width is 4″, what is the length?
10. $1234 - 987 = a$

Be Careful!

If you were a bus driver and there were 24 people on the bus, 6 got on and 4 got off, then 7 got on and 5 got off, could you tell the age of the bus driver?

One addend a whole number

Study the number line and the equations.



The top arrow shows:

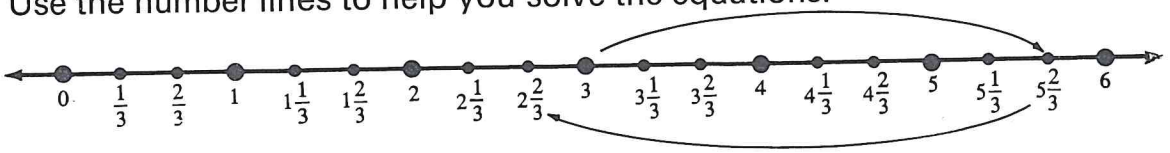
$$\begin{aligned} 4 + 1\tfrac{1}{5} &= 4 + \left(1 + \tfrac{1}{5}\right) \\ &= (4 + 1) + \tfrac{1}{5} \\ &= 5 + \tfrac{1}{5} \\ &= 5\tfrac{1}{5} \end{aligned}$$

The bottom arrow shows:

$$\begin{aligned} 5\tfrac{1}{5} - 4 &= \left(4 + 1\tfrac{1}{5}\right) - 4 \\ &= \left(1\tfrac{1}{5} + 4\right) - 4 \\ &= 1\tfrac{1}{5} + (4 - 4) \\ &= 1\tfrac{1}{5} \end{aligned}$$

EXERCISES

Use the number lines to help you solve the equations.



1. $3 + 2\tfrac{2}{3} = 3 + \left(a + \tfrac{2}{3}\right)$

$= \left(3 + a\right) + \tfrac{2}{3}$

$= b + \tfrac{2}{3}$

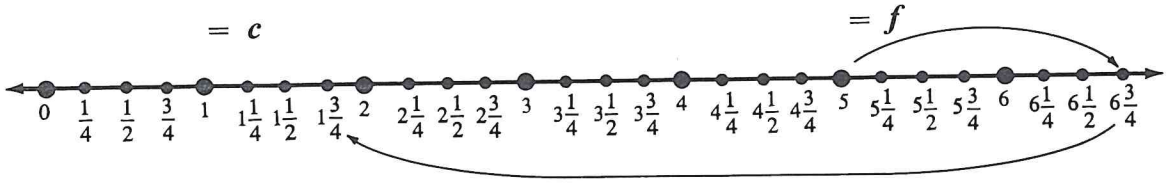
$= c$

2. $5\tfrac{2}{3} - 3 = \left(d + 2\tfrac{2}{3}\right) - 3$

$= \left(2\tfrac{2}{3} + d\right) - 3$

$= e + (d - 3)$

$= f$



3. $5 + 1\tfrac{3}{4} = 5 + (1 + g)$

$= (5 + 1) + g$

$= h + \tfrac{3}{4}$

$= i$

4. $6\tfrac{3}{4} - 5 = \left(j + 1\tfrac{3}{4}\right) - 5$

$= \left(1\tfrac{3}{4} + j\right) - 5$

$= k + (j - 5)$

$= l$

The sum or one addend greater than 1

Study the equations.

To add $4\frac{1}{5} + \frac{2}{5}$:

$$\begin{aligned} 4\tfrac{1}{5} + \tfrac{2}{5} &= \left(4 + \tfrac{1}{5}\right) + \tfrac{2}{5} \\ &= 4 + \left(\tfrac{1}{5} + \tfrac{2}{5}\right) \\ &= 4 + \tfrac{3}{5} \\ &= 4\tfrac{3}{5} \end{aligned}$$

To subtract $3\frac{5}{7} - \frac{2}{7}$:

$$\begin{aligned} 3\tfrac{5}{7} - \tfrac{2}{7} &= \left(3 + \tfrac{5}{7}\right) - \tfrac{2}{7} \\ &= 3 + \left(\tfrac{5}{7} - \tfrac{2}{7}\right) \\ &= 3 + \tfrac{3}{7} \\ &= 3\tfrac{3}{7} \end{aligned}$$

EXERCISES

Solve the equations.

1. $3\frac{1}{3} + \frac{2}{3} = (3 + a) + \frac{2}{3}$

$= 3 + \left(a + \frac{2}{3}\right)$

$= 3 + b$

$= c$

2. $7\frac{2}{5} + \frac{2}{5} = (7 + d) + \frac{2}{5}$

$= 7 + \left(d + \frac{2}{5}\right)$

$= 7 + e$

$= f$

3. $6\frac{4}{5} - \frac{2}{5} = (6 + g) - \frac{2}{5}$

$= 6 + \left(g - \frac{2}{5}\right)$

$= 6 + h$

$= i$

4. $4\frac{6}{7} - \frac{3}{7} = (4 + j) - \frac{3}{7}$

$= 4 + \left(\frac{j}{7} - \frac{3}{7}\right)$

$= 4 + k$

$= l$

Name the sums or differences.

5. $2\frac{1}{5} + \frac{2}{5}$

6. $3\frac{1}{4} + \frac{1}{4}$

7. $5\frac{2}{7} + \frac{4}{7}$

8. $\frac{1}{6} + \frac{5}{6}$

9. $10\frac{3}{8} + \frac{1}{8}$

10. $9\frac{5}{9} + \frac{2}{9}$

11. $8\frac{2}{3} - \frac{1}{3}$

12. $7\frac{5}{8} - \frac{3}{8}$

13. $2\frac{4}{5} - \frac{3}{5}$

14. $5\frac{8}{9} - \frac{2}{9}$

15. $3\frac{11}{12} - \frac{9}{12}$

16. $9\frac{7}{10} - \frac{3}{10}$

One addend greater than 1

Study these methods for naming the sum $4\frac{3}{5} + \frac{3}{5}$.

A

$$4\frac{3}{5} + \frac{3}{5} = 4 + \left(\frac{3}{5} + \frac{3}{5}\right)$$
$$= 4 + 1\frac{1}{5}$$
$$= 5\frac{1}{5}$$

B

$$\begin{array}{r} 4\frac{3}{5} \\ +\ \frac{3}{5} \\ \hline 4\frac{6}{5} \end{array} = 4 + 1\frac{1}{5} = 5\frac{1}{5}$$

EXERCISES

Copy and complete the equations.

1. $4\frac{5}{8} + \frac{3}{8} = 4 + \left(a + \frac{3}{8}\right)$
$= 4 + b$
$= c$

2. $6\frac{1}{3} + \frac{2}{3} = 6 + \left(d + \frac{2}{3}\right)$
$= 6 + e$
$= f$

3. $3\frac{5}{8} + \frac{7}{8} = 3 + \left(g + \frac{7}{8}\right)$
$= 3 + h$
$= i$

4. $5\frac{3}{4} + \frac{3}{4} = 5 + \left(j + \frac{3}{4}\right)$
$= 5 + k$
$= l$

5. $2\frac{4}{5} + \frac{3}{5} = 2 + \left(m + \frac{3}{5}\right)$
$= 2 + n$
$= p$

6. $6\frac{5}{6} + \frac{2}{6} = 6 + \left(q + \frac{2}{6}\right)$
$= 6 + r$
$= s$

Name the sums.

7. $1\frac{1}{2} + \frac{1}{2}$

8. $4\frac{1}{3} + \frac{2}{3}$

9. $3\frac{4}{5} + \frac{1}{5}$

10. $6\frac{4}{5} + \frac{2}{5}$

11. $7\frac{5}{9} + \frac{3}{8}$

12. $5\frac{1}{4} + \frac{3}{4}$

13. $4\frac{2}{3} + \frac{2}{3}$

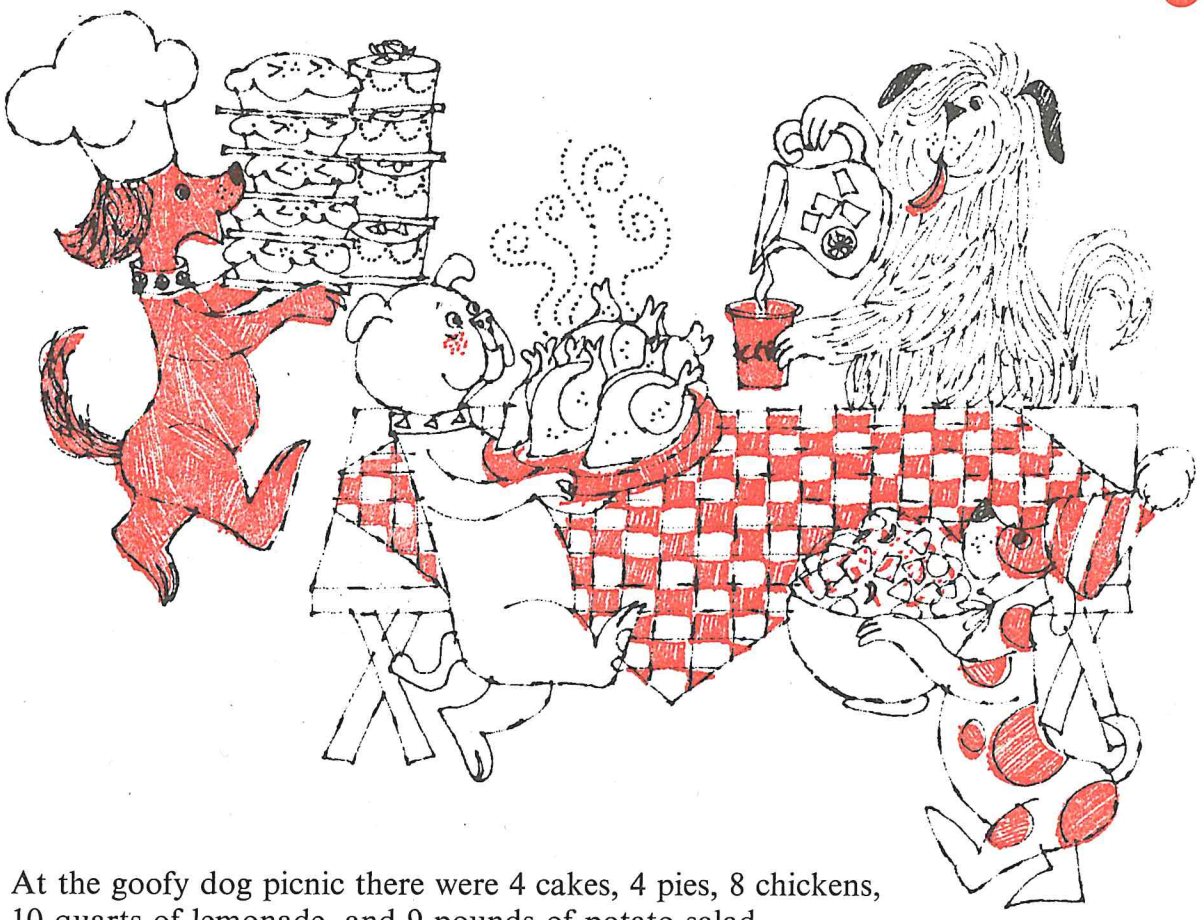
14. $3\frac{7}{8} + \frac{3}{8}$

15. $4\frac{5}{9} + \frac{7}{9}$

16. $9\frac{3}{5} + \frac{2}{5}$

17. $12\frac{3}{8} + \frac{7}{8}$

18. $5\frac{5}{6} + \frac{5}{6}$



At the goofy dog picnic there were 4 cakes, 4 pies, 8 chickens, 10 quarts of lemonade, and 9 pounds of potato salad.

1. The goofy dogs ate 2 chocolate cakes and $1\frac{1}{3}$ yellow cakes. How much cake did the dogs eat?

2. The goofy dogs ate $3\frac{2}{5}$ cherry pies and $\frac{1}{5}$ of an apple pie. How much pie did the dogs eat?

3. If the goofy dogs ate $6\frac{5}{8}$ chickens at dinner time and another $\frac{7}{8}$ of a chicken later, how much chicken did they eat?

4. The goofy dogs drank $5\frac{9}{10}$ quarts of lemonade. The pink dogs drank $\frac{7}{10}$ quarts of the lemonade. How many quarts did the rest of the dogs drink?

5. The goofy dogs ate $6\frac{7}{8}$ pounds of potato salad. The green dogs ate $\frac{5}{8}$ of a pound. How many pounds of potato salad did the rest of the dogs eat?

Both addends greater than 1

Study these methods for naming the sum $1\frac{1}{5} + 2\frac{3}{5}$.

A

$$1\tfrac{1}{5} + 2\tfrac{3}{5} = 3 + \frac{1+3}{5}$$
$$= 3 + \tfrac{4}{5}$$
$$= 3\tfrac{4}{5}$$

B

$$\begin{array}{r} 1\frac{1}{5} \\ +\ 2\frac{3}{5} \\ \hline 3\frac{4}{5} \end{array}$$

Study these methods for naming the difference $3\frac{4}{5} - 1\frac{1}{5}$.

A

$$3\tfrac{4}{5} - 1\tfrac{1}{5} = 2 + \frac{4-1}{5}$$
$$= 2 + \tfrac{3}{5}$$
$$= 2\tfrac{3}{5}$$

B

$$\begin{array}{r} 3\frac{4}{5} \\ -\ 1\frac{1}{5} \\ \hline 2\frac{3}{5} \end{array}$$

EXERCISES

Use method A to name the sums or differences.

1. $2\frac{2}{7} + 1\frac{4}{7}$ **2.** $2\frac{3}{8} + 5\frac{1}{8}$ **3.** $7\frac{3}{10} + 2\frac{5}{10}$ **4.** $4\frac{3}{5} - 1\frac{2}{5}$ **5.** $8\frac{5}{6} - 3\frac{2}{6}$ **6.** $5\frac{7}{8} - 4\frac{2}{8}$

Use method B to name the sums or differences.

7. $6\frac{1}{8} + 3\frac{5}{8}$ **9.** $4\frac{1}{5} + 2\frac{4}{5}$ **11.** $8\frac{5}{12} + 9\frac{1}{12}$ **13.** $7\frac{4}{5} - 6\frac{1}{5}$ **15.** $3\frac{7}{8} - 1\frac{4}{8}$ **17.** $3\frac{3}{4} - 2\frac{2}{4}$

8. $9\frac{2}{9} + 6\frac{5}{9}$ **10.** $7\frac{3}{7} + 8\frac{3}{7}$ **12.** $5\frac{11}{16} + 8\frac{3}{16}$ **14.** $8\frac{5}{6} - 7\frac{4}{6}$ **16.** $3\frac{7}{9} - 1\frac{2}{9}$ **18.** $7\frac{7}{8} - 4\frac{2}{8}$

To name the sum $1\frac{4}{5} + 2\frac{3}{5}$ we need another step:

$$1\tfrac{4}{5} + 2\tfrac{3}{5} = 3 + \frac{4+3}{5} = 3 + \tfrac{7}{5} = 3 + 1\tfrac{2}{5} = 4\tfrac{2}{5}$$

Name the sums.

19. $3\frac{2}{3} + 2\frac{2}{3}$ **20.** $4\frac{4}{5} + 2\frac{2}{5}$ **21.** $3\frac{3}{5} + 2\frac{2}{5}$ **22.** $6\frac{7}{8} + 2\frac{7}{8}$ **23.** $4\frac{7}{10} + 7\frac{9}{10}$ **24.** $3\frac{7}{12} + 4\frac{11}{12}$

Study these methods for naming the sum $4\frac{1}{4} + 2\frac{1}{3}$ and the difference $6\frac{1}{3} - 4\frac{1}{4}$.

A	B	A	B
$4\frac{1}{4} + 2\frac{1}{3}$	$4\frac{1}{4} = 4\frac{3}{12}$	$6\frac{1}{3} - 4\frac{1}{4}$	$6\frac{1}{3} = 6\frac{4}{12}$
$= 4\frac{3}{12} + 2\frac{4}{12}$	$+2\frac{1}{3} = 2\frac{4}{12}$	$= 6\frac{4}{12} - 4\frac{3}{12}$	$-4\frac{1}{4} = 4\frac{3}{12}$
$= 6 + \frac{3+4}{12} = 6\frac{7}{12}$	$6\frac{7}{12}$	$= 2 + \frac{4-3}{12} = 2\frac{1}{12}$	$2\frac{1}{12}$

EXERCISES

Use method A to name the sums or differences.

1. $5\frac{1}{2} + 4\frac{1}{8}$
2. $7\frac{5}{12} + 1\frac{1}{4}$
3. $5\frac{5}{6} + 3\frac{1}{3}$
4. $3\frac{7}{9} + 2\frac{5}{18}$
5. $4\frac{5}{6} + 2\frac{5}{8}$
6. $9\frac{9}{10} + 3\frac{7}{15}$
7. $4\frac{1}{2} - 2\frac{1}{4}$
8. $4\frac{1}{2} - 1\frac{1}{3}$
9. $7\frac{5}{8} - 2\frac{1}{4}$
10. $6\frac{3}{8} - 4\frac{1}{4}$
11. $9\frac{2}{3} - 4\frac{1}{4}$
12. $9\frac{5}{6} - 3\frac{1}{2}$

Use method B to name the differences.

13. $6\frac{3}{4} - 3\frac{1}{2}$
14. $5\frac{2}{3} - 3\frac{1}{2}$
15. $8\frac{1}{3} - 3\frac{1}{4}$
16. $5\frac{4}{5} - 2\frac{1}{2}$
17. $3\frac{7}{8} - 1\frac{3}{4}$
18. $6\frac{1}{2} - 3\frac{1}{5}$

To name the sum $1\frac{3}{4} + 3\frac{2}{3}$ we need another step.

$$1\frac{3}{4} + 3\frac{2}{3} = 1\frac{9}{12} + 3\frac{8}{12} = 4 + \frac{9+8}{12} = 4 + \frac{17}{12} = 5 + \frac{5}{12}$$

Name the sums.

19. $4\frac{3}{4} + 2\frac{1}{3}$
20. $4\frac{5}{8} + 3\frac{3}{4}$
21. $4\frac{2}{3} + 2\frac{1}{2}$
22. $2\frac{7}{8} + 4\frac{1}{2}$
23. $8\frac{3}{4} + 9\frac{1}{3}$
24. $3\frac{7}{10} + 4\frac{3}{5}$

Subtraction with mixed numerals

Study these methods for naming the difference $5\frac{1}{4} - 2\frac{3}{4}$.

A

$$\begin{array}{rl} & {\scriptstyle 4}\ \ {\scriptstyle \frac{5}{4}} \\ 5\frac{1}{4} = & \not{5} + \not{\frac{1}{4}} \\ \underline{-\ 2\frac{3}{4}} = & \underline{-\ 2 - \frac{3}{4}} \\ & 2 + \frac{2}{4} = 2\frac{1}{2} \end{array}$$

B

$$\begin{array}{r} 4\frac{5}{4} \\ \not{5}\not{\frac{1}{4}} \\ \underline{-\ 2\frac{3}{4}} \\ 2\frac{2}{4} = 2\frac{1}{2} \end{array}$$

EXERCISES

Copy and complete the equations.

1. $4\frac{1}{4} - \frac{3}{4} = 3 + \frac{a}{4} - \frac{3}{4} = b$
2. $2\frac{1}{5} - \frac{2}{5} = 1 + \frac{c}{5} - \frac{2}{5} = d$
3. $4\frac{1}{8} - \frac{5}{8} = 3 + \frac{e}{8} - \frac{5}{8} = f$
4. $2\frac{1}{5} - \frac{4}{5} = 1 + \frac{i}{5} - \frac{4}{5} = h$
5. $3\frac{1}{6} - \frac{5}{6} = 2 + \frac{n}{6} - \frac{5}{6} = j$
6. $5\frac{1}{8} - \frac{7}{8} = 4 + \frac{r}{8} - \frac{7}{8} = l$

Use method A to name the differences.

7. $\begin{array}{rl} & {\scriptstyle 3}\ \ {\scriptstyle \frac{a}{3}} \\ 4\frac{1}{3} = & \not{4} + \not{\frac{1}{3}} \\ \underline{-\ 1\frac{2}{3}} = & \underline{-\ 1 - \frac{2}{3}} \\ & 2 + \frac{c}{3} = b \end{array}$

8. $\begin{array}{rl} & {\scriptstyle d}\ \ {\scriptstyle \frac{6}{5}} \\ 6\frac{1}{5} = & \not{6} + \not{\frac{1}{5}} \\ \underline{-\ 2\frac{3}{5}} = & \underline{-\ 2 - \frac{3}{5}} \\ & 3 + \frac{e}{5} = f \end{array}$

9. $\begin{array}{rl} & {\scriptstyle k}\ \ {\scriptstyle \frac{5}{4}} \\ 9\frac{1}{4} = & \not{9} + \not{\frac{1}{4}} \\ \underline{-\ 5\frac{3}{4}} = & \underline{-\ 5 - \frac{3}{4}} \\ & 3 + \frac{r}{4} = l \end{array}$

Use method B to name the differences.

10. $\begin{array}{r} 6\frac{a}{5} \\ \not{7}\not{\frac{2}{5}} \\ \underline{-\ 3\frac{3}{5}} \\ 3\frac{c}{5} \end{array}$

11. $\begin{array}{r} 4\frac{n}{9} \\ \not{5}\not{\frac{2}{9}} \\ \underline{-\ 1\frac{5}{9}} \\ 3\frac{r}{9} \end{array}$

12. $\begin{array}{r} 7\frac{u}{3} \\ \not{8}\not{\frac{1}{3}} \\ \underline{-\ 6\frac{2}{3}} \\ 1\frac{v}{3} \end{array}$

13. $\begin{array}{r} f\frac{a}{9} \\ \not{9}\not{\frac{2}{9}} \\ \underline{-\ 2\frac{4}{9}} \end{array}$

14. $\begin{array}{r} 9\frac{e}{10} \\ \not{10}\not{\frac{3}{10}} \\ \underline{-\ 6\frac{7}{10}} \end{array}$

Complete the number sentences.
Write $<$, $>$, or $=$ to replace each $\bigcirc$.

1. $\frac{1}{4} + \frac{1}{4} \bigcirc \frac{1}{2}$
2. $\frac{1}{5} + \frac{1}{5} \bigcirc \frac{3}{5}$
3. $\frac{2}{7} + \frac{8}{7} \bigcirc 1$
4. $\frac{3}{4} - \frac{1}{4} \bigcirc \frac{1}{2}$
5. $\frac{3}{8} + \frac{1}{2} \bigcirc \frac{5}{8}$
6. $\frac{3}{4} + \frac{1}{8} \bigcirc \frac{7}{8}$
7. $\frac{1}{2} - \frac{1}{3} \bigcirc \frac{5}{6}$
8. $\frac{9}{6} - \frac{4}{6} \bigcirc \frac{5}{6}$
9. $\frac{1}{3} + \frac{1}{4} \bigcirc \frac{5}{12}$
10. $1\frac{2}{5} - \frac{4}{5} \bigcirc \frac{3}{5}$
11. $3\frac{2}{3} + \frac{4}{3} \bigcirc 4\frac{2}{3}$
12. $3 + 2\frac{2}{3} \bigcirc 5\frac{1}{3}$
13. $5\frac{1}{5} - 4 \bigcirc 1\frac{2}{5}$
14. $4\frac{1}{7} + \frac{3}{7} \bigcirc 4\frac{5}{7}$
15. $6\frac{3}{4} - 5 \bigcirc 1\frac{1}{2}$

Complete the number sentences.
Write $+$ or $-$ to replace each $\triangle$.

16. $\frac{3}{4} \triangle \frac{1}{4} = \frac{1}{2}$
17. $\frac{3}{10} \triangle \frac{4}{10} = \frac{7}{10}$
18. $\frac{8}{9} \triangle \frac{5}{9} = \frac{1}{3}$
19. $\frac{2}{3} \triangle \frac{1}{4} = \frac{11}{12}$
20. $\frac{15}{8} \triangle \frac{9}{8} = \frac{3}{4}$
21. $5\frac{1}{5} \triangle 4 = 1\frac{1}{5}$
22. $2\frac{2}{3} \triangle \frac{1}{3} = 3$
23. $3\frac{1}{4} \triangle \frac{3}{4} = 4$
24. $1\frac{3}{7} \triangle \frac{3}{7} = 1$
25. $1\frac{3}{7} \triangle \frac{6}{7} = \frac{4}{7}$
26. $2\frac{2}{9} \triangle \frac{8}{9} = 3\frac{1}{9}$
27. $6\frac{5}{8} \triangle \frac{7}{8} = 5\frac{3}{4}$
28. $4\frac{3}{5} \triangle \frac{4}{5} = 5\frac{2}{5}$
29. $3\frac{6}{7} \triangle \frac{5}{7} = 3\frac{1}{7}$
30. $1\frac{3}{8} \triangle 2\frac{5}{8} = 4$

Name the sums or differences.

31. $\frac{1}{3} + \frac{5}{3}$
32. $\frac{3}{4} + \frac{3}{4}$
33. $\frac{9}{8} - \frac{7}{8}$
34. $\frac{5}{6} + \frac{4}{6}$
35. $\frac{2}{3} + \frac{1}{4}$
36. $\frac{4}{5} - \frac{1}{10}$
37. $1\frac{5}{7} - \frac{3}{7}$
38. $\frac{2}{3} + \frac{5}{6}$
39. $3\frac{1}{5} + \frac{2}{5}$
40. $8\frac{1}{7} + \frac{6}{7}$
41. $9\frac{4}{5} - \frac{3}{5}$
42. $1\frac{2}{3} + 3\frac{2}{3}$
43. $\frac{5}{6} - \frac{2}{3}$
44. $\frac{1}{2} + \frac{3}{4}$

Practicing addition and subtraction

Name the sums.

1. $3\frac{2}{3} + 1\frac{2}{3}$

2. $5\frac{1}{6} + 3\frac{5}{6}$

3. $4\frac{3}{4} + 2\frac{3}{4}$

4. $5\frac{5}{7} + 4\frac{6}{7}$

5. $8\frac{7}{10} + 5\frac{6}{10}$

6. $7\frac{5}{8} + 6\frac{6}{8}$

7. $3\frac{6}{9} + 4\frac{5}{9}$

8. $2\frac{8}{11} + 8\frac{4}{11}$

9. $8\frac{4}{5} + 8\frac{3}{5}$

10. $7\frac{3}{8} + 9\frac{7}{8}$

11. $3\frac{4}{5} + 10\frac{4}{5}$

12. $15\frac{2}{3} + 2\frac{2}{3}$

Name the differences.

13. $8\frac{5}{7} - 5\frac{2}{7}$

14. $13\frac{1}{3} - 7\frac{2}{3}$

15. $7\frac{3}{4} - 3\frac{1}{4}$

16. $8\frac{2}{5} - 5\frac{4}{5}$

17. $9\frac{2}{3} - 2\frac{1}{3}$

18. $6\frac{1}{4} - 2\frac{3}{4}$

19. $11\frac{8}{11} - 5\frac{3}{11}$

20. $9\frac{3}{5} - 3\frac{4}{5}$

21. $13\frac{9}{10} - 6\frac{2}{10}$

22. $12\frac{1}{6} - 4\frac{5}{6}$

23. $9\frac{5}{6} - 4\frac{5}{6}$

24. $14\frac{3}{8} - 7\frac{5}{8}$

Name the differences.

25. $8\frac{2}{5} - 2\frac{4}{5}$

26. $7\frac{2}{7} - 3\frac{3}{7}$

27. $11\frac{1}{2} - 7\frac{3}{4}$

28. $13\frac{2}{9} - 4\frac{8}{9}$

29. $18\frac{3}{10} - 7\frac{7}{10}$

30. $12\frac{4}{9} - 6\frac{5}{9}$

Solve the problem.

31. Leonard is $4\frac{3}{4}$ feet tall and his younger brother is $2\frac{1}{4}$ feet tall. What is the sum of their heights?

Practicing addition and subtraction

Complete the tables.

1.

+	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{2}{5}$	$\frac{3}{7}$
$\frac{2}{3}$	*a*	*c*	*e*	*i*	*n*	*r*

2.

+	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{6}$	$\frac{6}{7}$
$\frac{3}{4}$	*s*	*u*	*v*	*x*	*z*	*a*

3.

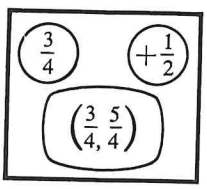
−	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{4}$
$\frac{1}{6}$	*c*	*e*	*i*	*n*	*r*	*s*

4.

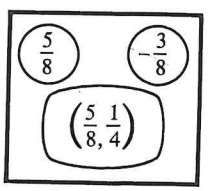
−	$\frac{7}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{5}$
$\frac{3}{8}$	*u*	*v*	*x*	*z*	*a*	*c*

Use the function TVs to complete the sets of number pairs.

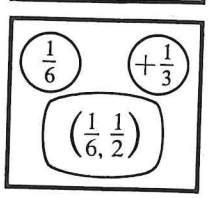
5. $\left\{\left(\frac{3}{4}, \frac{5}{4}\right), \left(\frac{2}{3}, a\right), \left(\frac{1}{4}, b\right), \left(\frac{3}{8}, c\right), \left(\frac{1}{6}, d\right), \left(\frac{7}{10}, e\right)\right\}$



6. $\left\{\left(\frac{5}{8}, \frac{1}{4}\right), \left(\frac{3}{4}, f\right), \left(\frac{1}{2}, g\right), \left(\frac{2}{3}, h\right), \left(\frac{7}{6}, i\right), \left(\frac{7}{10}, j\right)\right\}$



7. $\left\{\left(\frac{1}{6}, \frac{1}{2}\right), \left(\frac{1}{2}, k\right), \left(\frac{2}{3}, l\right), \left(1\frac{1}{3}, m\right), \left(1\frac{5}{6}, n\right), \left(3\frac{2}{3}, p\right)\right\}$



Name the sums or differences.

8. $47\frac{2}{3} + 28\frac{1}{6}$

9. $95\frac{4}{5} + 77\frac{3}{4}$

10. $154\frac{9}{10} + 96\frac{5}{6}$

11. $749\frac{2}{7} + 378\frac{7}{9}$

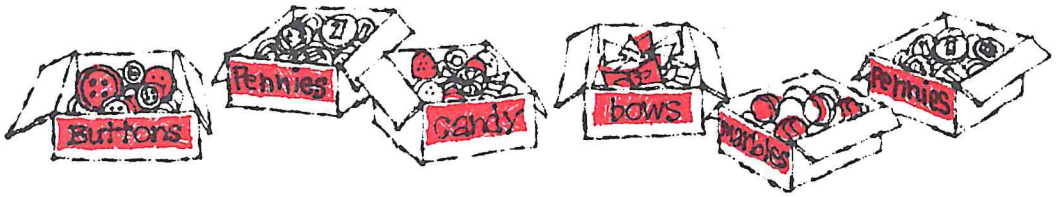
12. $96\frac{5}{8} - 29\frac{1}{4}$

13. $52\frac{3}{5} - 19\frac{1}{3}$

14. $128\frac{7}{10} - 65\frac{1}{6}$

15. $608\frac{2}{3} - 187\frac{7}{8}$

Probability



There are buttons in 1 of the 6 boxes, that is, in $\frac{1}{6}$ of the boxes. There are pennies in 2 of the 6 boxes, that is, in $\frac{2}{6}$ or $\frac{1}{3}$ of the boxes. If you were to choose one of the boxes, the **probability** (**chance**) that it would contain buttons would be 1 out of 6, that is, $\frac{1}{6}$; the chance that it would contain pennies would be 2 out of 6, that is, $\frac{2}{6}$ or $\frac{1}{3}$. Once again we are using a fractional number to compare a subset with a set.

EXERCISES

Solve the problems.

1. If there were pennies in 3 of the boxes above, what is the chance that any box you choose would contain pennies?
2. If there are 9 apples in a basket, 4 green and 5 ripe, what is the chance you would get a ripe apple if you took one without looking?
3. If there are 7 packages, all wrapped the same way, 5 with candy in them, what is the chance you would pick a package without candy in it?
4. What is the chance that you would pick a blue sweater, without looking, from a drawer in which there are 5 sweaters, 3 of them blue?
5. If you know 40 of the 45 basic addition-subtraction facts, what is the chance you might be asked to state a fact you do not know?

Consider $A = \{\square\ \square\ \square\ \triangle\ \triangle\ \triangle\ \triangle\ \triangle\ \bigcirc\ \bigcirc\ \bigcirc\ \bigcirc\}$. If you were to touch one of the objects in A without looking, what is the probability that the object you touch would be

6. a square?
7. a triangle?
8. a circle?
9. a triangle or a circle?
10. a circle or a square?
11. a square or a triangle?

Half of the numbers in $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ are even. If you choose a number from A without thinking, the probability of choosing an even number is $\frac{1}{2}$. We may write this as an equation: $P(\text{even number}) = \frac{1}{2}$.

EXERCISES

Write an equation to solve each of these problems.

1. What is the probability that the number you choose from set A above is odd? a multiple of 3? a multiple of 4? not a multiple of 4?

2. For $B = \{21, 22, 23, 24, 25, 26, 27, 28, 29\}$, what is the probability of choosing an even number? a multiple of 3? a multiple of 7? a number greater than 25?

3. There are 50 whole numbers less than 50. Fifteen of these are primes. If you choose a whole number less than 50, what is the probability that it is a prime? is not a prime?

4. There are 100 whole numbers less than 100. Twenty-five of these are primes. If you choose a whole number less than 100, what is the probability that it is a prime? is not a prime?

5. The sets in this chart contain marbles, some red, some black, and perhaps some of a third color. Copy and complete the chart.

total number of marbles	8	15	32	20	24	20	40	16
number of red marbles	4	6	20	6	8	5	24	0
number of black marbles	4	9	12	10	12	15	16	16
***P*(red marbles)**	***a***	***a***	***a***	***a***	***a***	***a***	***a***	***a***
***P*(black marbles)**	***b***	***b***	***b***	***b***	***b***	***b***	***b***	***b***
***P*(marbles of another color)**	***c***	***c***	***c***	***c***	***c***	***c***	***c***	***c***

Two of the balls are red, 3 are gray and 5 are black. If you reach into the box and take a ball without looking, the probability of getting a red ball is $\frac{2}{10}$, of getting a gray ball is $\frac{3}{10}$. $\frac{2}{10} < \frac{3}{10}$ The probability of getting a red ball is less than the probability of getting a gray ball.

$2 + 5 = 7$ of the balls are red *or* black. The probability of getting a red ball or a black ball is $\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$.

EXERCISES

Solve the problems from the example above.

1. Which is greater, the probability of getting a red or a black ball? of getting a gray or a black ball?
2. What is the probability of getting a red or a gray ball? of getting a gray or a black ball?
3. What is the probability of getting a red, a gray, or a black ball?
4. We see that there are no yellow balls. What is the probability of getting a yellow ball?

Solve the problems.

In a drawer there are 12 pairs of socks, 3 green, 4 blue, 5 red. If you reach into the drawer without looking,

5. what is the probability of getting a pair of green socks? a pair of red socks? a pair of blue socks?
6. what is the probability of getting a pair of green or blue socks? a pair of green or red socks? a pair of blue or red socks?
7. what is the probability of getting a pair of green, blue, or red socks?
8. what is the probability of getting a pair of purple socks?

For $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, the probability that a number is a multiple of 3 is $\frac{3}{10}$, the probability that a number is not a multiple of 3 is $\frac{7}{10}$. Since $\frac{7}{10} > \frac{3}{10}$, the probability of choosing a number from A which is not a multiple of 3 is greater than the probability of choosing a number which is a multiple of 3. Since $\frac{7}{10} + \frac{3}{10} = 1$, the probability that a number in A is a multiple of 3 or is not a multiple of 3 is $\frac{3}{10} + \frac{7}{10} = 1$. Since no member of A is both a multiple of 3 and not a multiple of 3, the probability of this is $\frac{0}{10} = 0$.

EXERCISES

Solve the problems.

1. For $B = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, what is the probability of choosing a multiple of 3? a multiple of 4?
2. For set B, which is greater, the probability of choosing a multiple of 3 or a multiple of 4?
3. For set B, is there a greater probability of choosing a prime number or a number which is not prime?

For $C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, name the probabilities.

4. P(choosing 4)
5. P(choosing a number less than 4)
6. P(choosing 5 or 6)
7. P(choosing a number other than 5 or 6)
8. P(a number greater than 8)
9. P(a number less than 8)
10. P(a number less than 10)
11. P(a number greater than 10)

For $A = \{$ ■ ■ ▲ ▲ ● $\}$, name the probabilities.

12. P(choosing ■)
13. P(choosing ▲)
14. P(choosing ●)
15. P(■ or ●)
16. P(■ or ▲)
17. P(● or ▲)
18. P(■, ▲, or ●)
19. P(◆)
20. P(■) + P(▲) + P(●)

Complete the sentences.

21. If something is certain to happen, its probability is ____?____.
22. If something cannot happen, its probability is ____?____.

CHECKING UP

Solve the equations.

1. $\frac{4}{8} + \frac{3}{8} = a$
2. $\frac{6}{7} + \frac{4}{7} = b$
3. $\frac{3}{8} + \frac{3}{4} = c$
4. $\frac{2}{3} + \frac{1}{5} = d$
5. $\frac{7}{10} - \frac{4}{10} = e$
6. $\frac{9}{8} - \frac{3}{8} = f$
7. $\frac{5}{6} - \frac{1}{2} = g$
8. $\frac{4}{9} - \frac{1}{6} = h$

Name the sums.

9. $2\frac{3}{5} + \frac{1}{5}$
10. $3\frac{1}{4} + 2\frac{1}{4}$
11. $6\frac{1}{3} + 7$
12. $5\frac{2}{7} + 2\frac{5}{7}$
13. $6\frac{5}{6} + 2\frac{5}{6}$
14. $2\frac{3}{8} + 3\frac{7}{8}$
15. $4\frac{3}{8} + 4\frac{1}{4}$
16. $7\frac{1}{2} + 4\frac{2}{5}$
17. $8\frac{2}{3} + 7\frac{5}{6}$
18. $9\frac{3}{10} + 9\frac{11}{15}$
19. $7\frac{3}{7} + 9\frac{4}{5}$
20. $2\frac{7}{8} + \frac{9}{10}$

Name the differences.

21. $5\frac{2}{3} - 1\frac{1}{3}$
22. $4\frac{7}{8} - 2\frac{3}{8}$
23. $6\frac{7}{9} - 2\frac{4}{9}$
24. $12\frac{7}{10} - 3\frac{9}{10}$
25. $13\frac{4}{11} - 10\frac{4}{11}$
26. $12\frac{2}{5} - 7\frac{3}{5}$
27. $10\frac{2}{9} - 7\frac{4}{9}$
28. $8\frac{3}{4} - 4\frac{1}{2}$
29. $9\frac{3}{5} - 2\frac{1}{3}$
30. $6\frac{1}{2} - 1\frac{5}{6}$
31. $4\frac{2}{5} - 1\frac{5}{8}$
32. $9\frac{1}{7} - \frac{3}{4}$

Solve the problems.

33. Sandi spent $\frac{1}{3}$ of her allowance for lunches at school, and $\frac{3}{8}$ of her allowance on records. What part of her allowance does she have left?

34. Imagine you are waiting at a bus stop for a friend to get off a bus from the north. Three buses from the north and 4 buses from the south arrive about the same time. What is the probability that your friend will get off the first bus? will get off the first bus from the north?

We may use the common factor property to add and subtract fractional numbers.

Solve the equations.

1. $\frac{2}{7} + \frac{3}{7} = \left(2 \times \frac{1}{7}\right) + \left(3 \times \frac{1}{7}\right)$

$= a \times \frac{1}{7}$

$= b$

2. $\frac{7}{10} - \frac{3}{10} = \left(7 \times \frac{1}{10}\right) - \left(3 \times \frac{1}{10}\right)$

$= e \times \frac{1}{10}$

$= f$

To name the sum or difference of 2 fractional numbers with the same denominator, find the sum or difference of their numerators and keep the same denominator.

Name the sums or differences.

3. $\frac{3}{7} + \frac{4}{7}$

4. $\frac{2}{9} + \frac{4}{9}$

5. $\frac{8}{9} - \frac{3}{9}$

6. $\frac{7}{12} - \frac{5}{12}$

7. $\frac{2}{3} + \frac{4}{3}$

8. $\frac{7}{10} + \frac{4}{10}$

9. $\frac{9}{11} - \frac{2}{11}$

We may name sums or differences by using expanded numerals or the working form.

Name the sums or differences.

10. $6\frac{3}{4} = 6 + \frac{3}{4}$

$-\ 2\frac{1}{4} = -\ 2 - \frac{1}{4}$

$a + b = c$

11. $2\frac{1}{7} = 2 + \frac{1}{7}$

$+\ 3\frac{4}{7} = 3 + \frac{4}{7}$

$d + e = f$

12. $5\frac{7}{9} = 5 + \frac{7}{9}$

$-\ 4\frac{5}{9} = -\ 4 - \frac{5}{9}$

$g + h = i$

Name the sums or differences.

13. $5\frac{1}{3} + 6\frac{1}{3}$

14. $6\frac{2}{7} + 3\frac{5}{7}$

15. $2\frac{3}{8} + 4\frac{3}{8}$

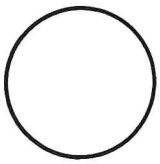
16. $7\frac{2}{5} - 3\frac{3}{5}$

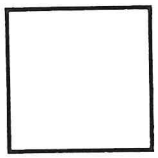
17. $13\frac{1}{4} - 4\frac{3}{4}$

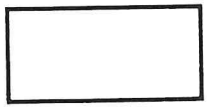
18. $15\frac{2}{9} - 8\frac{7}{9}$

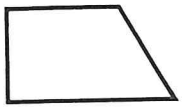
LOOKING WAY BACK

Name the plane figures.

1. 

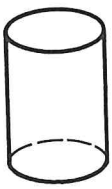
2. 

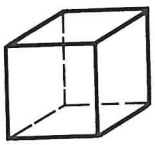
3. 

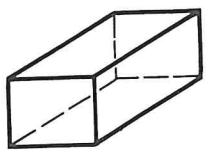
4. 

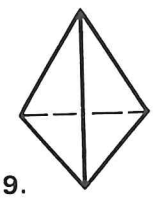
Name the space figures.

5.

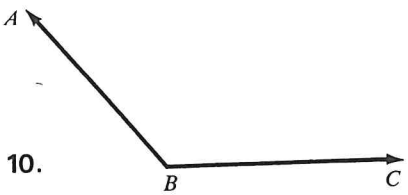
6. 

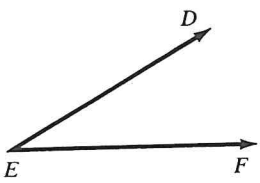
7. 

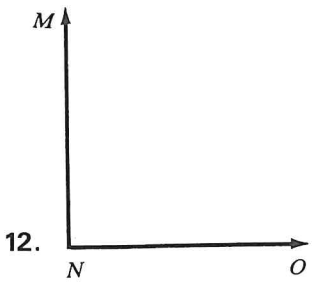
8. 

9. 

Measure the angles (in degrees).

10. 

11. 

12. 

Name the set of factors of each number.

13. 20 **14.** 36 **15.** 31 **16.** 48 **17.** 91 **18.** 120

Name the common factors of each pair of numbers.

19. 12 and 27 **20.** 15 and 30 **21.** 12 and 15 **22.** 36 and 48

Name the least common multiple of each pair of numbers.

23. 8 and 12 **24.** 10 and 15 **25.** 24 and 36 **26.** 7 and 11

Write a factor tree and the prime factorization for each number.

27. 120 **28.** 90 **29.** 60 **30.** 24 **31.** 64 **32.** 90

To add $4\frac{5}{8}$ and $5\frac{7}{8}$, you may think as follows: $4\frac{5}{8} + \frac{3}{8} = 5$, $5 + 5\frac{1}{2} = 10\frac{1}{2}$. Can you see how the $\frac{7}{8}$ is renamed as $\frac{3}{8} + \frac{1}{2}$?

Use this method to name the sums.

1. $1\frac{7}{8} + 3\frac{2}{8} = a$
2. $3\frac{2}{3} + 6\frac{2}{3} = b$
3. $5\frac{1}{6} + 7\frac{5}{6} = c$
4. $7\frac{3}{8} + 1\frac{7}{8} = d$
5. $2\frac{7}{9} + 3\frac{8}{9} = e$
6. $5\frac{7}{10} + 4\frac{3}{10} = f$

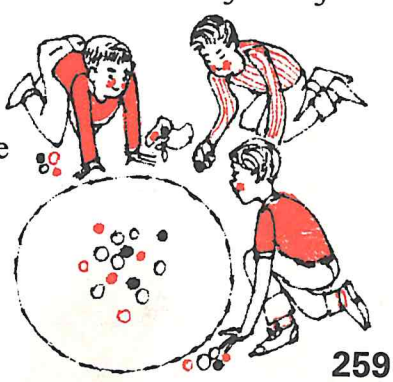
To subtract $2\frac{3}{4}$ from $5\frac{1}{4}$, you may think as follows: The difference, $5\frac{1}{4} - 2\frac{3}{4}$, will not be changed if we add $\frac{1}{4}$ to the $5\frac{1}{4}$ and also to the $2\frac{3}{4}$. $5\frac{1}{4} - 2\frac{3}{4} = 5\frac{1}{2} - 3 = 2\frac{1}{2}$

Use this method to name the differences.

7. $5\frac{1}{3} - 2\frac{2}{3} = a$
8. $6\frac{1}{4} - 2\frac{3}{4} = b$
9. $4\frac{3}{8} - 1\frac{7}{8} = c$
10. $9\frac{2}{9} - 4\frac{5}{9} = d$
11. $11\frac{3}{7} - 9\frac{5}{7} = e$
12. $7\frac{2}{5} - 4\frac{4}{5} = f$

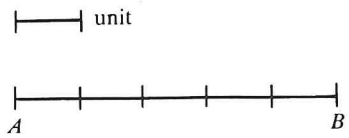
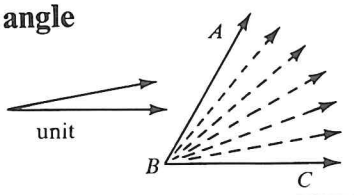
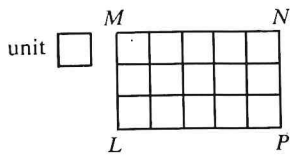
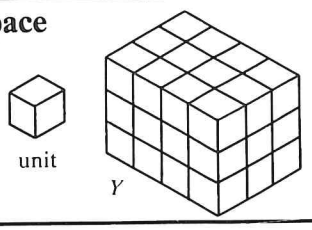
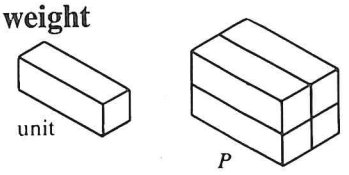
Just For Fun

13. Larry gave $\frac{1}{2}$ of his marbles to Mike. Mike gave Tony $\frac{1}{2}$ of the marbles Larry gave him. Tony gave Sam $\frac{1}{2}$ of the marbles that Mike gave him. What fractional number of Larry's marbles did Sam have?



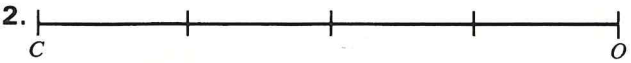
Measuring

Study this chart.

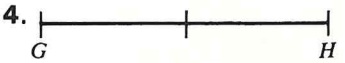
object	statement	unit	
		English	metric
line segment 	m($\overline{AB}$) = 5 The **length** of $\overline{AB}$ is 5 units.	inch foot yard mile	centimeter meter kilometer
angle 	m($\angle ABC$) = 6 The **size** of angle ABC is 6 units.	degree	degree
region 	m($MNPL$) = 15 The **area** of rectangle $MNPL$ is 15 square units.	square inch square foot square yard square mile	square centimeter hectare
space 	m(Y) = 36 The **volume** of rectangular prism Y is 36 cubic units.	cubic inch cubic foot cubic yard	cubic centimeter cubic meter
weight 	m(P) = 4 The **weight** of the butter is 4 units.	ounce pound ton	gram kilogram
liquid	m(S) = 2 The **capacity** of the container is 2 units.	pint quart gallon	liter

Name the measures of these line segments. The unit is

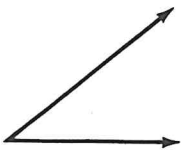
1.

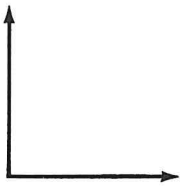
2. 

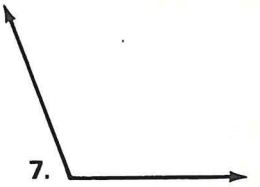
3.

4. 

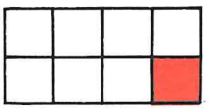
Name the measures of these angles. The unit is a degree.

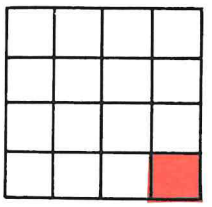
5. 

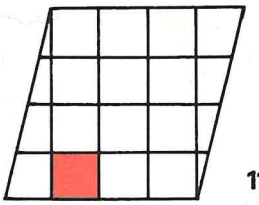
6. 

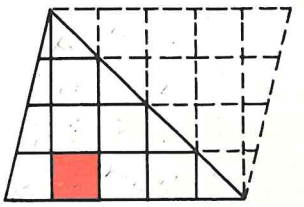
7. 

Name the areas of these regions. The unit is

8. 

9. 

10. 

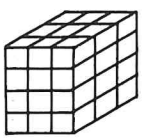
11. 

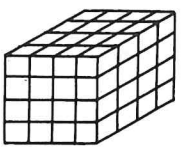
Name the area of

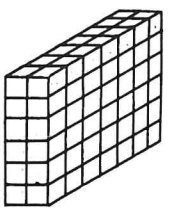
12. a square with sides of 12″

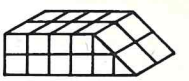
13. a triangle with a base of 14″ and a height of 11″

Name the volume in these closed surfaces. The unit is

14. 

15. 

16. 

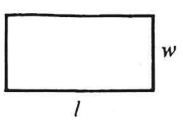
17. 

*Name the volume in

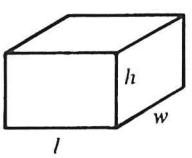
18. a cube with edges of 7″

19. a rectangular prism with edges 4″, 5″ and 6″

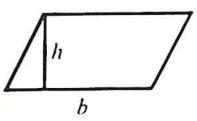
Write an equation for the measure of the area or volume in each figure.



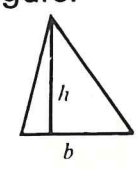
20. $A = a$



21. $V = b$

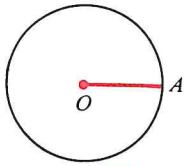


22. $A = c$

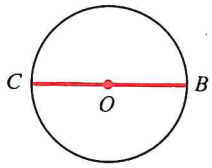


23. $A = d$

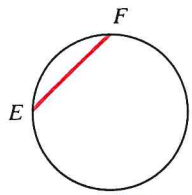
Working with circles



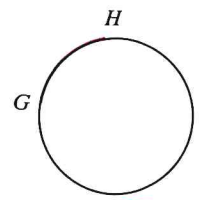
radius $\overline{OA}$



diameter $\overline{CB}$



chord $\overline{EF}$



arc $\overset{\frown}{GH}$

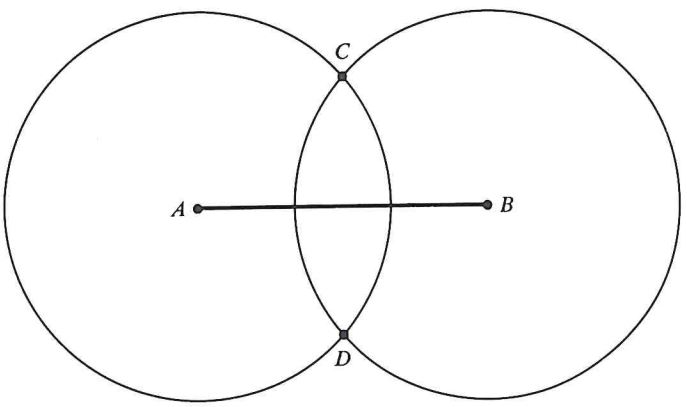
EXERCISES

Answer the questions.

1. How is a circle different from other simple closed curves?
2. Name the set of all points on the floor 6′ from where you are sitting.
3. Name the set of all points in one plane 3″ from a given point.

Follow these steps, then answer the questions.

4. Draw a line segment $\overline{AB}$ of length $1\frac{1}{2}$″
5. With A as the center draw a circle of radius 1″. (If you have forgotten how to draw a circle, see page 76.)
6. With B as the center draw a circle of radius 1″.



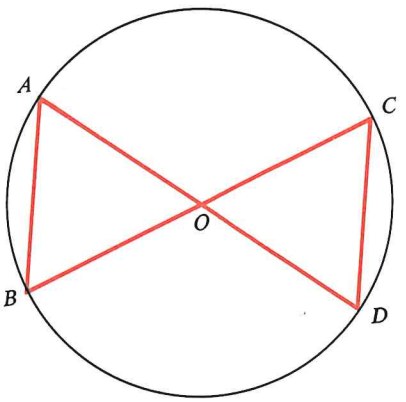
7. Label the points where the circles intersect, C and D.
8. How far is C from A?
9. from B?
10. How far is D from A?
11. from B?
12. Join the point C to A and to B to form a triangle. How long is each side of the triangle?

Answer the questions.

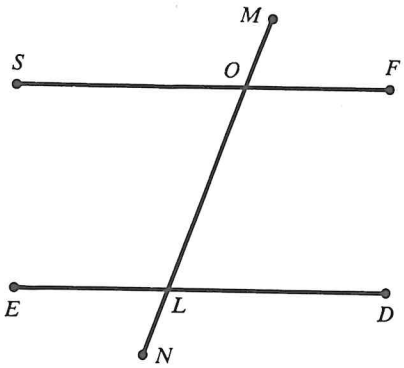
13. A line segment joining any 2 points on a circle is called a **chord.** Name the longest chord in a circle.
14. How many *arcs* are formed by any 2 points on a circle?

*15. How many arcs are formed by any 3 points on a circle?

Name the measures in inches or degrees, then answer the questions.



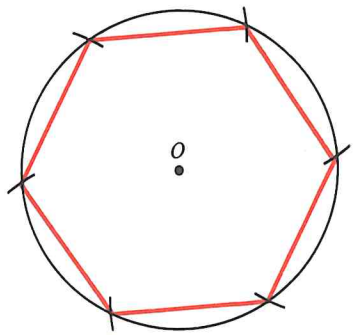
1. m($\overline{AD}$)
2. m($\overline{CB}$)
3. m($\angle COD$)
4. m($\angle BOA$)
5. m($\angle COA$)
6. m($\angle DOB$)
7. m($\overline{CD}$)
8. m($\overline{AB}$)
9. Is $\overline{AD}$ congruent to $\overline{CB}$?
10. Name the congruent angles in the figure.
11. Do you think that $\overset{\frown}{CD}$ is the same size as $\overset{\frown}{AB}$?
12. Is chord $\overline{CD}$ congruent to chord $\overline{AB}$?



Name the measures, in degrees.

13. m($\angle SOL$)
14. m($\angle OLD$)
15. m($\angle LOF$)
16. m($\angle OLE$)
17. Name the congruent angles in the figure.

Draw the figure, then answer the questions.

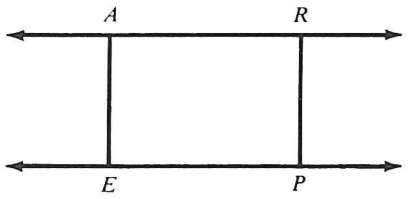


18. Draw a circle with radius of 1″.
19. Mark off 6 arcs on the circle using a radius of 1″.
20. Join the 6 points. This forms a **hexagon.** How many sides does it have?
21. Are the sides of the hexagon congruent?
22. Measure each angle. Are the angles of the hexagon congruent?

*23. Are any sides parallel?

Parallel line segments

$\overleftrightarrow{AR}$ and $\overleftrightarrow{EP}$ are parallel.
$\overline{AE}$ and $\overline{RP}$ are perpendicular to $\overline{AR}$ and $\overline{EP}$.
So $\overline{AE}$ and $\overline{RP}$ are parallel.
Because $\overline{AR}$ and $\overline{EP}$ are parallel, $m(\overline{AE}) = m(\overline{RP})$.
Because $\overline{AE}$ and $\overline{RP}$ are parallel, $m(\overline{AR}) = m(\overline{EP})$.

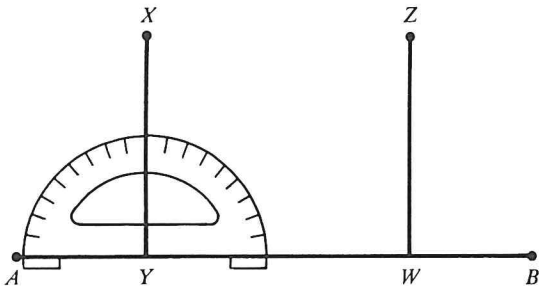


EXERCISES

We can draw two parallel line segments in many ways.

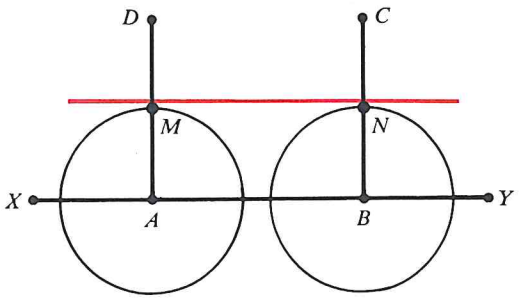
1. *Step 1:* Draw any line segment, $\overline{AB}$. Place your protractor with its side on $\overline{AB}$ and draw a line segment $\overline{XY}$ perpendicular to $\overline{AB}$.

Step 2: Draw another line segment, $\overline{ZW}$, perpendicular to $\overline{AB}$. $\overline{XY}$ and $\overline{ZW}$ are parallel.



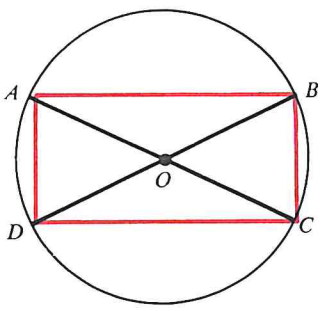
2. *Step 1:* Draw 2 line segments, $\overline{AD}$ and $\overline{BC}$, perpendicular to $\overline{XY}$. ($\overline{AD}$ and $\overline{BC}$ are parallel.)

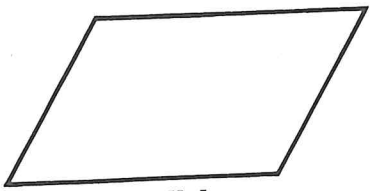
Step 2: Draw 2 circles of the same radius, with centers A and B, one intersecting $\overline{AD}$ at M, the other intersecting $\overline{BC}$ at N. Join M and N. $\overline{MN}$ is parallel to $\overline{AB}$.



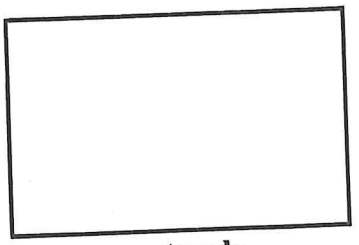
3. *Step 1:* Draw 2 line segments intersecting at O.

Step 2: Draw a circle of any radius with center O. If the circle intersects the line segments at A, B, C, and D, we can draw 2 pairs of parallel line segments: $\overline{AB}$ and $\overline{DC}$, $\overline{AD}$ and $\overline{BC}$.

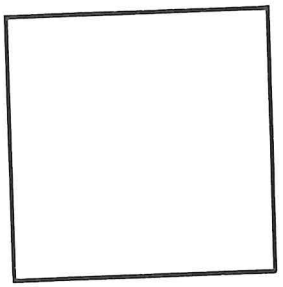




parallelogram



rectangle

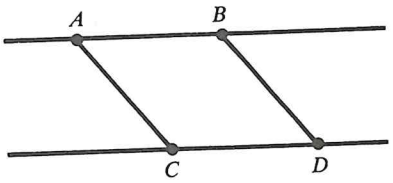


square

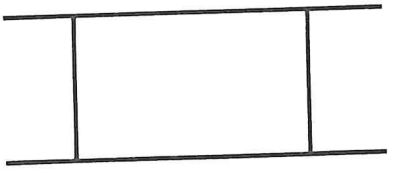
Parallelograms are quadrilaterals with opposite sides parallel and congruent. **Rectangles** and **squares** are special parallelograms.

EXERCISES

To draw parallelograms and rectangles, we may use the methods for drawing parallels.



1. *Step 1:* Draw 2 parallel line segments.
   Step 2: Mark off congruent line segments $\overline{AB}$ and $\overline{CD}$.
   Step 3: Join A and C, B and D.



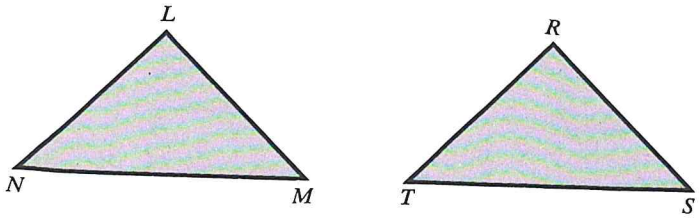
2. *Step 1:* Draw 2 parallel line segments.
   Step 2: Draw 2 line segments perpendicular to the line segments in step 1.

3. Use the steps in exercise 2 on page 264 to draw a rectangle with sides of 3″ and 2″.

Use the methods for drawing parallels and rectangles to draw

4. a rectangle with sides of 4″ and 2″
5. a square with sides of 3″
6. Name the area and perimeter of the figures in exercises 4 and 5.

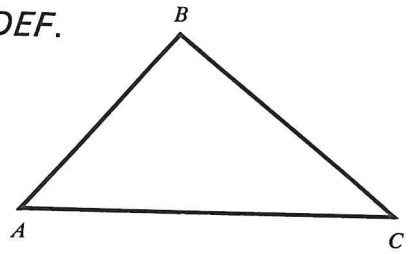
Angles and triangles

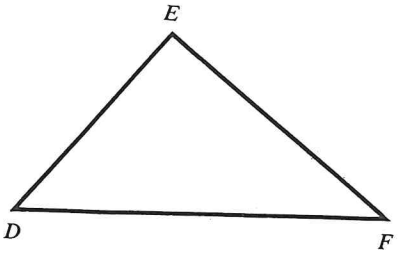


If 2 triangles are the same size and shape, they are **congruent.** Triangles LMN and RST are congruent.

EXERCISES

Name the measures for triangles ABC and DEF. The sides are measured in inches, the angles in degrees.

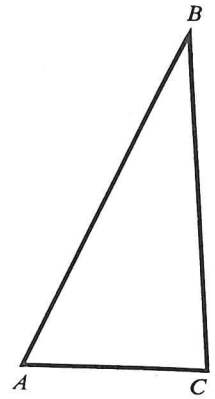




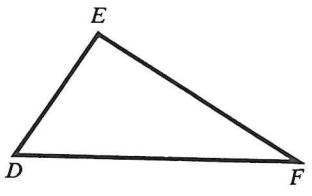
1. $m(\overline{AB})$
2. $m(\overline{DE})$
3. $m(\overline{BC})$
4. $m(\overline{EF})$
5. $m(\overline{CA})$
6. $m(\overline{FD})$
7. $m(\angle ABC)$
8. $m(\angle DEF)$
9. $m(\angle BCA)$
10. $m(\angle EFD)$
11. $m(\angle BAC)$
12. $m(\angle EDF)$
13. What can you say about the triangles ABC and DEF?

Name the measures of the angles and sides of each triangle.

14.



15.



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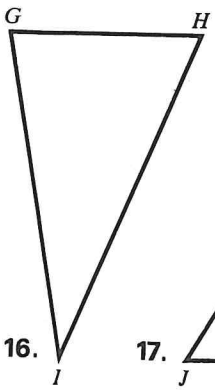
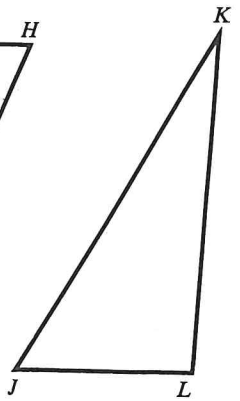
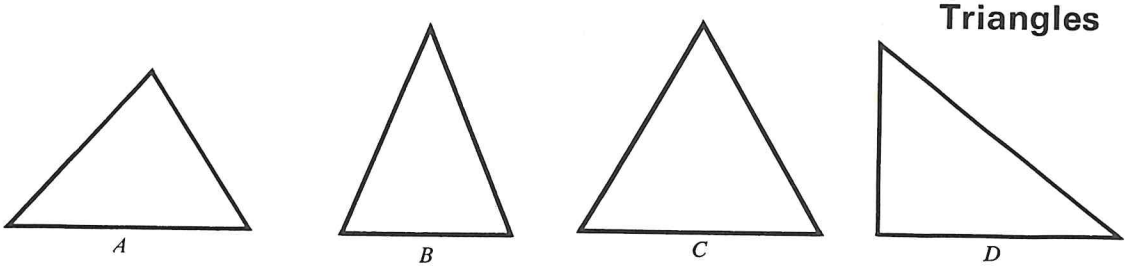


17.



18. Name the congruent triangles in exercises 14–17.



Triangles are formed by the union of 3 line segments with common end points. Triangle B has 2 congruent sides, C has 3 congruent sides, and D is a right triangle.

EXERCISES

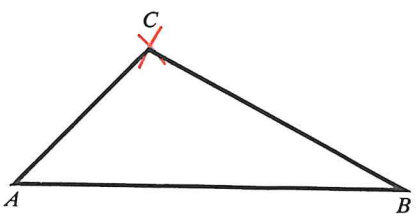
1. Use this method to draw a triangle if you know the lengths of the 3 sides. For example, to draw a triangle ABC with sides 1″, $1\frac{1}{2}$″, and 2″:

Step 1: Draw $\overline{AB}$ 2″ long.

Step 2: With center A draw an arc of a circle of radius 1″.

Step 3: With center B draw an arc of a circle of radius $1\frac{1}{2}$ ″.

Step 4: If the 2 arcs intersect in C, draw $\overline{AC}$ and $\overline{BC}$.

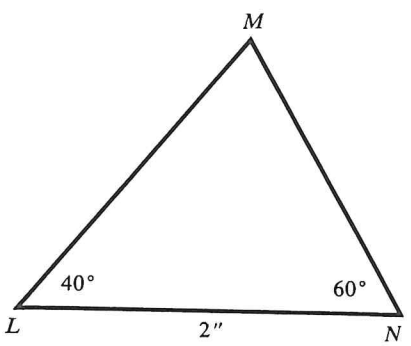


2. Use this method to draw a triangle RST if you know 2 angles and the side between them. For example, to draw a triangle congruent to LMN:

Step 1: Draw $\overline{RS}$ 2″ long.

Step 2: At R draw an angle of 40°.

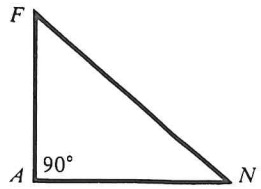
Step 3: At S draw an angle of 60°. The sides of the angles you have drawn meet in T.



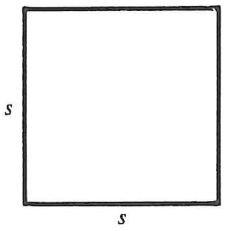
3. Use method 1 to draw a triangle with sides of 3″, 4″ and 2″.

4. Use method 2 to draw a triangle congruent to the one you drew in exercise 3.

Right triangles



Remember: A triangle which has a right angle is called a **right triangle.** $\angle FAN$ is 90°, a right angle.

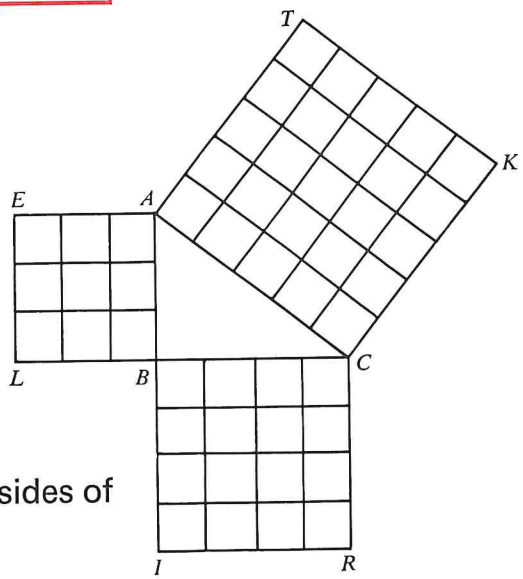


If a square has sides of length s, then the area (A) of the square region will be $s \times s = s^2$, square units.

EXERCISES

For triangle ABC, name

1. $m(\overline{AB})$ **2.** $m(\overline{BC})$ **3.** $m(\overline{AC})$



Complete the sentences.

4. The area of $LEAB$ is ___?___.

5. The area of $BCRI$ is ___?___.

6. The area of $LEAB$ + the area of $BCRI$ is ___?___.

7. The area of $ATKC$ is ___?___.

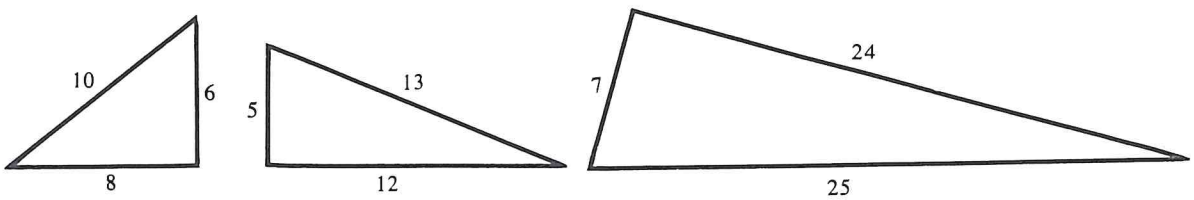
Name the area of a square region with sides of

8. 6″ **11.** 5″

9. 8″ **12.** 12″

10. 10″ **13.** 13″

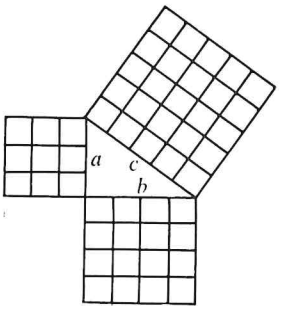
Study the right triangles, then solve the equations.



14. $6^2 + 8^2 = a$ **16.** $5^2 + 12^2 = c$ **18.** $24^2 + 7^2 = e$

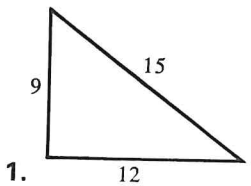
15. $10^2 = b$ **17.** $13^2 = d$ **19.** $25^2 = f$

A very famous Greek mathematician named **Pythagoras** discovered that for a *right triangle*, $c^2 = a^2 + b^2$, where a, b, and c are the measures of the sides and c is the measure of the side opposite the right angle. We call this the **Theorem of Pythagoras.**

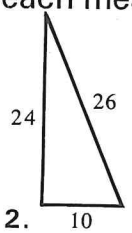


EXERCISES

Find the square of each measure for the right triangles.



1.



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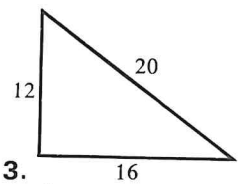
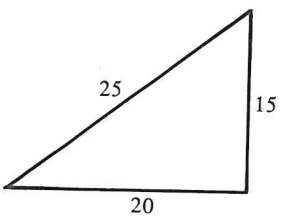


3.



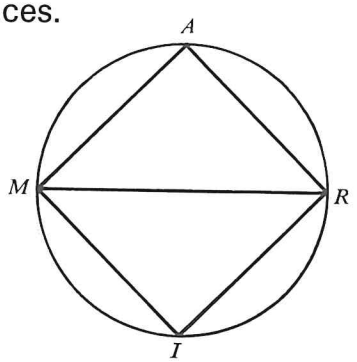
4.

Complete the sentence and answer the question.

5. The square of the measure of the side opposite the right angle of a right triangle equals the sum of the ____?____ of the measures of the other sides.

6. Does the square of the measure of the side opposite the right angle equal the sum of the squares of the measure of the other 2 sides in each case in exercise 1?

Study the figure, then complete the sentences.



7. $\overline{MR}$ is called a ____?____.

8. $\overline{MA}$, $\overline{AR}$, $\overline{MI}$ and $\overline{RI}$ are called ____?____ of the circle.

9. $m(\angle MAR) = a$

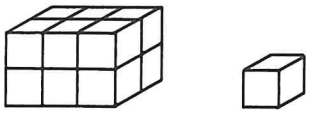
10. $m(\angle MIR) = b$

11. $\angle MAR$ and $\angle MIR$ are ____?____ angles.

12. The ancient Egyptians used to form a right angle by tying 13 knots in a rope at equal intervals and then forming a triangle with sides of 3, 4, and 5 units. Could they be sure that this formed a right angle?

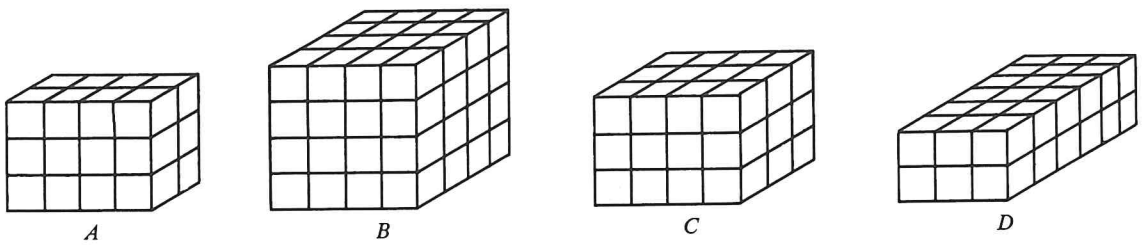
Naming volumes

There are (3×2) cubic units per row. There are 2 rows so there are $(3 \times 2) \times 2 = 12$ cubic units all together. The **volume** in this rectangular prism is 12 cubic units.



EXERCISES

How many cubic units are there in each rectangular prism?

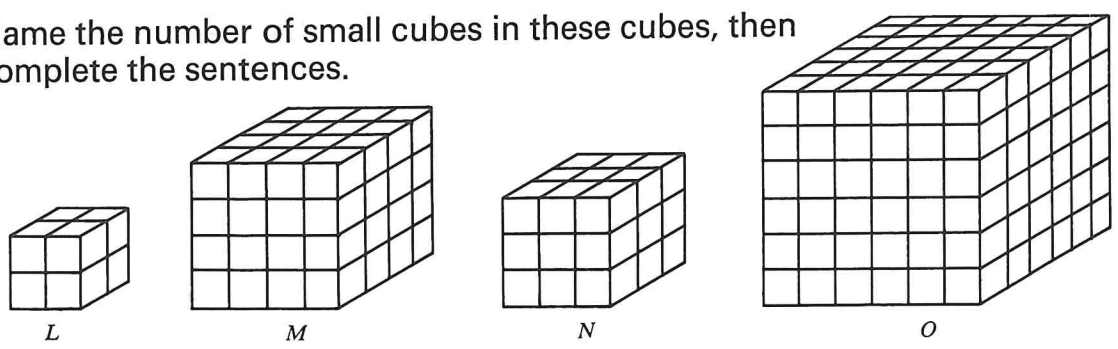


1. $m(A) = a$ **2.** $m(B) = b$ **3.** $m(C) = c$ **4.** $m(D) = d$

Complete the sentence.

5. For a rectangular prism, if l is the number of cubic units along the length, w is the number of cubic units along the width, and h is the number of cubic units along the height, then the product $l \times w \times h$ will name the number of cubic units in the ___?___. We can write this as $V = l \times w \times h$.

Name the number of small cubes in these cubes, then complete the sentences.



6. $m(L) = a$ **7.** $m(M) = b$ **8.** $m(N) = c$ **9.** $m(O) = d$

10. If s is the number of cubic units along the side of a cube, then the number of units of ___?___ in the cube is s^3.

11. We may write $V = s^3$ and say "V equals s ___?___".

Pair each statement with the correct equation.
Write the letter to pair with each of the numbers.
For example, write *(1, c)*.

1.	area of a rectangular region	**a.**	$V = l \times w \times h$
2.	area of a triangular region	**b.**	$A = b \times h$
3.	area of a square region	**c.**	$A = l \times w$
4.	area of a parallelogram region	**d.**	$V = s^3$
5.	volume in a cube	**e.**	$A = s^2$
6.	volume in a rectangular prism	**f.**	$A = \frac{1}{2} \times (b \times h)$

Name the area of

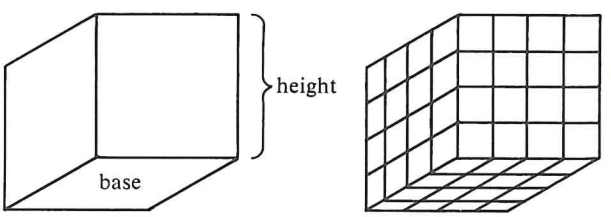
7. a rectangular region with length 26″ and width 18″
8. a triangular region with base 13″ and height 11″
9. the region of a parallelogram with base 47″ and height 39″
10. a square region with $s = 27$, in inches

Name the volume in

11. a rectangular prism with $l = 7$, $w = 9$, $h = 3$, in inches
12. a cube with $s = 6$, in feet
13. a rectangular prism with $l = 12$, $w = 5$, $h = 8$, in inches
14. a cube with sides of 9″

Study the closed surfaces.

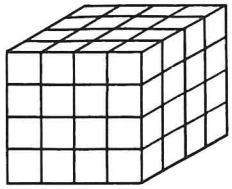
15. Name the *area* of the base of the rectangular prism.
16. Name the height of the prism.
17. Name the volume in the prism.



Complete the sentence.

18. We can think of the volume in a rectangular prism as the product of the measure of the height of the prism and the number of units of ___?___ in the base.

Areas and surfaces



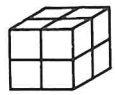
Each face of the cube is the same size and shape. We say that the faces are **congruent.**

The area of 1 face is 16 square units. The area of 6 faces is 96 square units. The total area of the faces of this cube is 96 square units. This is called the **surface area** of the cube.

EXERCISES

Name the surface area of each cube.

1.



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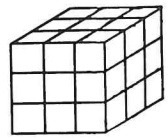
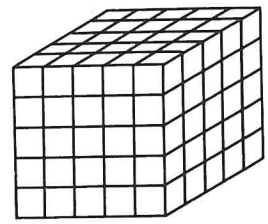
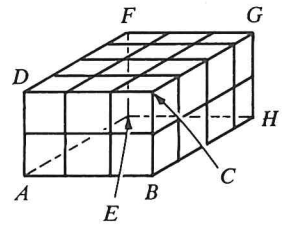


3.



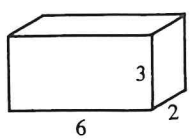
4.

Name these measures for the rectangular prism.

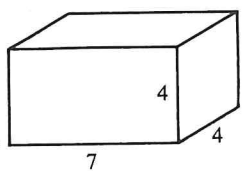


5. m($ABCD$)
6. m($EFGH$)
7. m($ABHE$)
8. m($DCGF$)
9. m($BCGH$)
10. m($ADFE$)
11. m(*surface area*)
12. Name the 3 sets of congruent line segments in the prism.
13. Name the 3 pairs of congruent faces in the prism.

Name the measure of the surface area of each prism.



14.



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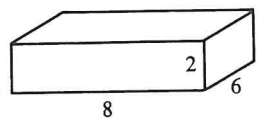

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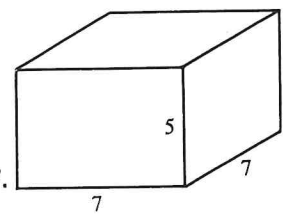


17.

Name the volume in

18. the cubes in exercises 1–4

19. the prisms in exercises 14–17

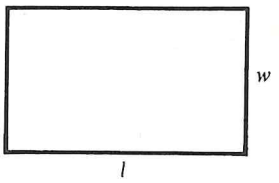
Equations for areas and volume

l = number of units of area along length

w = number of units of area along width

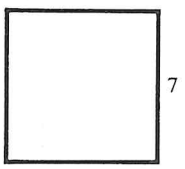
A = total number of units of area

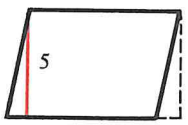
$A = l \times w$ The area is $(l \times w)$ square units.

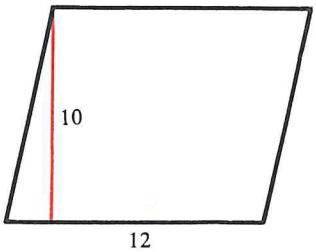


Name the areas of the regions.

1.

2. 

3. 

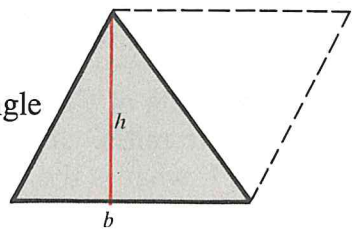
4. 

b = number of units of area along base

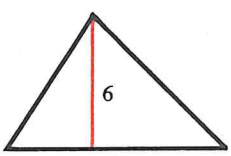
h = number of units of area along height

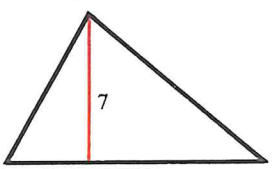
A = total number of units of area in shaded triangle

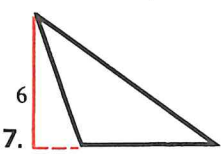
$A = \frac{1}{2} \times b \times h$

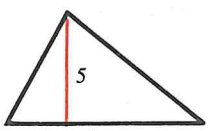


Name the areas of the triangular regions.

5. 

6. 

7. 

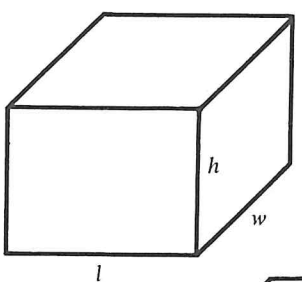
8. 

l = number of units of volume along length

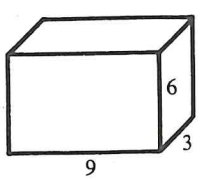
w = number of units of volume along width

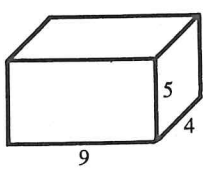
h = number of units of volume along height

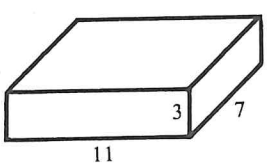
$V = l \times w \times h$ The volume is V units.

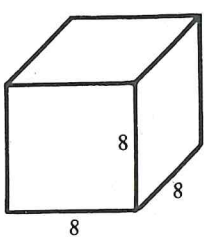


Name the volumes in the closed surfaces.

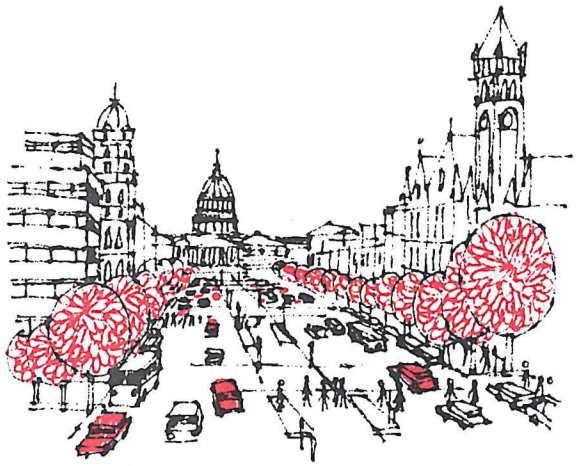
9. 

10. 

11. 

12. 

Areas and volumes



1. Washington, D. C. has a land area of 61 square miles. If there are 12,600 persons per square mile, what is the city's population?

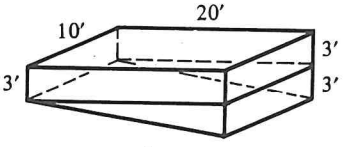
2. The state of Colorado has a land area of 103,884 square miles. If the population per square mile is 17, what is the total population of the state?

3. A refrigerator with a capacity of 18 cubic feet costs $414. What is the cost of the refrigerator per cubic foot? Is this more expensive per cubic foot than a 12 cubic foot refrigerator for $300?

4. The Masters family is moving to Texas. The mover estimates that their furniture will take up 700 cubic feet of space. Would a truck with a container 6′ wide, 8′ high, and 16′ long hold all of the furniture?

*5. Mrs. Masters is going to buy wall-to-wall carpeting for her new living room. If the room is 12 feet wide and 18 feet long, how much carpeting will she need? If it costs $8.50 a square yard, what is the total cost?

6. Name the volume in this swimming pool by:
   a. naming the volume in the rectangular prism,
   b. naming the volume in the triangular prism,
   c. naming the total of these volumes.



Name the sums.

1. $\begin{array}{r} 72 \\ 63 \\ +\ 88 \\ \hline \end{array}$ **2.** $\begin{array}{r} 345 \\ 276 \\ +\ 639 \\ \hline \end{array}$ **3.** $\begin{array}{r} 584 \\ 376 \\ +\ 891 \\ \hline \end{array}$ **4.** $\begin{array}{r} 325 \\ 792 \\ +\ 856 \\ \hline \end{array}$ **5.** $\begin{array}{r} 904 \\ 378 \\ +\ 418 \\ \hline \end{array}$ **6.** $\begin{array}{r} 5436 \\ 2198 \\ +\ 9157 \\ \hline \end{array}$

Name the differences.

7. $\begin{array}{r} 90 \\ -\ 37 \\ \hline \end{array}$ **8.** $\begin{array}{r} 435 \\ -\ 116 \\ \hline \end{array}$ **9.** $\begin{array}{r} 608 \\ -\ 293 \\ \hline \end{array}$ **10.** $\begin{array}{r} 802 \\ -\ 546 \\ \hline \end{array}$ **11.** $\begin{array}{r} 701 \\ -\ 394 \\ \hline \end{array}$ **12.** $\begin{array}{r} 5721 \\ -\ 2843 \\ \hline \end{array}$

Name the products.

13. $\begin{array}{r} 38 \\ \times\ 76 \\ \hline \end{array}$ **14.** $\begin{array}{r} 528 \\ \times\ 8 \\ \hline \end{array}$ **15.** $\begin{array}{r} 987 \\ \times\ 9 \\ \hline \end{array}$ **16.** $\begin{array}{r} 527 \\ \times\ 64 \\ \hline \end{array}$ **17.** $\begin{array}{r} 392 \\ \times\ 573 \\ \hline \end{array}$ **18.** $\begin{array}{r} 7284 \\ \times\ 26 \\ \hline \end{array}$

Name the quotients and remainders.

19. $7\overline{)602}$ **20.** $8\overline{)390}$ **21.** $23\overline{)805}$ **22.** $42\overline{)3216}$ **23.** $79\overline{)6726}$

Name the sums or differences.

24. $\begin{array}{r} 2\frac{2}{4} \\ +\ \frac{3}{4} \\ \hline \end{array}$ **25.** $\begin{array}{r} 3\frac{2}{3} \\ +\ 1\frac{5}{6} \\ \hline \end{array}$ **26.** $\begin{array}{r} 6\frac{6}{7} \\ -\ 3\frac{4}{7} \\ \hline \end{array}$ **27.** $\begin{array}{r} 7\frac{11}{12} \\ -\ 4\frac{3}{4} \\ \hline \end{array}$ **28.** $\begin{array}{r} 8\frac{5}{6} \\ +\ 4\frac{1}{3} \\ \hline \end{array}$ **29.** $\begin{array}{r} 6\frac{2}{3} \\ +\ 5\frac{1}{4} \\ \hline \end{array}$

Complete the sentences. Write $<$, $>$, or $=$ to replace each $\bigcirc$.

30. $7648 + 3352 \bigcirc 10{,}000$

31. $12{,}428 - 6754 \bigcirc 6000$

32. $64 \times (36 + 42) \bigcirc (64 \times 36) + (64 \times 42)$

33. $\frac{1}{2} + \frac{1}{3} \bigcirc \frac{3}{4} - \frac{1}{3}$ **34.** $3\frac{1}{2} + 2\frac{7}{9} \bigcirc 2\frac{2}{3} + 3\frac{1}{6}$ **35.** $\frac{1}{3} + \frac{1}{4} + \frac{2}{5} \bigcirc 1$

Solve the problems.

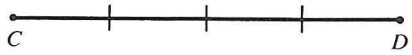
36. The Dawson Ice Cream Company makes 3976 pints of ice cream every hour. How many pints do they make in **a.** 8 hours, **b.** 24 hours?

37. The Tasty Lollipop Company made 25,697 lollipops one day and 19,854 the next day. How many lollipops did they make in the two days?

Ratio

$m(\overline{AB}) = 3$
$m(\overline{CD}) = 4$

$\dfrac{m(\overline{AB})}{m(\overline{CD})} = \dfrac{3}{4}$



The measures of $\overline{AB}$ and $\overline{CD}$ may be compared by means of a **ratio.** We say that the ratio is 3 to 4.

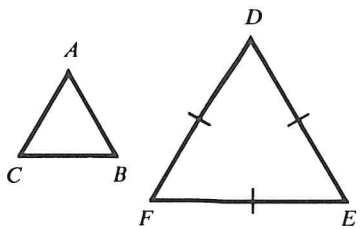
EXERCISES

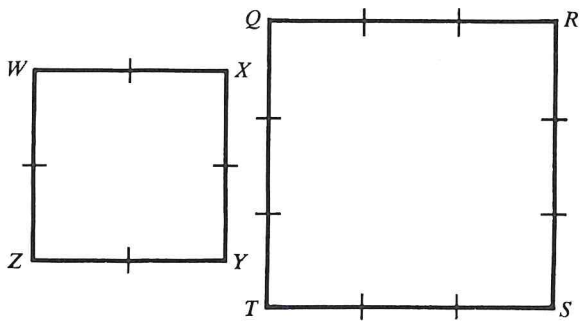
Name the ratios.

1. $\dfrac{m(\overline{AB})}{m(\overline{CD})} = a$

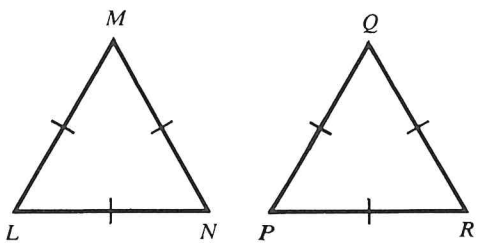
2. $\dfrac{m(\overline{EF})}{m(\overline{CD})} = b$

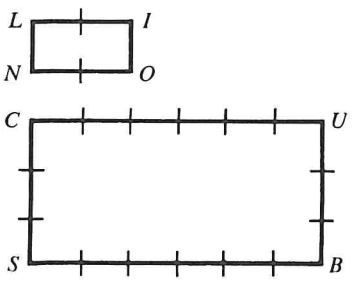
Name the ratios.





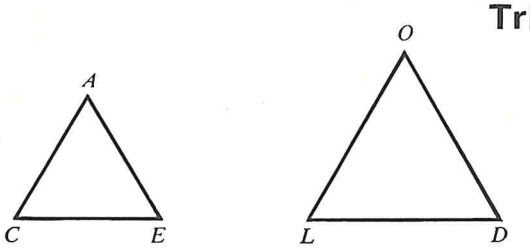
3. $\dfrac{m(\overline{AB})}{m(\overline{DE})} = \dfrac{a}{b}$ **4.** $\dfrac{m(\overline{BC})}{m(\overline{EF})} = \dfrac{c}{d}$ **5.** $\dfrac{m(\overline{XY})}{m(\overline{RS})} = \dfrac{e}{f}$ **6.** $\dfrac{m(\overline{XW})}{m(\overline{RQ})} = \dfrac{g}{h}$





7. $\dfrac{m(\overline{LM})}{m(\overline{PQ})} = \dfrac{i}{j}$ **8.** $\dfrac{m(\overline{MN})}{m(\overline{QR})} = \dfrac{k}{l}$ **9.** $\dfrac{m(\overline{LN})}{m(\overline{CS})} = \dfrac{m}{n}$ **10.** $\dfrac{m(\overline{NO})}{m(\overline{SB})} = \dfrac{r}{s}$

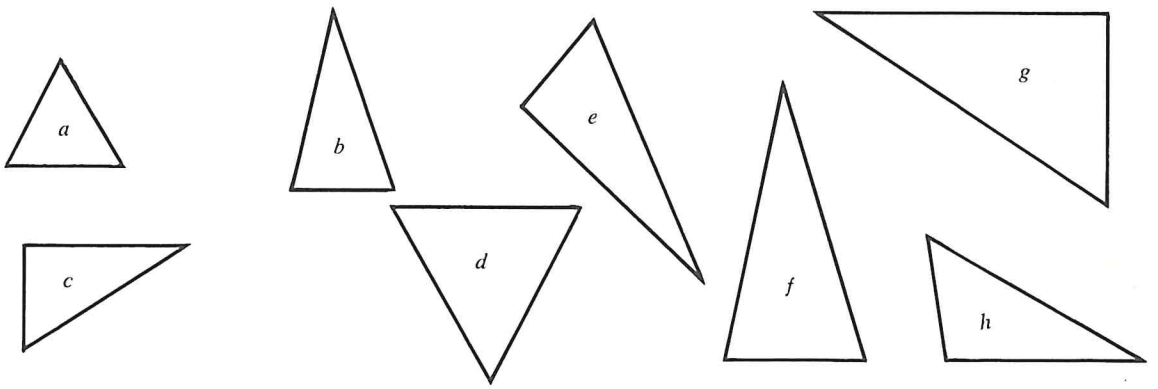
Triangles that look alike



Triangles ACE and OLD have the same shape but are not the same size. They are examples of **similar triangles.**

EXERCISES

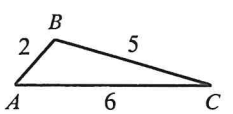
Pair the triangles which appear to be similar.



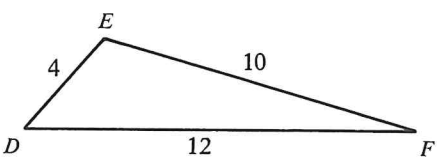
We may compare the lengths of the sides of similar triangles by using *ratios.*

Name the ratios.

1. $\dfrac{m(\overline{AB})}{m(\overline{DE})} = \dfrac{a}{b}$ **2.** $\dfrac{m(\overline{BC})}{m(\overline{EF})} = \dfrac{c}{d}$



3. $\dfrac{m(\overline{AC})}{m(\overline{DF})} = \dfrac{e}{f}$



4. Name the ratio of the perimeter of ABC to the perimeter of DEF.

For triangles ABC and DEF, name the measures.

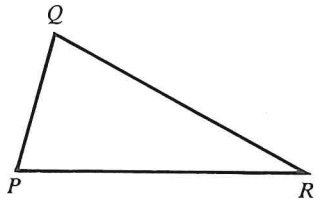
5. $m(\angle BAC) = a$ **7.** $m(\angle BCA) = c$ **9.** $m(\angle CBA) = e$

6. $m(\angle EDF) = b$ **8.** $m(\angle EFD) = d$ **10.** $m(\angle FED) = f$

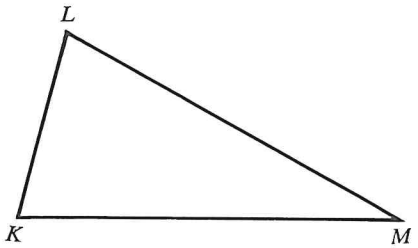
11. Name the congruent angles.

Similar polygons

If the angles in 2 triangles can be matched to form 3 pairs of congruent angles, the triangles are **similar.**

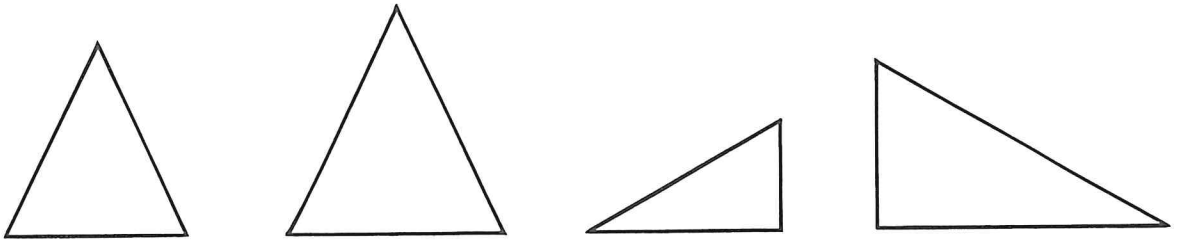


In the triangles at the right:
$m(\angle PQR) = m(\angle KLM)$
$m(\angle RPQ) = m(\angle MKL)$
$m(\angle QRP) = m(\angle LMK)$
Triangles PQR and KLM are similar.

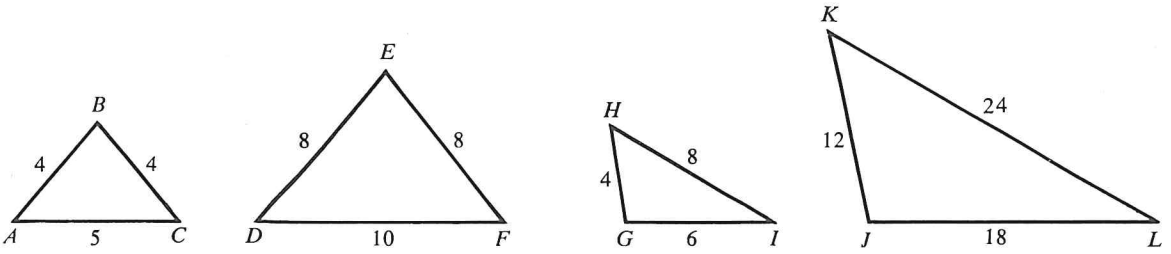


EXERCISES

Use your protractor to measure the angles in each pair of triangles.



1. Are the triangles in the 2 pairs similar?



Use the figures above to name the ratios.

2. $\dfrac{m(\overline{AB})}{m(\overline{DE})} = a$

3. $\dfrac{m(\overline{BC})}{m(\overline{EF})} = b$

4. $\dfrac{m(\overline{CA})}{m(\overline{FD})} = c$

5. $\dfrac{m(\overline{GH})}{m(\overline{JK})} = d$

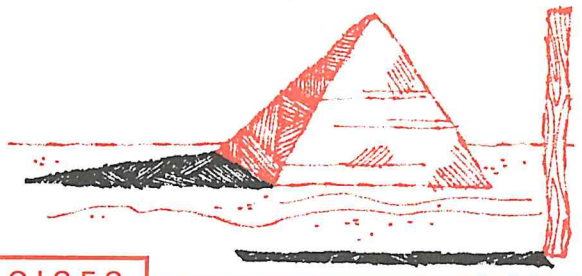
6. $\dfrac{m(\overline{HI})}{m(\overline{KL})} = e$

7. $\dfrac{m(\overline{IG})}{m(\overline{LJ})} = f$

Complete the sentence.

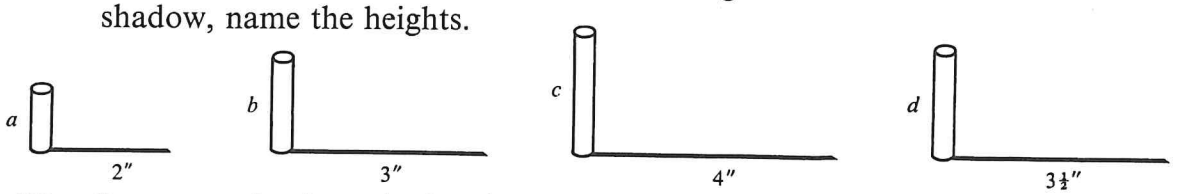
8. The ratios of matching sides of similar triangles are ____?____.

Eratosthenes found the height of the Great Pyramid in Egypt by using the idea that the ratio of its height to the length of its shadow would be the same as the ratio of the height of a stick to its shadow at the same time.



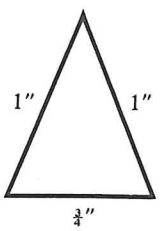
EXERCISES

1. Do you know something else about Eratosthenes? Look in Chapter 6.
2. What is the ratio of the height of the Washington Monument to its shadow, if the height of the stick is twice the length of its shadow?
3. If the height of each stick is one-half the length of its shadow, name the heights.



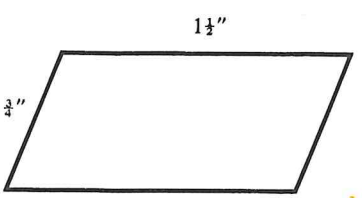
We often use ratios in scale drawing.

Jane wants to copy this triangle. She wants the sides of her triangle to be 4 times the sides of this triangle.



4. What lengths will she use?
5. Draw a triangle like Jane's.

Steve wants to copy this parallelogram. He wants the sides of his parallelogram to be 4 times the sides of the parallelogram shown.

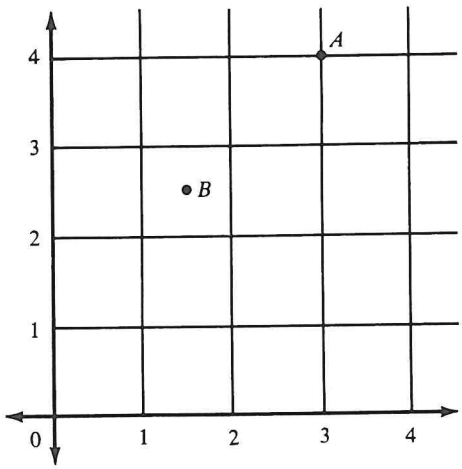


6. What lengths will he use?
7. Draw a parallelogram like Steve's.
8. Draw a triangle with sides 3″, 4″, and 5″. Then draw a triangle with sides half these lengths.

Points on a number plane

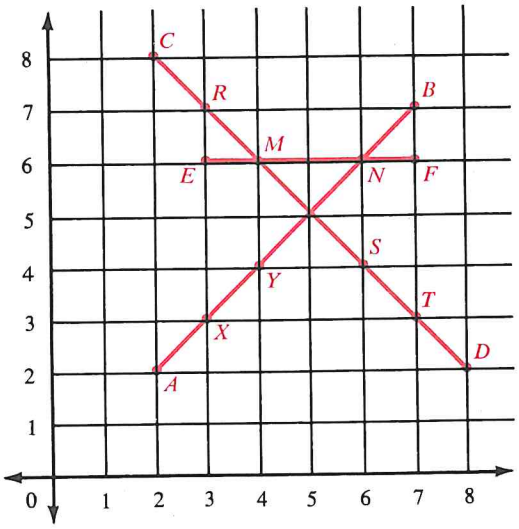
A plane is a set of points. If we name the points with number pairs, we call the plane a **number plane.**

This graph shows part of a number plane. The point A is named by the pair of whole numbers $(3, 4)$. The point B is named by the pair of fractional numbers $(1\frac{1}{2}, 2\frac{1}{2})$.



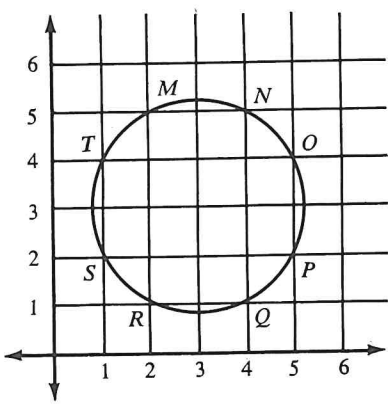
EXERCISES

In the number plane on the right, use pairs of whole numbers to name



1. the points X and Y on the line segment $\overline{AB}$
2. the points R, S, and T on the line segment $\overline{CD}$
3. four points on the line segment $\overline{EF}$
4. the point where $\overline{AB}$ and $\overline{EF}$ intersect
5. the point where $\overline{CD}$ and $\overline{EF}$ intersect

6. The circle passes through the 8 points labeled M to T. Name the number pairs for each of these points.

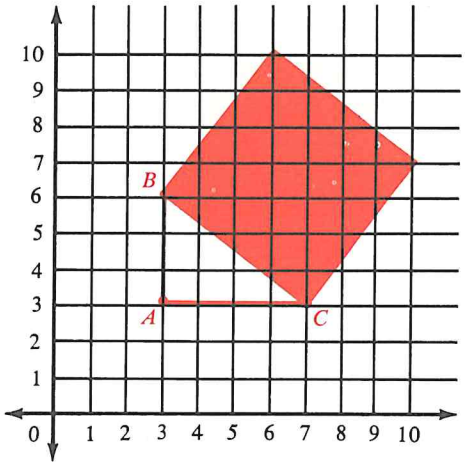


For the circle in the number plane, name the number of units between

7. M and N
8. P and O
9. O and T
10. S and P
11. M and R
12. N and Q

Working in the number plane

ABC is a right triangle in the number plane. The Theorem of Pythagoras tells us that the area of the red region is 9 + 16 or 25 square units.



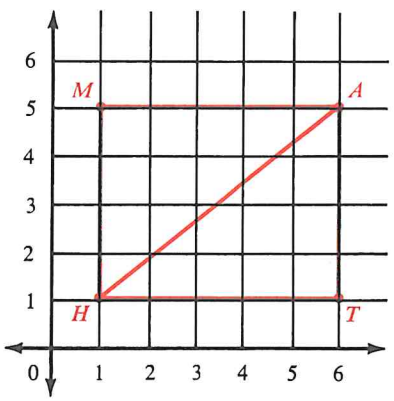
EXERCISES

Use the number plane above to help you answer the questions.

1. Name the number pairs for points *A*, *B*, and *C*.
2. Since $m(\overline{AB}) = 3$ and $m(\overline{AC}) = 4$, what is $m(\overline{BC})$?
3. Name the area of triangular region *ABC*.

Use the number plane to help you answer the questions.

4. Name the area of rectangular region *MATH*.
5. The diagonal $\overline{HA}$ separates rectangle *MATH* into 2 triangles. What kind of triangles are they?
6. Name the areas of triangular regions *MAH* and *THA*.
7. Is triangle *MAH* congruent to triangle *THA*?

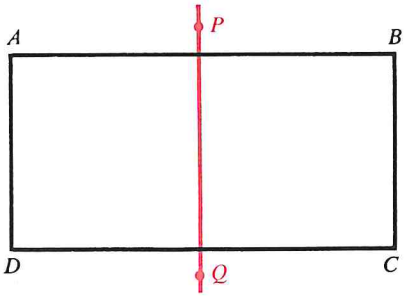


Draw a number plane with 10 units on each axis.

8. Locate and label the points. **a.** (0, 1) **b.** (3, 4) **c.** (7, 8) **d.** (9, 10)
9. Join the points and name the figure.
10. Name the ordered pairs for 6 other points on the figure.
11. Write an equation for the set of 10 ordered pairs.

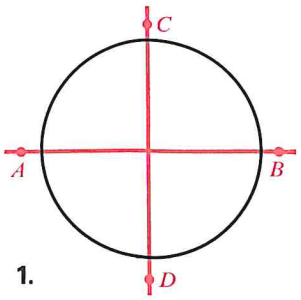
Symmetry

$\overline{PQ}$ separates rectangle $ABCD$ into 2 matching parts. We say that $ABCD$ is **symmetrical** about $\overline{PQ}$. $\overline{PQ}$ is called a **line of symmetry.**

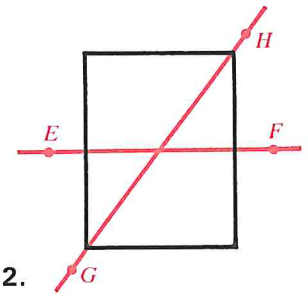


EXERCISES

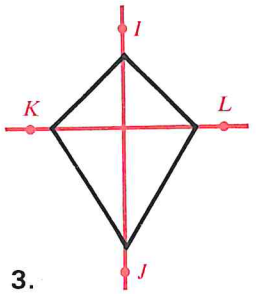
Study each figure, then name each line of symmetry.



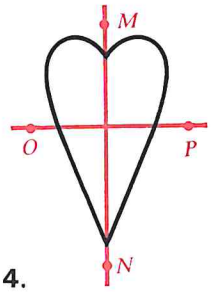
1.



2.

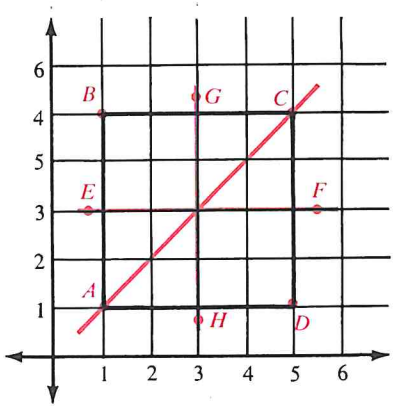


3.



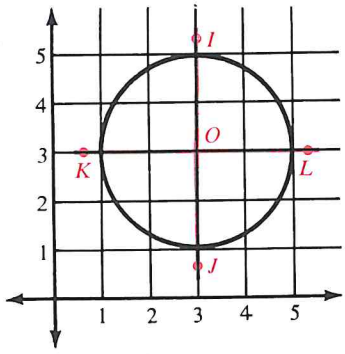
4.

For square $ABCD$ in the number plane



5. is $\overline{EF}$ a line of symmetry?
6. is $\overline{GH}$ a line of symmetry?
7. is $\overline{AC}$ a line of symmetry?
8. is triangle ABC congruent to triangle ADC?

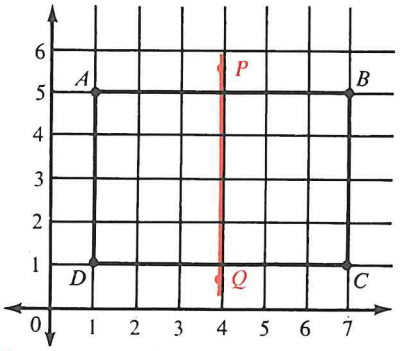
For the circle in the number plane



9. name the number pair for the center.
10. is $\overline{IJ}$ a line of symmetry?
11. is $\overline{KL}$ a line of symmetry?
12. Is any diameter a line of symmetry for a circle?

Symmetry in the number plane

$\overline{PQ}$ is a line of symmetry for $ABCD$. The points in $\overline{AD}$ can be matched with the points in $\overline{BC}$ so that each pair of points is the same distance from $\overline{PQ}$.



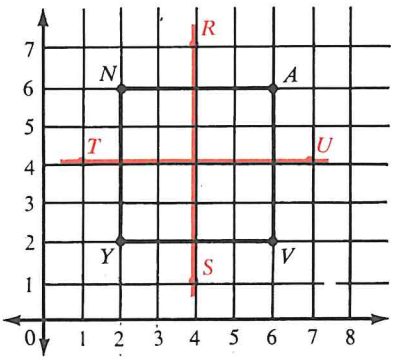
EXERCISES

For the figure $ABCD$ above

1. is point A the same distance from $\overline{PQ}$ as the point B?
2. is the point D the same distance from $\overline{PQ}$ as the point C?

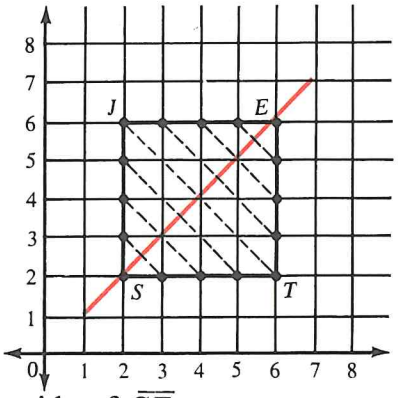
Study the number plane, then answer the questions.

3. Is $NAVY$ symmetrical about $\overline{RS}$?
4. Is $NAVY$ symmetrical about $\overline{TU}$?
5. Would $NAVY$ be symmetrical about diagonal $\overline{AY}$?
6. about diagonal $\overline{NV}$?



Study the number plane, then answer the questions.

7. Is the square $JETS$ symmetrical about diagonal $\overline{SE}$?
8. We can match (2, 3) with (3, 2) and (3, 6) with (6, 3). What point would match (2, 4)? (2, 5)? (4, 6)? (5, 6)?
9. Would point J match point T? What are the ordered pairs for these points?



*10. If (x, y) is the ordered pair for a point on one side of $\overline{SE}$, what would be its matching point on the other side of $\overline{SE}$?

Averages

The data shows the test scores of 6 students. The **average** is 498 ÷ 6 or 83. The average tells the score that each student would have had if all scores were the same. The scores **range** from 95 to 65. The range is 95 − 65 or 30.

Test 1
90
79
65
95
84
85
Total 498

EXERCISES

Name the averages and the ranges for the data.

1. 64, 38, 92, 97, 59

2. 97, 96, 95, 94, 93

3. 250, 150, 200, 300

4. 1, 2, 3, 4, 5, 6, 7, 8, 9

We often think of an average as one number to stand for the data we have. Find the averages of these data.

5.
95
86
89
78

6.
$2.54
1.75
3.28
2.06
2.92

7.
158
13
12
14
18

8.
1000
90
80
50

9. Can you tell why the average is *not* a good representative of the data in 7 and 8?

10. Name the range for each set of data.

***11.** Joe is 5 feet tall. If he knows that the average depth of the stream behind his house is 4 feet, can he be sure that for this reason he can safely wade across the stream? If not, why not?

12. Pete averaged 16 points a game. If he played in 9 games, how many points did he score in all?

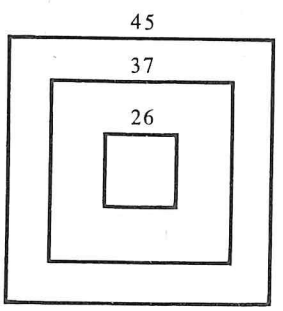
13. A pitcher in a big league game threw 81 pitches in 9 innings. What was his average number of pitches per inning? Do you think that his team *probably* won the game?

1. Elaine received scores of 93, 72, 85, 67, 98, and 83 on her math tests. What was the range of her scores? What was her average score?

2. For 5 days, Mr. Adams at the bus company kept track of the number of passengers who used the Central Avenue bus at rush hour. The numbers were 67, 84, 73, 69, and 82. How many passengers were there on the average?

3. Mrs. Allen said that the average attendance at 7 basketball games was 278. How many people attended all together?

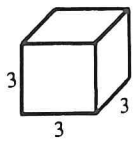
4. Mary sold Girl Scout cookies for 65¢ a box. She sold 27, 42, 19, and 36 boxes on four different days. **a.** What is the average number of boxes she sold per day? **b.** How much money did she collect all together?

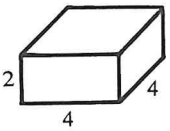


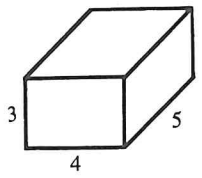
***5.** One square has a side of 26″, one of 37″ and one of 45″. **a.** What is the average length of the sides? **b.** What is the average perimeter? **c.** What is the average area?

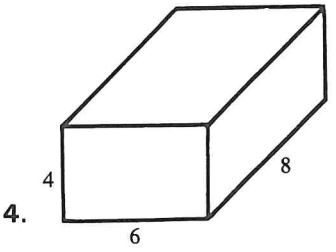
CHECKING UP

Name the surface area and volume of each closed surface.

1. 

2. 

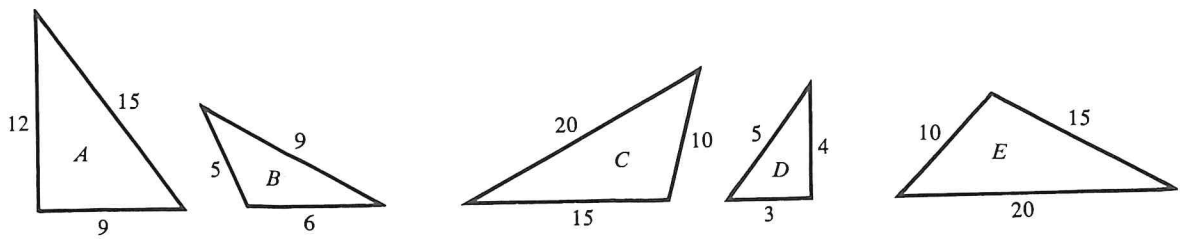
3. 

4. 

Use the Theorem of Pythagoras to determine if triangles with the following measures are right triangles.

5. 6, 8, 10 **6.** 5, 12, 13 **7.** 6, 9, 11 **8.** 7, 24, 25

Use these triangles to answer the questions.



9. Name the triangles which are congruent.

10. Name the triangles which are similar and the ratios of their sides.

11. Name the triangles which are right triangles.

12. Name the areas of the regions in A and D.

Complete the sentences.

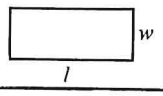
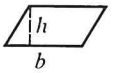
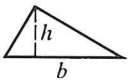
13. If 2 triangles are the same size and shape, the triangles are ___?___.

14. If the angles of 2 triangles can be matched to form 3 pairs of congruent angles, the triangles are ___?___.

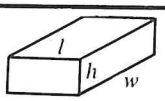
15. Name the average and range of the following data.
61, 78, 56, 24, 37, 82, 46, 73, 29

Equations can be used to help us find the measures of plane regions and space. Copy and complete the tables.

1.

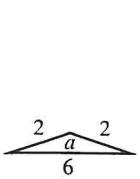
region	figure	equation
rectangle		*a*
square		*b*
c		$A = b \times h$
triangle		*d*

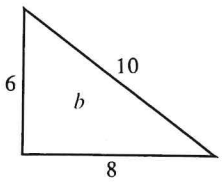
2.

space	figure	equation
rectangular prism		*e*
f		$V = s^3$

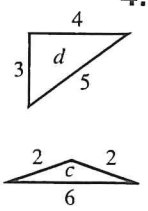
We may compare the size and shape of plane figures. In the figures below

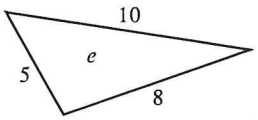
3. pair the congruent figures.





4. pair the similar figures.





> For a right triangle with sides of measure a, b, and c (where c is the measure of the side opposite the right angle), $c^2 = a^2 + b^2$.

Which of these are right triangles if the measures of the sides are

5. 4, 3, 5?

6. 10, 24, 26?

7. 6, 14, 15?

8. 11, 60, 61?

LOOKING WAY BACK

Name the numbers at the right which are divisible by

100	123	101
369	963	396
900	225	418
777	93	888

1. 2 **3.** 4 **5.** 6

2. 3 **4.** 5 **6.** 9

7. Which of the numbers are primes?

For the numbers 16, 24, and 36

8. list the set of factors for each number

9. list the sets of multiples, less than 150, for each number

10. draw a factor tree for each number

11. name the prime factorization of each number

12. name the GCF of 16 and 24 **13.** name the LCM of 16 and 36.

Solve the equations.

14. $6437 + 7985 = a$ **16.** $19{,}467 - 8396 = c$ **18.** $769 \times 45 = e$

15. $9281 - 6085 = b$ **17.** $4328 \times 8 = d$ **19.** $538 \times 397 = f$

Name the quotients and remainders.

20. $9\overline{)414}$ **21.** $8\overline{)590}$ **22.** $18\overline{)954}$ **23.** $84\overline{)7056}$ **24.** $36\overline{)2075}$

Solve the equations.

25. $\frac{1}{2} = \frac{a}{6}$ **26.** $\frac{3}{4} = \frac{c}{12}$ **27.** $\frac{1}{3} = \frac{e}{9}$ **28.** $\frac{5}{6} = \frac{i}{18}$ **29.** $\frac{2}{7} = \frac{n}{21}$

Name the sums or differences.

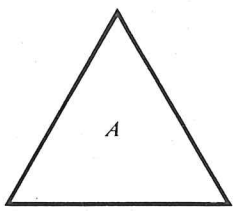
30. $\frac{1}{2} + \frac{2}{3} = a$ **32.** $\frac{2}{5} + \frac{3}{10} = c$ **34.** $\frac{7}{8} - \frac{3}{4} = e$

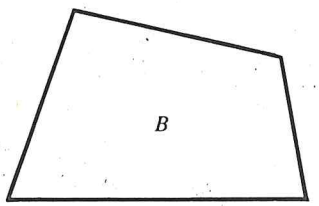
31. $\frac{1}{4} + \frac{3}{4} = b$ **33.** $\frac{5}{7} - \frac{1}{3} = d$ **35.** $\frac{3}{4} - \frac{5}{8} = f$

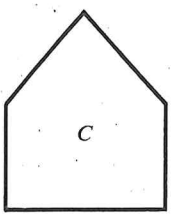
Name the sums or differences.

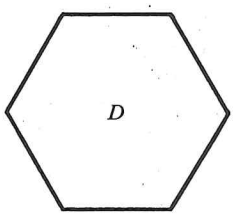
36. $3\frac{1}{7} - 1$ **37.** $5\frac{1}{3} - 2\frac{2}{3}$ **38.** $6\frac{7}{8} + 2\frac{3}{4}$ **39.** $9\frac{2}{3} - 4\frac{2}{5}$ **40.** $4\frac{1}{8} + 3\frac{1}{2}$

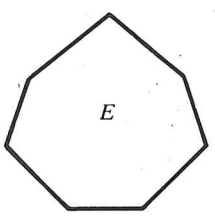
Measure all the angles of each polygon, then complete the table.

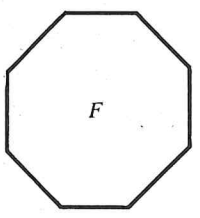










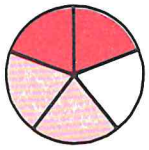


1.

figure		no. of sides	sum of angle measures
A	triangle	3	$a = b \times 180$
B	quadrilateral	4	$c = d \times 180$
C	pentagon	5	$e = f \times 180$
D	hexagon	6	$g = h \times 180$
E	septagon	7	$i = j \times 180$
F	octagon	8	$k = l \times 180$

2. Copy the figures above on a piece of paper. Separate each polygonal region into triangular regions by drawing diagonals. In each case name the number of triangles you get, and compare your results with ***b, d, f, h, j,*** and ***l*** in exercise 1.

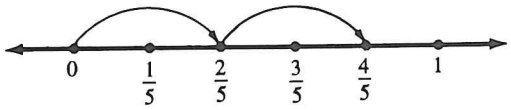
Joining equivalent subsets



$\frac{2}{5}$ of the region is red.

$\frac{2}{5}$ of the region is pink.

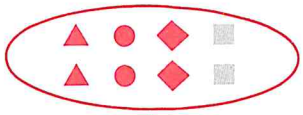
$\frac{2}{5} + \frac{2}{5} = 2 \times \frac{2}{5} = \frac{4}{5}$ of the region is red or pink.



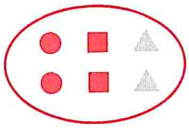
The arrows show:
$\frac{2}{5} + \frac{2}{5} = 2 \times \frac{2}{5} = \frac{4}{5}$

EXERCISES

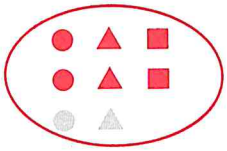
Use the sets to help you solve the equations.



1. $\frac{2}{8} + \frac{2}{8} + \frac{2}{8} = \frac{c}{8} = \frac{e}{4}$



2. $\frac{2}{6} + \frac{2}{6} = \frac{e}{6} = \frac{n}{3}$



3. $\frac{3}{8} + \frac{3}{8} = \frac{r}{8} = \frac{s}{4}$

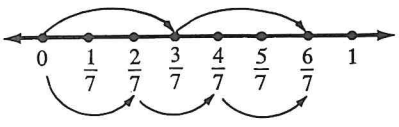
Use the regions to help you solve the equations.

4. $\frac{2}{6} + \frac{2}{6} + \frac{2}{6} = \frac{a}{6} = b$

5. $\frac{2}{7} + \frac{2}{7} = \frac{c}{7}$

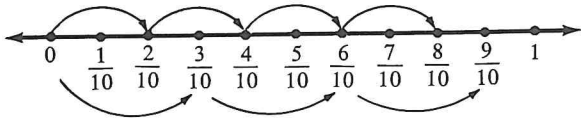
6. $\frac{3}{10} + \frac{3}{10} + \frac{3}{10} = \frac{s}{10}$

Use the number lines to help you solve the equations.



7. $\frac{3}{7} + \frac{3}{7} = \frac{a}{7}$

8. $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{c}{7}$



9. $\frac{2}{10} + \frac{2}{10} + \frac{2}{10} + \frac{2}{10} = \frac{e}{10} = \frac{i}{5}$

10. $\frac{3}{10} + \frac{3}{10} + \frac{3}{10} = \frac{e}{10}$

The associative property

Study the equations.

A

$$4 \times \frac{2}{9} = 4 \times \left(2 \times \frac{1}{9}\right)$$
$$= (4 \times 2) \times \frac{1}{9}$$
$$= 8 \times \frac{1}{9}$$
$$= \frac{8}{9}$$

B

$$\frac{2}{9} \times 4 = 4 \times \frac{2}{9}$$
$$= \frac{8}{9}$$

EXERCISES

1. What property of multiplication is used in the A equations?

2. What property of multiplication is used in the B equations?

Copy and complete the equations.

3.
$$3 \times 20 = 3 \times (2 \times 10)$$
$$= (3 \times 2) \times 10$$
$$= \quad a \quad \times 10$$
$$= \quad b$$

4.
$$2 \times 40 = 2 \times (4 \times 10)$$
$$= (2 \times 4) \times 10$$
$$= \quad c \quad \times 10$$
$$= \quad d$$

5.
$$3 \times \frac{2}{7} = 3 \times \left(2 \times \frac{1}{7}\right)$$
$$= (3 \times 2) \times \frac{1}{7}$$
$$= \quad e \quad \times \frac{1}{7}$$
$$= \quad f$$

6.
$$2 \times \frac{4}{9} = 2 \times \left(4 \times \frac{1}{9}\right)$$
$$= (2 \times 4) \times \frac{1}{9}$$
$$= \quad g \quad \times \frac{1}{9}$$
$$= \quad h$$

Complete the number charts.

7.

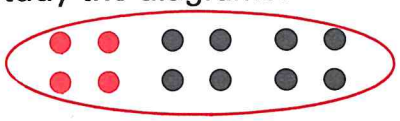
factor	4	2	2	3	2	3	5
factor	2	2	2	2	5	3	2
factor	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{12}$
product	$\frac{8}{9}$	***a***	***b***	***c***	***d***	***e***	***f***

8.

factor	2	3	6	9
factor	2	3	4	6
factor	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$
product	***a***	***b***	***c***	***d***

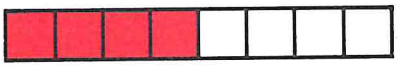
Separating sets

Study the diagrams.



One third of the circles are red.

$\frac{1}{3} \times 12 = 4$ $\quad 12 \div 3 = 4$



One half of the rod is red.

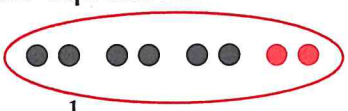
$\frac{1}{2} \times 8 = 4$ $\quad 8 \div 2 = 4$

EXERCISES

Study the sets and solve the equations.

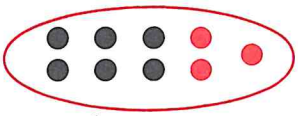
1. $\frac{1}{3} \times 6 = a$

$6 \div b = 2$



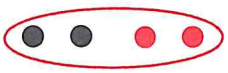
3. $\frac{1}{4} \times 8 = e$

$8 \div f = 2$



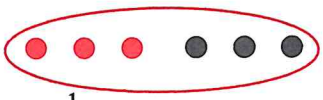
5. $\frac{1}{3} \times 9 = i$

$9 \div j = 3$



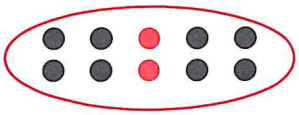
2. $\frac{1}{2} \times 4 = c$

$4 \div d = 2$



4. $\frac{1}{2} \times 6 = g$

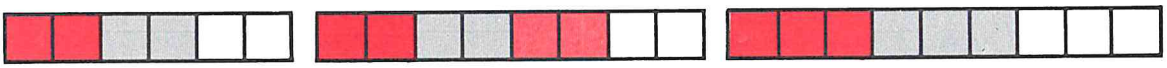
$6 \div h = 3$



6. $\frac{1}{5} \times 10 = k$

$10 \div l = 2$

Study the rods and solve the equations.



7. $\frac{1}{3} \times 6 = a$

$6 \div b = 2$

8. $\frac{1}{4} \times 8 = c$

$8 \div d = 2$

9. $\frac{1}{3} \times 9 = e$

$9 \div f = 3$

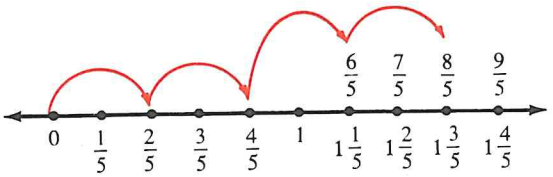
Answer the questions.

10. There are 6 apples in one package. There are ___?___ apples in $\frac{1}{2}$ of the package. There are ___?___ apples in $\frac{1}{3}$ of the package.

11. There are 9 bananas in one bunch. How many bananas in $\frac{1}{3}$ of the bunch?

12. If there are 10 pears in one package, how many pears in $\frac{1}{5}$ of the package?

Study the equations.

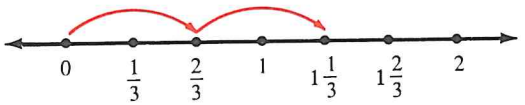


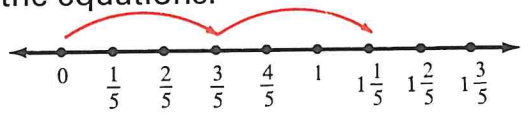
$4 \times \frac{2}{5} = \frac{8}{5} = 1\frac{3}{5}$

$$17 \times \frac{1}{3} = (15 + 2) \times \frac{1}{3}$$
$$= \left(15 \times \frac{1}{3}\right) + \left(2 \times \frac{1}{3}\right)$$
$$= 5 + \frac{2}{3}$$
$$= 5\frac{2}{3}$$

EXERCISES

Use the number lines to help you solve the equations.





1. $2 \times \frac{2}{3} = \frac{i}{3} = j$

2. $2 \times \frac{3}{5} = \frac{c}{5} = l$

Copy and complete the equations.

3. $4 \times \frac{2}{3} = 4 \times \left(2 \times \frac{1}{3}\right)$
$= (4 \times 2) \times \frac{1}{3}$
$= a \times \frac{1}{3}$
$= b$

4. $5 \times \frac{5}{6} = 5 \times \left(5 \times \frac{1}{6}\right)$
$= (5 \times 5) \times \frac{1}{6}$
$= c \times \frac{1}{6}$
$= d$

5. $3 \times \frac{2}{5} = 3 \times \left(2 \times \frac{1}{5}\right)$
$= (3 \times 2) \times \frac{1}{5}$
$= e \times \frac{1}{5}$
$= f$

6. $3 \times \frac{2}{4} = 3 \times \left(2 \times \frac{1}{4}\right)$
$= (3 \times 2) \times \frac{1}{4}$
$= g \times \frac{1}{4}$
$= h$

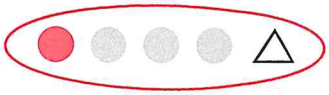
Name each of the products first as an expanded numeral, then as a mixed numeral.

7. $7 \times \frac{1}{3}$ **8.** $9 \times \frac{1}{4}$ **9.** $11 \times \frac{1}{5}$ **10.** $8 \times \frac{2}{5}$

Name the products.

11. $\frac{3}{5} \times 2$ **12.** $\frac{4}{7} \times 2$ **13.** $\frac{2}{9} \times 5$ **14.** $\frac{1}{4} \times 7$ **15.** $\frac{2}{5} \times 4$ **16.** $\frac{4}{5} \times 6$ **17.** $\frac{3}{4} \times 8$

Separating subsets

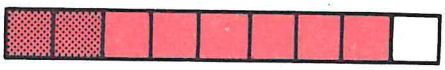


$\frac{4}{5}$ of the objects are circles.

$\frac{1}{4}$ of the circles are red.

$\frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$ of the objects are red.

$\frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$ $\quad$ $\frac{4}{5} \div 4 = \frac{1}{5}$



$\frac{8}{9}$ of the region is red.

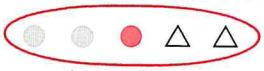
$\frac{1}{4}$ of the red region is dotted.

$\frac{1}{4} \times \frac{8}{9} = \frac{2}{9}$ of the region is red and dotted.

$\frac{1}{4} \times \frac{8}{9} = \frac{8}{9} \div 4 = \frac{2}{9}$

EXERCISES

Use the sets to help you solve the equations.

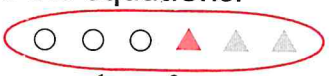


1. $\frac{1}{3} \times \frac{3}{5} = \frac{a}{5}$

2. $\frac{3}{5} \div b = \frac{1}{5}$

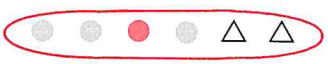
3. $\frac{1}{4} \times \frac{4}{5} = \frac{c}{5}$

4. $\frac{4}{5} \div d = \frac{1}{5}$



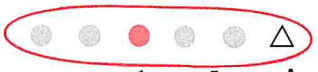
5. $\frac{1}{3} \times \frac{3}{6} = \frac{e}{6}$

6. $\frac{3}{6} \div f = \frac{1}{6}$



7. $\frac{1}{4} \times \frac{4}{6} = \frac{r}{6}$

8. $\frac{4}{6} \div h = \frac{1}{6}$



9. $\frac{1}{5} \times \frac{5}{6} = \frac{i}{6}$

10. $\frac{5}{6} \div j = \frac{1}{6}$

11. $\frac{1}{3} \times \frac{3}{7} = \frac{s}{7}$

12. $\frac{3}{7} \div l = \frac{1}{7}$

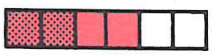
Use the regions to help you solve the equations.

13. $\frac{1}{2} \times \frac{4}{5} = \frac{a}{5}$

14. $\frac{4}{5} \div b = \frac{2}{5}$

15. $\frac{1}{3} \times \frac{6}{7} = \frac{c}{7}$

16. $\frac{6}{7} \div d = \frac{2}{7}$



17. $\frac{1}{2} \times \frac{4}{6} = \frac{e}{6}$

18. $\frac{4}{6} \div f = \frac{2}{6}$

19. $\frac{1}{2} \times \frac{4}{7} = \frac{s}{7}$

20. $\frac{4}{7} \div h = \frac{2}{7}$

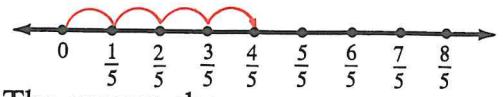
21. $\frac{1}{3} \times \frac{6}{8} = \frac{i}{8}$

22. $\frac{6}{8} \div j = \frac{2}{8}$

23. $\frac{1}{3} \times \frac{6}{9} = \frac{n}{9}$

24. $\frac{6}{9} \div l = \frac{2}{9}$

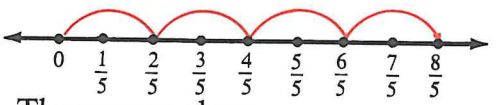
Study the number lines.



The arrows show:

$$\frac{4}{5} \div 4 = \frac{1}{5}$$

$$\frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$$



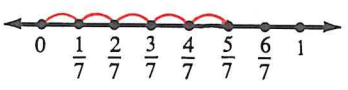
The arrows show:

$$\frac{8}{5} \div 4 = \frac{2}{5}$$

$$\frac{1}{4} \times \frac{8}{5} = \frac{2}{5}$$

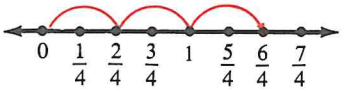
EXERCISES

Use the number lines to help you solve the equations.



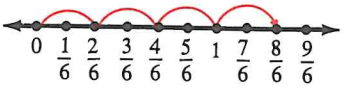
1. $\frac{5}{7} \div 5 = \frac{p}{7}$

2. $\frac{1}{s} \times \frac{5}{7} = \frac{1}{7}$



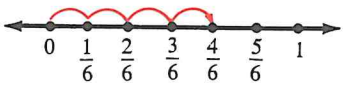
7. $\frac{6}{4} \div 3 = \frac{a}{4}$

8. $\frac{1}{e} \times \frac{6}{4} = \frac{2}{4}$



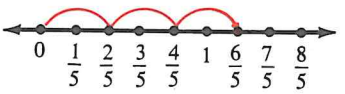
13. $\frac{8}{6} \div 4 = \frac{s}{6}$

14. $\frac{1}{n} \times \frac{8}{6} = \frac{2}{6}$



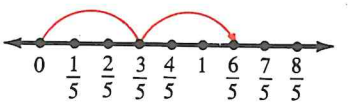
3. $\frac{4}{6} \div 4 = \frac{i}{6}$

4. $\frac{1}{u} \times \frac{4}{6} = \frac{1}{6}$



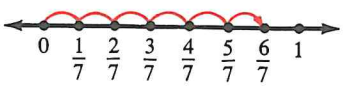
9. $\frac{6}{5} \div 3 = \frac{c}{5}$

10. $\frac{1}{r} \times \frac{6}{5} = \frac{2}{5}$



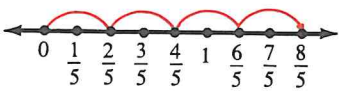
15. $\frac{6}{5} \div 2 = \frac{i}{5}$

16. $\frac{1}{u} \times \frac{6}{5} = \frac{3}{5}$



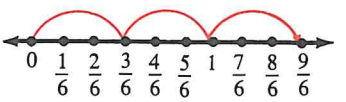
5. $\frac{6}{7} \div 6 = \frac{v}{7}$

6. $\frac{1}{z} \times \frac{6}{7} = \frac{1}{7}$



11. $\frac{8}{5} \div 4 = \frac{e}{5}$

12. $\frac{1}{r} \times \frac{8}{5} = \frac{2}{5}$



17. $\frac{9}{6} \div 3 = \frac{v}{6}$

18. $\frac{1}{x} \times \frac{9}{6} = \frac{3}{6}$

Using the associative property

Study the equations.

A

$$\begin{aligned}\frac{1}{4} \times \frac{4}{5} &= \frac{1}{4} \times \left(4 \times \frac{1}{5}\right) \\ &= \left(\frac{1}{4} \times 4\right) \times \frac{1}{5} \\ &= 1 \times \frac{1}{5} \\ &= \frac{1}{5}\end{aligned}$$

B

$$\begin{aligned}\frac{1}{4} \times \frac{8}{9} &= \frac{1}{4} \times \left(8 \times \frac{1}{9}\right) \\ &= \left(\frac{1}{4} \times 8\right) \times \frac{1}{9} \\ &= 2 \times \frac{1}{9} \\ &= \frac{2}{9}\end{aligned}$$

EXERCISES

Copy and complete the equations.

1. $\frac{1}{3} \times \frac{3}{5} = \frac{1}{3} \times \left(a \times \frac{1}{5}\right)$
$= \left(\frac{1}{3} \times 3\right) \times \frac{1}{5}$
$= b \times \frac{1}{5}$

2. $\frac{1}{3} \times \frac{6}{5} = \frac{1}{3} \times \left(c \times \frac{1}{5}\right)$
$= \left(\frac{1}{3} \times c\right) \times \frac{1}{5}$
$= d \times \frac{1}{5}$

3. $\frac{1}{5} \times \frac{5}{8} = \frac{1}{5} \times \left(e \times \frac{1}{8}\right)$
$= \left(\frac{1}{5} \times e\right) \times \frac{1}{8}$
$= f \times \frac{1}{8}$

4. $\frac{1}{5} \times \frac{15}{8} = \frac{1}{5} \times \left(g \times \frac{1}{8}\right)$
$= \left(\frac{1}{5} \times g\right) \times \frac{1}{8}$
$= h \times \frac{1}{8}$

Solve the equations.

5. $\frac{1}{3} \times \frac{6}{7} = \frac{a}{7}$

6. $\frac{1}{4} \times \frac{8}{7} = \frac{c}{7}$

7. $\frac{1}{2} \times \frac{2}{5} = \frac{i}{5}$

8. $\frac{1}{2} \times \frac{4}{5} = \frac{n}{5}$

9. $\frac{1}{3} \times \frac{9}{10} = \frac{s}{10}$

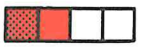
10. $\frac{1}{3} \times \frac{9}{8} = \frac{u}{8}$

Solve the problems.

11. Mary had 30 stamps. She put $\frac{4}{5}$ of her stamps in her stampbook and gave $\frac{1}{5}$ to her friend, Alice. How many stamps did Mary put in her book?

12. Alice put $\frac{1}{3}$ of the stamps Mary gave her on a card. How many stamps did Alice put on her card?

Study the region and the equations.



$\frac{1}{2} = \frac{2}{4}$ of the region is red.

$\frac{1}{2}$ of the red region is dotted.

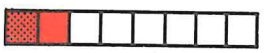
$\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{2}$ of $\frac{2}{4} = \frac{1}{4}$

$\frac{1}{4}$ of the region is red and dotted.

$$\begin{aligned}\frac{1}{2} \times \frac{1}{2} &= \frac{1}{2} \times \frac{2}{4} \\ &= \frac{1}{2} \times \left(2 \times \frac{1}{4}\right) \\ &= \left(\frac{1}{2} \times 2\right) \times \frac{1}{4} \\ &= 1 \times \frac{1}{4} \\ &= \frac{1}{4}\end{aligned}$$

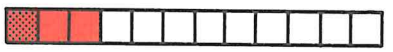
EXERCISES

Use the regions to help you solve the equations.



1. $\frac{1}{2} \times \frac{1}{4} = \frac{a}{8}$

2. $\frac{1}{2} \times \frac{3}{5} = \frac{c}{10}$



3. $\frac{1}{3} \times \frac{1}{4} = \frac{r}{12}$

Copy and complete the equations.

4.
$$\begin{aligned}\frac{1}{3} \times \frac{1}{2} &= \frac{1}{3} \times \frac{3}{6} \\ &= \frac{1}{3} \times \left(a \times \frac{1}{6}\right) \\ &= \left(\frac{1}{3} \times a\right) \times \frac{1}{i} \\ &= c \times \frac{1}{6} \\ &= d\end{aligned}$$

5.
$$\begin{aligned}\frac{1}{5} \times \frac{1}{2} &= \frac{2}{10} \times \frac{1}{2} \\ &= \left(\frac{1}{10} \times e\right) \times \frac{1}{2} \\ &= \frac{1}{10} \times \left(e \times \frac{1}{u}\right) \\ &= \frac{1}{10} \times g \\ &= h\end{aligned}$$

6.
$$\begin{aligned}\frac{1}{3} \times \frac{1}{4} &= \frac{4}{12} \times \frac{1}{4} \\ &= \left(\frac{1}{12} \times i\right) \times \frac{1}{4} \\ &= \frac{1}{v} \times \left(i \times \frac{1}{4}\right) \\ &= \frac{1}{12} \times k \\ &= l\end{aligned}$$

7.
$$\begin{aligned}\frac{1}{2} \times \frac{1}{4} &= \frac{1}{2} \times \frac{2}{8} \\ &= \frac{1}{2} \times \left(m \times \frac{1}{8}\right) \\ &= \left(\frac{1}{2} \times m\right) \times \frac{1}{n} \\ &= 1 \times p \\ &= q\end{aligned}$$

Using the associative property

To name the product $\frac{1}{2} \times \frac{3}{4}$ we remember that $\frac{3}{4} = \frac{6}{8}$. Then we can do our work as in the equations.

$$\begin{aligned}\frac{1}{2} \times \frac{3}{4} &= \frac{1}{2} \times \frac{6}{8} \\ &= \left(\frac{1}{2} \times 6\right) \times \frac{1}{8} \\ &= 3 \times \frac{1}{8} = \frac{3}{8}\end{aligned}$$

EXERCISES

Copy and complete the equations.

1. $\frac{1}{3} \times \frac{2}{5} = \frac{1}{3} \times \frac{6}{15}$
$= \left(\frac{1}{3} \times a\right) \times \frac{1}{15}$
$= b \times \frac{1}{15}$
$= c$

2. $\frac{1}{5} \times \frac{2}{3} = \frac{1}{5} \times \frac{10}{15}$
$= \left(\frac{1}{5} \times d\right) \times \frac{1}{15}$
$= e \times \frac{1}{15}$
$= f$

To name the product $\frac{1}{4} \times \frac{3}{5}$ we may use this method:

$$\begin{aligned}\frac{1}{4} \times \frac{3}{5} &= \frac{1}{4} \times \left(\frac{1}{5} \times 3\right) \\ &= \left(\frac{1}{4} \times \frac{1}{5}\right) \times 3 \\ &= \frac{1}{20} \times 3 \\ &= \frac{3}{20}\end{aligned}$$

Copy and complete the equations.

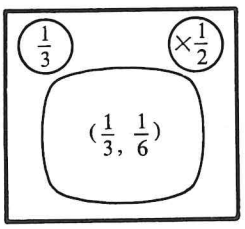
3. $\frac{1}{3} \times \frac{2}{5} = \frac{1}{3} \times \left(\frac{1}{5} \times a\right)$
$= \left(\frac{1}{3} \times b\right) \times a$
$= \frac{c}{15}$

4. $\frac{3}{8} \times \frac{1}{4} = (d \times e) \times \frac{1}{4}$
$= d \times \left(e \times \frac{1}{4}\right)$
$= f$

5. $\frac{1}{2} \times \frac{3}{7} = \frac{1}{2} \times (g \times h)$
$= \left(\frac{1}{2} \times g\right) \times h$
$= \frac{i}{14}$

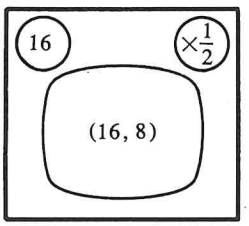
6. $\frac{2}{5} \times \frac{1}{3} = (2 \times j) \times \frac{1}{3}$
$= 2 \times (j \times k)$
$= l$

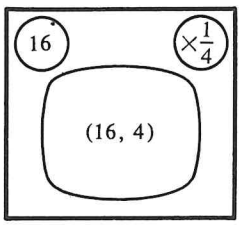
The function TV shows the tuned-in number $\frac{1}{3}$ and the tuned-in rule *multiply by* $\frac{1}{2}$. The screen shows the number pair $\left(\frac{1}{3}, \frac{1}{6}\right)$.



EXERCISES

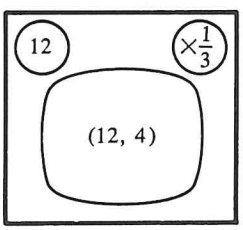
Use the rules shown on the function TVs to complete the sets of number pairs.

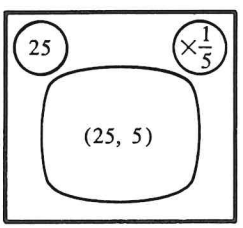




1. $\left\{(16, 8), (8, 4), (4, 2), (2, a), (1, b), \left(\frac{1}{2}, c\right), \left(\frac{1}{3}, d\right), \left(\frac{1}{4}, e\right)\right\}$

3. $\left\{(16, 4), (12, 3), (4, k), (1, l), \left(\frac{1}{2}, m\right), \left(\frac{1}{3}, n\right), \left(\frac{1}{4}, p\right)\right\}$





2. $\left\{(12, 4), (9, 3), (6, 2), (3, f), (1, g), \left(\frac{1}{2}, h\right), \left(\frac{1}{3}, i\right), \left(\frac{1}{4}, j\right)\right\}$

4. $\left\{(25, 5), (20, 4), (15, 3), (10, q), (5, r), (1, s), \left(\frac{1}{2}, t\right), \left(\frac{1}{3}, u\right)\right\}$

Complete the number patterns.

5.

$\times$	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
2	16	8	*a*	*b*	*c*	*d*

6.

$\times$	16	8	4	2	1	$\frac{1}{2}$
$\frac{1}{2}$	8	4	*f*	*g*	*h*	*i*

Sharpshooting

Answer the questions in A as quickly as you can. Then see if you can answer the questions in B even more quickly.

A

1. Name the greatest common factor of 24 and 36.
2. $46 + 8 + 9 + 16 = n$
3. $1\frac{1}{4} + \frac{7}{8} = n$
4. What is the difference between 386 and 485?
5. Name the whole number between (7×8) and (9×6).
6. How far will you travel in 6 hours at 45 miles per hour?
7. What change will you get from \$5 if you spend \$3.78?
8. How much will you earn in 5 hours at 45¢ an hour?
9. How many gallons are there in 256 pints?
10. Name the prime factorization for 84.

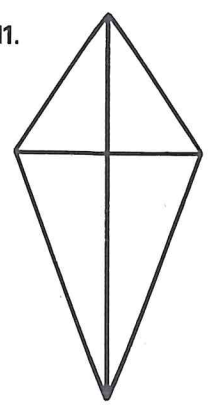
B

1. What is the total value of the 9 in 396,485?
2. Name the least common multiple of 6 and 15.
3. $65 \times 56 = n$
4. $1\frac{1}{4} - \frac{7}{8} = n$
5. $46{,}895 + 27{,}364 = n$
6. Name the product of 17 and 89.
7. What is the cost of 6 chairs at \$7.95 each?
8. If you spend \$2.80 in 8 days, how much is that per day on the average?
9. How many 35¢ sundaes can you buy for \$1.75?
10. What is the average of 35, 56, and 59?

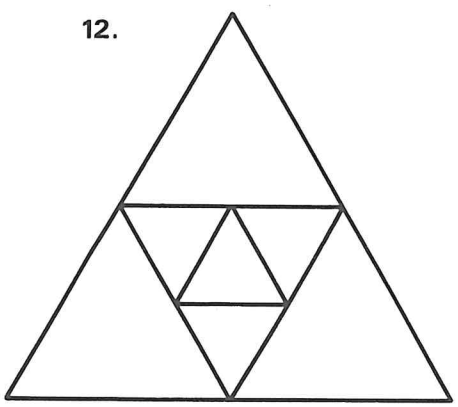
Be careful!

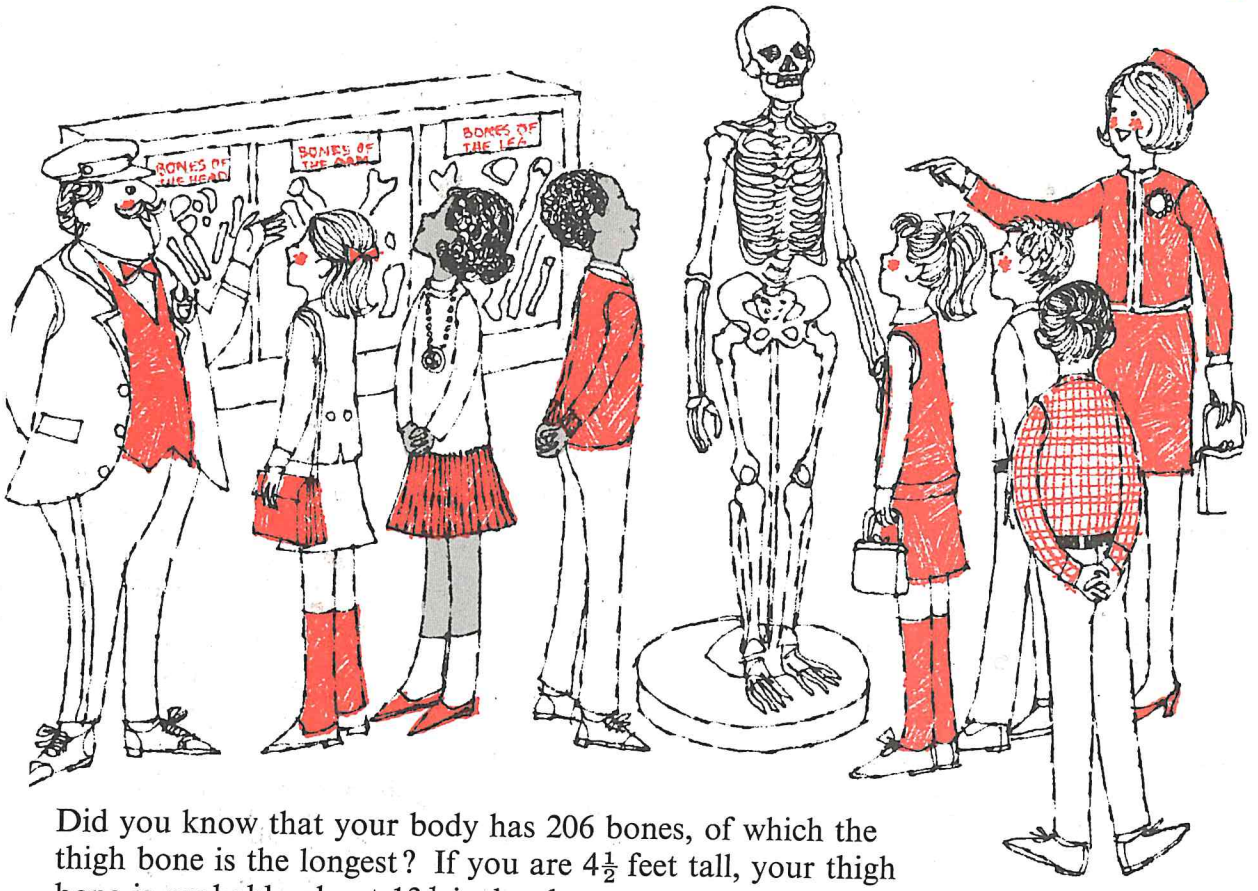
How many triangles are there in these figures?

11.



12.

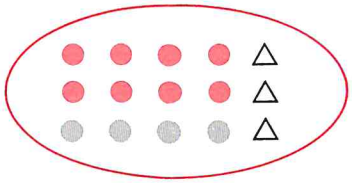




Did you know that your body has 206 bones, of which the thigh bone is the longest? If you are $4\frac{1}{2}$ feet tall, your thigh bone is probably about $13\frac{1}{2}$ inches long.

1. There are 29 bones in the head. The inner ear contains the smallest bone in the whole body. It is called the stirrup bone and is only $\frac{1}{8}$ inch long.
2. In your arm there are about 30 bones. Four-fifths or 24 of them are in either the wrist or hand. This means that $\frac{1}{5}$ of them are in the rest of the arm.
3. Your body has about 600 muscles. The smallest one is only about $\frac{1}{19}$ inch long and controls the smallest bone in your body (the stirrup bone).
4. Your heart beats about 70 times per minute. When you are playing hard, it may beat as often as 260 times in a minute.
5. When you are resting, you take about 18 breaths per minute or about $\frac{3}{10}$ of a breath every second.
6. If you do not have any teeth pulled, you will probably have 32 teeth by the time you are 18 years old.

Separating subsets



$\frac{4}{5}$ of the objects are circles.

$\frac{2}{3}$ of the circles are red.

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$ of the objects are red.

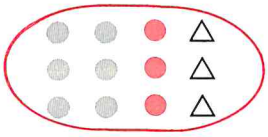
$\frac{3}{4}$ of the rod is red.

$\frac{2}{3}$ of the red part is dotted.

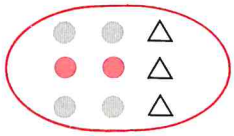
$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$ of the rod is dotted.

EXERCISES

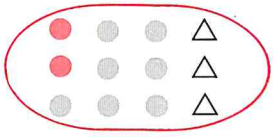
Use the sets to help you solve the equations.



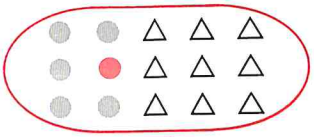
1. $\frac{1}{3} \times \frac{3}{4} = a$



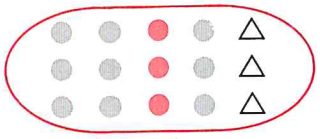
2. $\frac{1}{3} \times \frac{2}{3} = b$



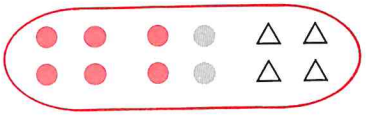
3. $\frac{2}{9} \times \frac{3}{4} = c$



4. $\frac{1}{6} \times \frac{2}{5} = d$



5. $\frac{1}{4} \times \frac{4}{5} = e$

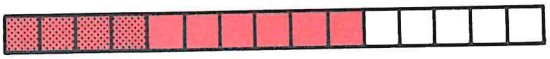


6. $\frac{3}{4} \times \frac{2}{3} = f$

Use the rods to help you solve the equations.

7. $\frac{2}{3} \times \frac{2}{3} = a$

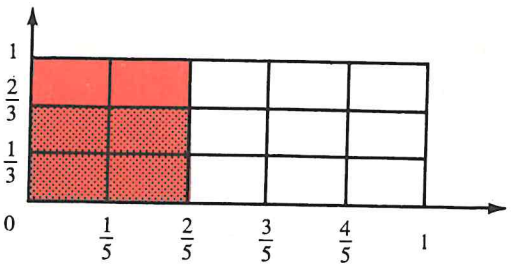
8. $\frac{3}{4} \times \frac{2}{3} = b$



9. $\frac{2}{5} \times \frac{2}{3} = c$

10. $\frac{2}{3} \times \frac{4}{5} = d$

Study the region and the equations.

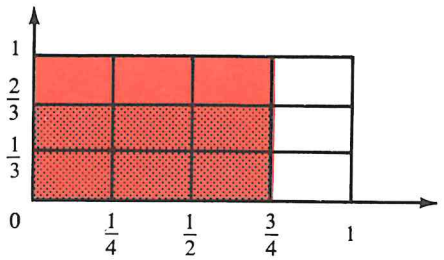


$\frac{2}{3}$ of $\frac{2}{5} = \frac{4}{15}$ of the region is red and dotted.

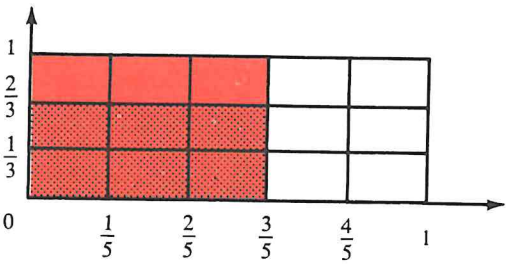
$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$

EXERCISES

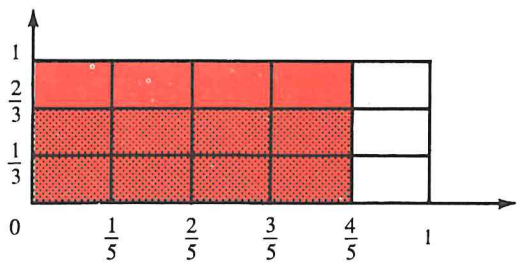
Use the regions to help you solve the equations.



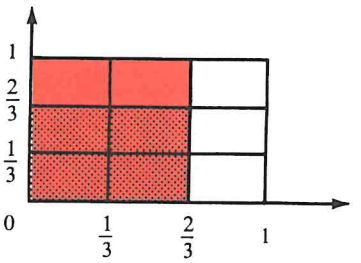
1. $\frac{2}{3} \times \frac{3}{4} = \frac{a}{12} = b$



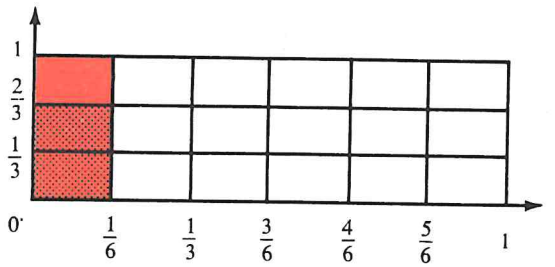
2. $\frac{2}{3} \times \frac{3}{5} = \frac{c}{15} = d$



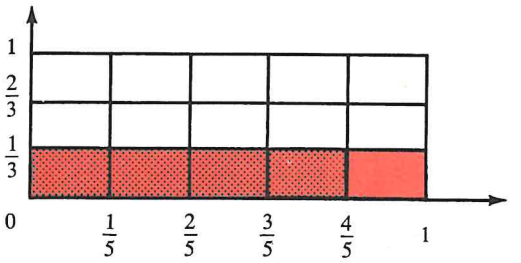
3. $\frac{2}{3} \times \frac{4}{5} = \frac{e}{15}$



4. $\frac{2}{3} \times \frac{2}{3} = \frac{n}{9}$



5. $\frac{2}{3} \times \frac{1}{6} = \frac{i}{18} = j$



6. $\frac{4}{5} \times \frac{1}{3} = \frac{n}{15}$

Using the associative property of multiplication

The product $\frac{2}{3} \times \frac{4}{5}$ has 4 factors which we regroup as these equations show.

$$\frac{2}{3} \times \frac{4}{5} = 2 \times \frac{1}{3} \times 4 \times \frac{1}{5}$$ Renaming as on page 297.

$$= 2 \times 4 \times \frac{1}{3} \times \frac{1}{5}$$ Changing the order of the factors.

$$= (2 \times 4) \times \left(\frac{1}{3} \times \frac{1}{5}\right)$$ Regrouping the factors.

$$= 8 \times \frac{1}{15}$$ Renaming the products.

$$= \frac{8}{15}$$ Renaming as on page 298.

EXERCISES

Copy and complete the equations.

1. $\frac{2}{3} \times \frac{2}{5} = 2 \times \frac{1}{3} \times 2 \times \frac{1}{5}$

$= 2 \times a \times \frac{1}{3} \times \frac{1}{5}$

$= (2 \times a) \times \left(\frac{1}{3} \times \frac{1}{5}\right)$

$= b \times \frac{1}{15}$

$= c$

2. $\frac{2}{5} \times \frac{3}{4} = 2 \times \frac{1}{5} \times 3 \times \frac{1}{4}$

$= 2 \times d \times \frac{1}{5} \times \frac{1}{4}$

$= (2 \times d) \times \left(\frac{1}{5} \times \frac{1}{4}\right)$

$= e \times \frac{1}{20}$

$= f$

3. $\frac{3}{4} \times \frac{2}{3} = (3 \times 2) \times \left(\frac{1}{4} \times \frac{1}{3}\right)$

$= h \times \frac{1}{12}$

$= i$

4. $\frac{2}{5} \times \frac{3}{5} = (2 \times j) \times \left(\frac{1}{5} \times \frac{1}{5}\right)$

$= k \times \frac{1}{25}$

$= l$

Solve the equations.

5. $\frac{2}{3} \times \frac{2}{5} = a \times \frac{1}{15}$

6. $\frac{2}{3} \times \frac{2}{5} = b \times \frac{4}{5}$

7. $\frac{3}{4} \times \frac{3}{5} = d \times \frac{3}{20}$

8. $\frac{3}{4} \times \frac{3}{5} = e \times \frac{9}{5}$

9. $\frac{4}{5} \times \frac{2}{5} = g \times \frac{8}{5}$

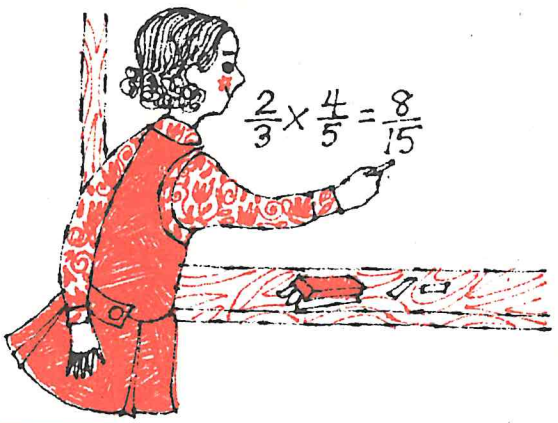
10. $\frac{4}{5} \times \frac{2}{5} = h \times \frac{2}{25}$

Multiplying fractional numbers

Nancy discovered this short method for multiplying fractional numbers.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

To multiply fractional numbers, name the product of the numerators and the product of the denominators.



EXERCISES

Complete the chart.

1.

factors	$\frac{2}{3} \times \frac{4}{5}$	$\frac{3}{4} \times \frac{1}{2}$	$\frac{2}{3} \times \frac{2}{5}$	$\frac{4}{5} \times \frac{3}{5}$	$\frac{2}{5} \times \frac{2}{7}$	$\frac{2}{3} \times \frac{4}{7}$
product of numerators	8	a	d	g	j	m
product of denominators	15	b	e	h	k	n
total product	$\frac{8}{15}$	c	f	i	l	p

Copy and complete the equations.

2. $\frac{1}{3} \times \frac{5}{6} = \frac{1 \times a}{3 \times c} = \frac{e}{18}$

3. $\frac{2}{5} \times \frac{3}{5} = \frac{i \times 3}{5 \times n} = \frac{r}{25}$

4. $\frac{3}{7} \times \frac{3}{4} = \frac{s \times 3}{7 \times u} = \frac{v}{28}$

5. $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times a}{z \times 7} = \frac{c}{21}$

6. $\frac{4}{5} \times \frac{1}{3} = \frac{e \times i}{5 \times 3} = \frac{4}{n}$

7. $\frac{2}{3} \times \frac{2}{5} = \frac{2 \times 2}{r \times s} = u$

Name the products.

8. $\frac{1}{3} \times \frac{2}{5}$

9. $\frac{2}{3} \times \frac{5}{8}$

10. $\frac{2}{5} \times \frac{3}{5}$

11. $\frac{3}{4} \times \frac{3}{5}$

12. $\frac{4}{9} \times \frac{2}{3}$

13. $\frac{3}{7} \times \frac{2}{5}$

Practice with fractional numbers

Complete the sentences. Write $<$, $>$, or $=$ to replace each $\bigcirc$.

1. $\frac{1}{3} + \frac{1}{3} \bigcirc \frac{1}{2}$
2. $\frac{1}{2} + \frac{1}{3} \bigcirc \frac{2}{3}$
3. $\frac{2}{5} + \frac{3}{5} \bigcirc 1$
4. $\frac{2}{8} + \frac{2}{8} \bigcirc 3 \times \frac{2}{8}$
5. $\frac{2}{3} + \frac{1}{3} \bigcirc \frac{3}{4}$
6. $\frac{1}{2} \times 8 \bigcirc \frac{1}{4}$
7. $\frac{1}{3} \times 6 \bigcirc 2$
8. $\frac{1}{5} \times \frac{5}{6} \bigcirc \frac{1}{3}$
9. $\frac{1}{4} \div 3 \bigcirc \frac{1}{3}$
10. $\frac{6}{7} \div 7 \bigcirc \frac{2}{7}$
11. $\frac{8}{5} \div 4 \bigcirc \frac{3}{5}$
12. $\frac{1}{3} \times \frac{1}{2} \bigcirc \frac{1}{3}$

Complete the equations. Write $+$, $-$, or $\times$ to replace each $\triangle$.

13. $4 \triangle \frac{2}{9} = \frac{8}{9}$
14. $\frac{1}{3} \triangle \frac{1}{4} = \frac{1}{12}$
15. $8 \triangle \frac{1}{2} = 4$
16. $\frac{1}{5} \triangle \frac{2}{5} = \frac{3}{5}$
17. $\frac{1}{2} \triangle \frac{1}{6} = \frac{1}{3}$
18. $\frac{2}{3} \triangle \frac{1}{4} = \frac{11}{12}$
19. $\frac{4}{7} \triangle \frac{1}{7} = \frac{3}{7}$
20. $\frac{2}{3} \triangle \frac{2}{3} = \frac{4}{9}$
21. $\frac{1}{2} \triangle \frac{3}{5} = \frac{3}{10}$
22. $\frac{1}{3} \triangle \frac{1}{4} = \frac{1}{12}$
23. $\frac{3}{4} \triangle 12 = 9$
24. $\frac{4}{5} \triangle \frac{1}{10} = \frac{7}{10}$

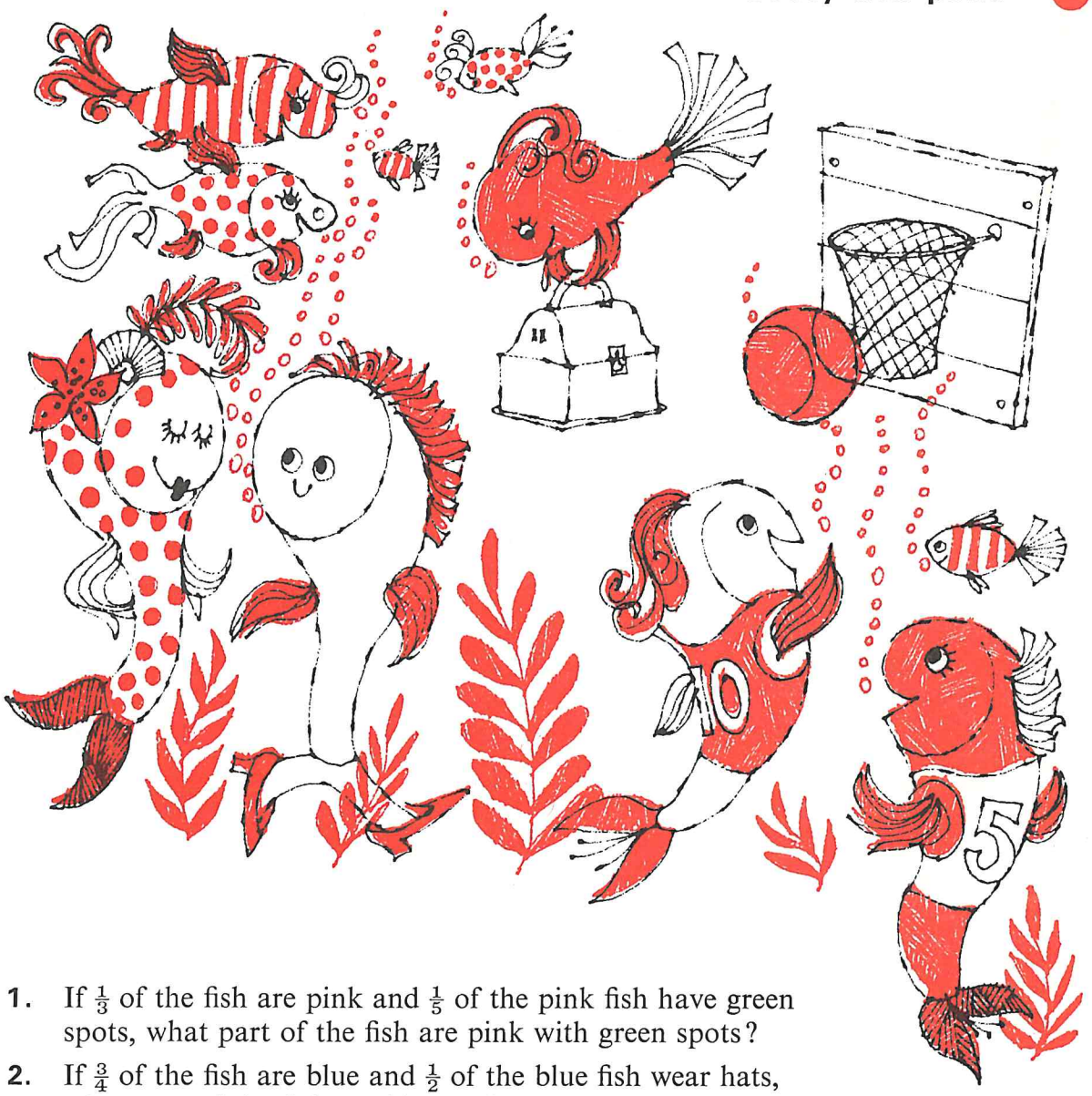
Name the sums or differences.

25. $4\frac{1}{4} + \frac{1}{2}$
26. $\frac{3}{4} + \frac{3}{4}$
27. $2\frac{1}{3} + \frac{2}{3}$
28. $3\frac{1}{4} + 2\frac{5}{8}$
29. $84\frac{5}{6} + 63\frac{1}{6}$
30. $43\frac{7}{8} + 16\frac{5}{8}$
31. $\frac{7}{8} - \frac{3}{4}$
32. $4\frac{1}{4} - 1\frac{7}{8}$
33. $6\frac{1}{2} - \frac{3}{4}$
34. $12\frac{3}{5} - \frac{7}{10}$
35. $64\frac{1}{3} - 32\frac{2}{3}$
36. $97\frac{1}{2} - 90\frac{3}{4}$

Name the products.

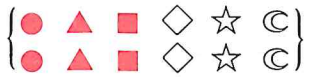
37. $\frac{3}{4} \times \frac{2}{3} = a$
38. $\frac{6}{5} \times \frac{5}{6} = b$
39. $\frac{7}{8} \times \frac{1}{2} = c$
40. $\frac{3}{4} \times \frac{3}{8} = d$
41. $\frac{5}{9} \times \frac{2}{3} = e$
42. $\frac{4}{11} \times \frac{7}{8} = f$
43. $\frac{3}{7} \times \frac{7}{9} = g$
44. $\frac{5}{8} \times \frac{3}{10} = h$

Goofy fish pond



1. If $\frac{1}{3}$ of the fish are pink and $\frac{1}{5}$ of the pink fish have green spots, what part of the fish are pink with green spots?
2. If $\frac{3}{4}$ of the fish are blue and $\frac{1}{2}$ of the blue fish wear hats, what part of the fish are blue and wear hats?
3. Two-thirds of the fish are brown. Two-thirds of the brown fish play basketball. What part of the fish are brown and play basketball?
4. If $\frac{2}{5}$ of the fish are green and $\frac{2}{3}$ of the green fish wear shoes, what part of the fish are green and wear shoes?
5. Three-fourths of the fish go to school. One-third of the fish that go to school bring their lunches. What part of the fish bring their lunches to school?

Multiplication and division are opposites



To name $\frac{1}{2}$ of 12 we can divide by 2.

$12 \div 2 = 6$

One half of 12 is 6.

$\frac{1}{2} \times 12 = 6$

> Instead of multiplying by $\frac{1}{2}$, we can divide by 2.

EXERCISES

Use the sets to help you solve the equations.

1. $\frac{1}{4} \times 8 = 8 \div a = b$

2. $\frac{1}{3} \times 12 = 12 \div c = d$

3. $\frac{1}{2} \times 10 = 10 \div e = f$

4. $\frac{1}{4} \times 12 = 12 \div g = h$

5. $\frac{1}{6} \times 12 = 12 \div i = j$

6. $\frac{1}{5} \times 10 = 10 \div k = l$

Complete the sentence.

7. To name a product with one factor like $\frac{1}{4}$ and the other factor a whole number, we ___?___ the whole number by the denominator of the fraction (4).

Solve the equations.

8. $\frac{1}{2} \times 8 = 8 \div a = b$

9. $\frac{1}{3} \times 6 = 6 \div c = d$

10. $\frac{1}{5} \times 15 = 15 \div e = f$

11. $\frac{1}{4} \times 12 = 12 \div g = h$

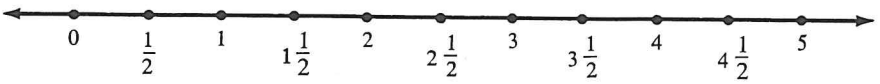
12. $10 \times \frac{1}{2} = 10 \div i = j$

13. $8 \times \frac{1}{4} = 8 \div k = l$

14. $\frac{1}{2} \times 12 = 12 \div m = n$

15. $9 \times \frac{1}{3} = 9 \div p = q$

Multiplication and division are opposites



To name the number of halves in 5 we multiply the number of halves in 1 by 5.

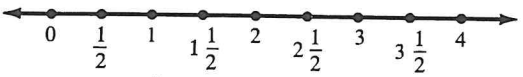
There are 10 halves in 5.

$$1 \div \frac{1}{2} = 2$$
$$5 \times 2 = 10$$
$$5 \div \frac{1}{2} = 10$$

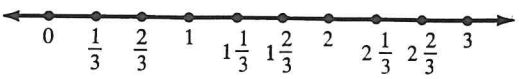
Instead of dividing by $\frac{1}{2}$ we can multiply by 2.

EXERCISES

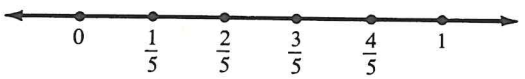
Use the number lines to help you solve the equations.



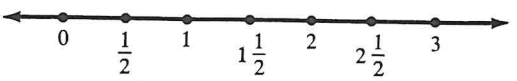
1. $3 \div \frac{1}{2} = a$ **2.** $3 \times b = 6$



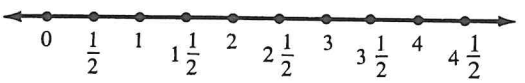
7. $3 \div \frac{1}{3} = g$ **8.** $3 \times h = 9$



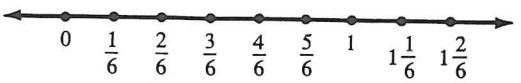
3. $1 \div \frac{1}{5} = c$ **4.** $1 \times d = 5$



9. $6 \div \frac{1}{2} = i$ **10.** $6 \times j = 12$



5. $4 \div \frac{1}{2} = e$ **6.** $4 \times f = 8$



11. $1 \div \frac{1}{6} = k$ **12.** $1 \times l = 6$

*Solve the equations.

13. $3 \times 4 = 12$
$3 \div a = 12$

14. $2 \times 5 = 10$
$2 \div b = 10$

15. $6 \times 2 = 12$
$6 \div c = 12$

16. $5 \times 3 = 15$
$5 \div d = 15$

17. $4 \times 4 = 16$
$4 \div e = 16$

18. $1 \times 5 = 5$
$1 \div f = 5$

Complete the sentence.

19. To name a quotient with the divisor of the form $\frac{1}{n}$ we ___?___ the whole number by the denominator (n).

Division with fractional numbers

Study these equations.

A. $\frac{1}{2} \times 6 = 6 \div 2 = 3$

B. $\frac{1}{2} \times \frac{3}{4} = \frac{3}{4} \div 2 = \frac{3}{8}$

C. $6 \div \frac{1}{2} = 6 \times 2 = 12$

D. $\frac{1}{2} \div \frac{1}{2} = \frac{1}{2} \times 2 = 1$

> In A and B, instead of multiplying by $\frac{1}{2}$, we divided by 2.
>
> In C and D, instead of dividing by $\frac{1}{2}$, we multiplied by 2.

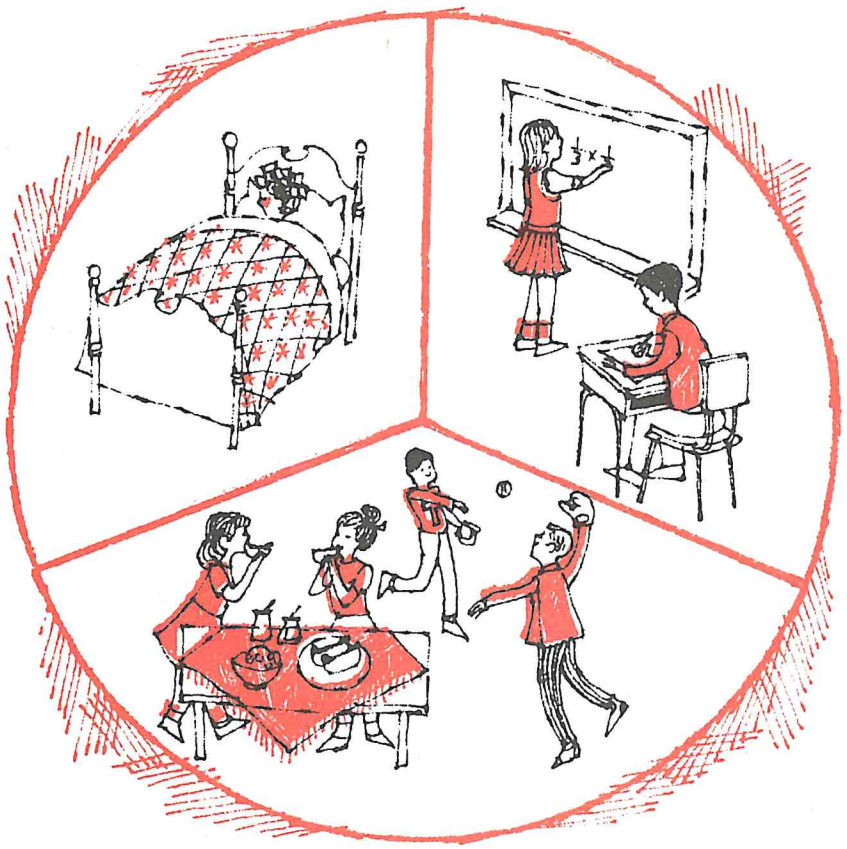
EXERCISES

Name the products.

1. $\frac{1}{2} \times 2$
2. $\frac{1}{3} \times 3$
3. $4 \times \frac{1}{4}$
4. $5 \times \frac{1}{5}$
5. $\frac{1}{6} \times 6$
6. $6 \times \frac{1}{6}$
7. $\frac{1}{7} \times 7$
8. $8 \times \frac{1}{8}$
9. $\frac{2}{3} \times \frac{3}{2}$
10. $\frac{3}{4} \times \frac{4}{3}$
11. $\frac{5}{6} \times \frac{6}{5}$
12. $\frac{2}{5} \times \frac{5}{2}$
13. $\frac{5}{8} \times \frac{8}{5}$
14. $\frac{4}{5} \times \frac{5}{4}$
15. $\frac{7}{8} \times \frac{8}{7}$
16. $\frac{9}{10} \times \frac{10}{9}$

Change each of the divisions to a multiplication, then name the product.

17. $4 \div \frac{1}{3}$
18. $6 \div \frac{1}{5}$
19. $8 \div \frac{1}{2}$
20. $4 \div \frac{1}{5}$
21. $12 \div \frac{1}{5}$
22. $8 \div \frac{1}{9}$
23. $7 \div \frac{1}{4}$
24. $6 \div \frac{1}{7}$
25. $\frac{1}{4} \div \frac{1}{4}$
26. $\frac{1}{4} \div \frac{1}{8}$
27. $\frac{1}{3} \div \frac{1}{3}$
28. $\frac{1}{3} \div \frac{1}{6}$
29. $\frac{1}{2} \div \frac{1}{4}$
30. $\frac{1}{4} \div \frac{1}{12}$
31. $\frac{1}{5} \div \frac{1}{10}$
32. $\frac{1}{10} \div \frac{1}{12}$
33. $\frac{2}{5} \div \frac{1}{5}$
34. $\frac{1}{5} \div \frac{3}{4}$
*35. $\frac{2}{5} \div \frac{3}{8}$
*36. $\frac{2}{5} \div \frac{4}{5}$
*37. $\frac{3}{8} \div \frac{1}{2}$
*38. $\frac{3}{8} \div \frac{5}{8}$
*39. $\frac{3}{8} \div \frac{7}{8}$
*40. $\frac{3}{8} \div \frac{3}{4}$



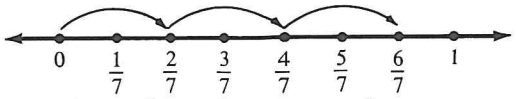
Did you know that we spend about $\frac{1}{3}$ of our lives sleeping, $\frac{1}{3}$ working, and $\frac{1}{3}$ playing and eating?

Here are some more facts with fractional numbers.

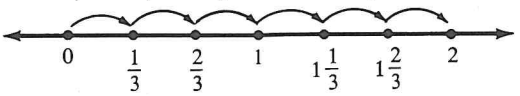
1. About $\frac{9}{10}$ of your body is made up of water.
2. The muscle in your body makes up about $\frac{2}{5}$ of your weight. If you weigh 80 pounds, about 32 pounds is muscle.
3. The length of the thigh bone in your leg is about $\frac{1}{4}$ of your height. If you are 51 inches tall, your thigh bone is about 13 inches long.
4. About $\frac{2}{3}$ of the earth's surface is covered with water.
5. The United States covers about $\frac{1}{17}$ of the earth's land area.
6. About $\frac{1}{3}$ of the people in the world today are over 50 years of age.
7. About $\frac{1}{4}$ of the people living in the world today are Chinese.

CHECKING UP

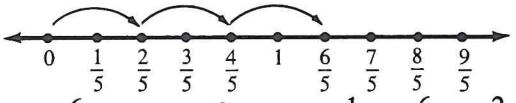
Use the number lines to help you solve the equations.



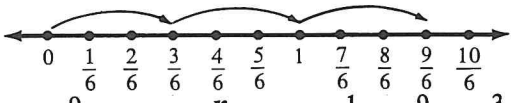
1. $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} = a \times \frac{2}{7} = b$



2. $5 \times \frac{1}{3} = \frac{c}{3} = d$

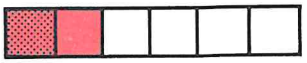


3. $\frac{6}{5} \div 3 = \frac{e}{5}$ **4.** $\frac{1}{i} \times \frac{6}{5} = \frac{2}{5}$

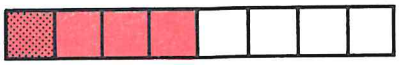


5. $\frac{9}{6} \div 3 = \frac{n}{6}$ **6.** $\frac{1}{s} \times \frac{9}{6} = \frac{3}{6}$

Study the rods and solve the equations.

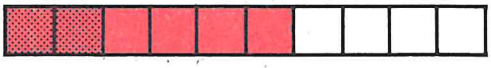


7. $\frac{1}{2} \times \frac{1}{3} = a$ **8.** $\frac{1}{3} \div b = \frac{1}{6}$



9. $\frac{1}{4} \times \frac{1}{2} = c$ **10.** $\frac{1}{2} \div d = \frac{1}{8}$

11. $\frac{1}{3} \times \frac{2}{3} = e$ **12.** $\frac{2}{3} \div f = \frac{2}{9}$



13. $\frac{1}{3} \times \frac{3}{5} = g$ **14.** $\frac{3}{5} \div h = \frac{1}{5}$

Copy and complete the equations.

15.
$$\frac{2}{5} \times \frac{3}{4} = 2 \times \frac{1}{5} \times 3 \times \frac{1}{4}$$
$$= 2 \times a \times \frac{1}{5} \times \frac{1}{4}$$
$$= (2 \times a) \times \left(\frac{1}{5} \times \frac{1}{4}\right)$$
$$= b \times \frac{1}{20}$$
$$= c$$

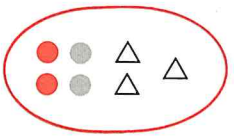
16.
$$\frac{3}{5} \times \frac{3}{5} = 3 \times \frac{1}{5} \times 3 \times \frac{1}{5}$$
$$= 3 \times d \times \frac{1}{5} \times \frac{1}{5}$$
$$= (3 \times d) \times \left(\frac{1}{5} \times \frac{1}{5}\right)$$
$$= e \times \frac{1}{25}$$
$$= f$$

Name the products.

17. $\frac{2}{3} \times \frac{4}{7}$ **19.** $\frac{4}{5} \times \frac{2}{3}$ **21.** $\frac{5}{8} \times \frac{2}{3}$ **23.** $\frac{4}{9} \times \frac{1}{3}$

18. $\frac{2}{5} \times \frac{3}{5}$ **20.** $\frac{1}{3} \times \frac{3}{8}$ **22.** $\frac{2}{5} \times \frac{4}{7}$ **24.** $\frac{5}{6} \times \frac{2}{5}$

Separating subsets helps us multiply and divide with fractional numbers.



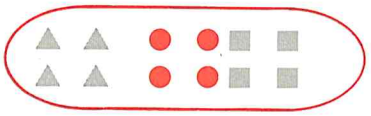
$\frac{1}{2} \times \frac{4}{7} = \frac{2}{7}$

$\frac{4}{7} \div 2 = \frac{2}{7}$

Solve the equations.

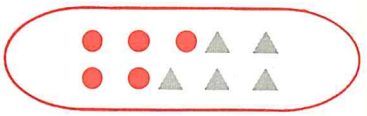
1. $\frac{1}{4} \times 8 = a$

2. $8 \div b = 2$



3. $\frac{1}{3} \times 12 = c$

4. $12 \div d = 4$



5. $\frac{1}{2} \times 10 = e$

6. $10 \div f = g$

The associative property can be used to multiply fractional numbers.

Solve the equations.

7. $\frac{2}{5} \times \frac{3}{4} = 2 \times \frac{1}{5} \times 3 \times \frac{1}{4}$

$= 2 \times a \times \frac{1}{5} \times \frac{1}{4}$

$= (2 \times a) \times \left(\frac{1}{5} \times \frac{1}{4}\right)$

$= b \times \frac{1}{20}$

$= c$

8. $\frac{4}{5} \times \frac{2}{3} = 4 \times \frac{1}{5} \times 2 \times \frac{1}{3}$

$= 4 \times d \times \frac{1}{5} \times \frac{1}{3}$

$= (4 \times d) \times \left(\frac{1}{5} \times \frac{1}{3}\right)$

$= e \times \frac{1}{15}$

$= f$

To multiply fractional numbers, find the product of the numerators and the product of the denominators.

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

Solve the equations.

9. $\frac{2}{5} \times \frac{3}{5} = a$

10. $\frac{1}{3} \times \frac{4}{7} = b$

11. $\frac{4}{5} \times \frac{1}{2} = c$

12. $\frac{2}{3} \times \frac{5}{6} = d$

13. $\frac{3}{4} \times \frac{1}{2} = e$

14. $\frac{3}{4} \times \frac{3}{5} = f$

LOOKING WAY BACK

Name the sums or differences.

1. $39 + 87$ **2.** $91 - 58$ **3.** $372 + 259$ **4.** $523 - 378$ **5.** $2652 + 8579$ **6.** $6910 - 2754$

7. $3856 + 4703$ **8.** $9516 - 8498$ **9.** $5739 + 1810$ **10.** $8203 - 2354$ **11.** $213000 + 578230$

12. $317 + 265 + 314$ **13.** $289 + 367 + 854$ **14.** $4561 + 3920 + 5178$ **15.** $92034 + 61072 + 45816$ **16.** $31400 + 20034 + 18025$

17. $38 + 54 + 92 + 63 + 84$ **18.** $231 + 456 + 792 + 837 + 598$ **19.** $371 + 922 + 476 + 516 + 283$ **20.** $2976 + 5321 + 4689 + 6572 + 1905$ **21.** $85462 + 37674 + 2918 + 5462$

Name the products.

22. 39×12 **25.** 397×19 **28.** 542×173 **31.** 712×819 **34.** 234×456

23. 57×16 **26.** 254×56 **29.** 695×100 **32.** 596×217 **35.** 902×437

24. 83×22 **27.** 981×35 **30.** 842×376 **33.** 412×609 **36.** 688×317

Name the quotients and remainders.

37. $8\overline{)598}$ **40.** $7\overline{)954}$ **43.** $12\overline{)369}$ **46.** $26\overline{)978}$

38. $32\overline{)640}$ **41.** $18\overline{)785}$ **44.** $52\overline{)1040}$ **47.** $80\overline{)1840}$

39. $23\overline{)487}$ **42.** $49\overline{)503}$ **45.** $9\overline{)888}$ **48.** $72\overline{)1532}$

To name the sum $\left(\frac{1}{2} \times 15\right) + \left(\frac{1}{2} \times 25\right)$, we may use the distributive property.

$$\begin{aligned}\left(\frac{1}{2} \times 15\right) + \left(\frac{1}{2} \times 25\right) &= \frac{1}{2} \times (15 + 25) \\ &= \frac{1}{2} \times 40 \\ &= 20\end{aligned}$$

Use the distributive property to name the sums.

1. $\left(\frac{1}{3} \times 10\right) + \left(\frac{1}{3} \times 20\right)$
2. $\left(\frac{1}{2} \times 9\right) + \left(\frac{1}{2} \times 7\right)$
3. $\left(\frac{1}{5} \times 14\right) + \left(\frac{1}{5} \times 16\right)$
4. $\left(\frac{1}{4} \times 13\right) + \left(\frac{1}{4} \times 11\right)$
5. $\left(\frac{1}{5} \times 12\right) + \left(\frac{1}{5} \times 13\right)$
6. $\left(\frac{1}{6} \times 13\right) + \left(\frac{1}{6} \times 17\right)$

To name the product $\frac{1}{2} \times 42$, we may use the distributive property in this way:

$$\begin{aligned}\frac{1}{2} \times 42 &= \frac{1}{2} \times (40 + 2) \\ &= \left(\frac{1}{2} \times 40\right) + \left(\frac{1}{2} \times 2\right) \\ &= 20 + 1 \\ &= 21\end{aligned}$$

Use the distributive property to name the products.

7. $\frac{1}{3} \times 33$
8. $\frac{1}{2} \times 52$
9. $\frac{1}{4} \times 48$
10. $\frac{1}{5} \times 55$
11. $\frac{1}{2} \times 34$
12. $\frac{1}{3} \times 66$
13. $\frac{1}{4} \times 48$
14. $\frac{1}{3} \times 39$
*15. $\frac{1}{5} \times 27$

CHAPTER 11 • Numbers, Fractional Numbers, Decimal Fractions

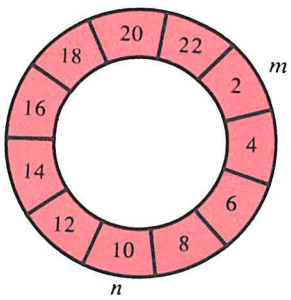
Addition

Addition is the renaming of two addends as a sum.	$16 + 19 = 35$ addends, sum
The order of the addends does not affect the sum. (This is the commutative property.)	$16 + 19 = 19 + 16$
We may group the center addend with either the first or the last addend. (This is one way of stating the associative property.)	$(6 + 4) + 9 = 6 + (4 + 9)$ $10 + 9 = 6 + 13$ $19 = 19$

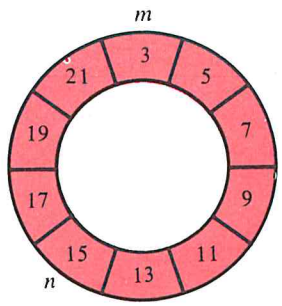
EXERCISES

Name the sums of the addends in the red regions.
Check your work by adding twice, first beginning with the number labeled *m*, then with the number labeled *n*.

1.



2.



Complete the equations to show the commutative property.

3. $13 + 23 = a + b$

4. $c + d = 15 + 17$

5. $24 + e = 35 + f$

6. $78 + 16 = g + h$

7. $i + j = 46 + 29$

8. $k + 41 = l + 37$

Complete the equations to show the associative property.

9. $(8 + 3) + 2 = a + (b + c)$

10. $(13 + d) + 16 = e + (15 + f)$

11. $50 + (20 + 30) = (g + h) + i$

12. $j + (24 + k) = (35 + l) + 46$

We have learned several ways to set up our additions. Here is one which was used some centuries ago. We show the method in 3 forms.

A	B	C
4682 + 3954 7000 1500 130 6 8636	4682 + 3954 7 15 13 6 8636	4682 + 3954 ~~7~~~~3~~6 86

EXERCISES

Use method A to name the sums.

1. 5798 + 2531 **2.** 6713 + 3997 **3.** 3924 + 5897 **4.** 9258 + 3476 **5.** 7926 + 1385 **6.** 82476 + 39798

Use method B to name the sums.

7. 6547 + 3581 **8.** 8274 + 3961 **9.** 5423 + 7869 **10.** 2376 + 8415 **11.** 3915 + 4238 **12.** 64782 + 71969

Use method C to name the sums.

13. 4983 + 5764 **14.** 1392 + 9587 **15.** 3627 + 1894 **16.** 2874 + 9356 **17.** 8175 + 2943 **18.** 58894 + 88248

Use your regular method to name the sums.

19. 34
79
68
54
23
+ 17

20. 57
92
81
64
47
+ 38

21. 68
15
42
37
59
+ 78

22. 54176
29358
+ 37154

23. 7219589
5998732
+ 4835761

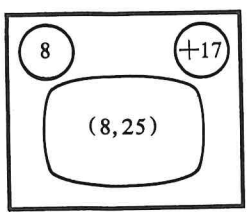
24. 98347657
32185427
+ 47639816

Subtraction

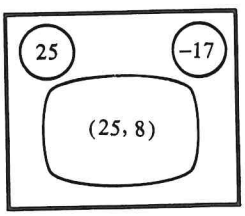
Subtraction is the renaming of a sum and one addend as the other addend. This missing addend is usually called the difference.	$\underset{\text{sum}}{14} - \underset{\text{addend}}{6} = \underset{\text{addend (difference)}}{8}$
Order is important in subtraction.	$14 - 6 \neq 6 - 14$
Subtraction is the opposite of addition.	$14 - 6 = 8$ $6 + 8 = 14$

EXERCISES

Complete the sets of ordered pairs.



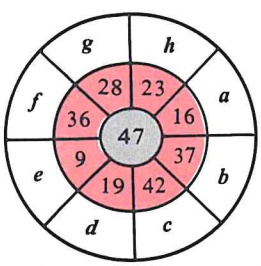
1. $\{(8, 25), (13, a), (17, b), (42, c), (96, d), (109, e)\}$



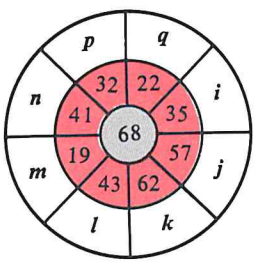
2. $\{(25, 8), (30, f), (34, g), (59, h), (113, i), (126, j)\}$

Name the difference between the sum given in the gray region and the addends in the red region.

3.



4.



Complete the number patterns.

5.

+	90	80	z	i	u
10	100	90	80	70	60

6.

+	89	79	m	n	a
11	100	90	80	70	60

We have learned several ways to set up our subtractions. Here is one like the one for addition on page 317.

A	B	C
4682 − 2946 2000 1700 40 36 1736	4682 − 2946 2 17 4 36 1736	4682 − 2946 1736 1 3

EXERCISES

Use method A to name the differences.

1. 3756 − 2481 **2.** 5942 − 3765 **3.** 8347 − 2691 **4.** 7658 − 4732 **5.** 4726 − 3981 **6.** 43206 − 17594

Use method B to name the differences.

7. 7654 − 3921 **8.** 4381 − 2522 **9.** 6734 − 2985 **10.** 8134 − 4276 **11.** 1365 − 279 **12.** 70813 − 27548

Use method C to name the differences.

13. 9372 − 4851 **14.** 7651 − 3772 **15.** 5413 − 2764 **16.** 6543 − 2877 **17.** 4183 − 2576 **18.** 51160 − 38264

Use your regular method to name these differences.

19. 943 − 176 **21.** 51478 − 35769 **23.** 863576 − 479851 **25.** 3271648 − 1185492 **27.** 63029765 − 42954879

20. 837 − 568 **22.** 39901 − 16987 **24.** 57913 − 24688 **26.** 406352 − 188475 **28.** 321694 − 178065

Multiplication

Multiplication is the renaming of 2 factors as a product.	$3 \times 29 = 87$ factors, product
The order of the factors does not affect the product. (This is the commutative property.)	$3 \times 29 = 29 \times 3$
We may group the center factor with the first or last factor. (This is one way of stating the associative property.)	$(3 \times 4) \times 5 = 3 \times (4 \times 5)$ $12 \times 5 = 3 \times 20$ $60 = 60$
If 2 addends have a common factor, we may **a.** multiply the sum of the other factors by the common factor or **b.** rename each addend and find their sum. (This is the common factor or distributive property.)	**a.** $(3 \times 4) + (5 \times 4) = (3 + 5) \times 4$ $= 8 \times 4$ $= 32$ **b.** $(3 \times 4) + (5 \times 4) = 12 + 20$ $= 32$

EXERCISES

Complete the equations to show the commutative property.

1. $5 \times 18 = a \times b$

2. $17 \times c = 19 \times d$

3. $52 \times 84 = e \times f$

4. $g \times 23 = h \times 62$

Complete the equations to show the associative property.

5. $(2 \times 7) \times 18 = a \times (b \times c)$

6. $9 \times (d \times 61) = (e \times 54) \times f$

Complete the equations to show the distributive property.

7. $(6 \times 7) + (5 \times 7) = (a + b) \times c$

8. $(d \times 8) + (13 \times 8) = (12 + e) \times f$

9. $(23 \times g) + (h \times i) = (j + 17) \times 9$

10. $(4 \times 13) + (8 \times 13) = k \times l$

Name the products.

11. $9 \times 14 = a$ **12.** $7 \times 85 = b$ **13.** $13 \times 62 = c$ **14.** $16 \times 8 = d$

We have learned several ways to set up our multiplications. Here is one like the one for addition on page 317.

A	B	C
4763 × 9 36000 6300 540 27 42867	4763 × 9 36 63 54 27 42867	4763 × 9 ~~36347~~ 4286

EXERCISES

Use method A to name the products.

1. 2729 × 8 **2.** 5416 × 7 **3.** 7853 × 6 **4.** 8237 × 9 **5.** 2945 × 5 **6.** 38562 × 7

Use method B to name the products.

7. 1893 × 4 **8.** 6547 × 7 **9.** 3289 × 3 **10.** 4160 × 5 **11.** 9076 × 6 **12.** 25965 × 5

Use method C to name the products.

13. 4736 × 9 **14.** 5981 × 4 **15.** 6073 × 8 **16.** 9451 × 6 **17.** 8732 × 2 **18.** 86437 × 8

Use your regular method to name the products.

19. 97 × 62 **21.** 235 × 28 **23.** 1476 × 34 **25.** 54183 × 39 **27.** 327684 × 25 **29.** 605437 × 67

20. 85 × 43 **22.** 605 × 39 **24.** 3589 × 27 **26.** 29352 × 44 **28.** 639158 × 51 **30.** 78295 × 78

Division

Division may be the renaming of a product (the dividend) and one factor (the divisor) as a factor (the quotient).

$$\underset{\text{product (dividend)}}{54} \div \underset{\text{factor (divisor)}}{6} = \underset{\text{factor (quotient)}}{9}$$

Division is the opposite of multiplication.

$6 \times 12 = 72$
$72 \div 6 = 12$

In some divisions, there is a remainder. We cannot solve the equation $55 \div 6 = n$ with a whole number. $55 - (6 \times 9) = 1$

If 2 addends have a common divisor, we may **a.** divide the sum of the products by the common factor or **b.** rename each addend and find their sum. (This is the distributive property.)

a. $(36 \div 4) + (24 \div 4) = (36 + 24) \div 4$
$= 60 \div 4$
$= 15$

b. $(36 \div 4) + (24 \div 4) = 9 + 6$
$= 15$

EXERCISES

Complete the number chart.

1.

dividend	42	55	78	25	61	85	17	36	95	47
divisor	6	9	10	4	8	9	5	6	9	5
quotient	7	6	*a*	*c*	*e*	*z*	*r*	*i*	*m*	*n*
remainder	0	1	*n*	*x*	*s*	*u*	*v*	*a*	*x*	*w*

Solve the equations.

2. $50 - (6 \times 8) = a$

3. $67 - (8 \times b) = 3$

4. $c - (9 \times 3) = 0$

5. $38 - (d \times 6) = 2$

6. $23 - (e \times f) = 2$

7. $g - (9 \times 8) = 5$

Use the distributive property to solve the equations.

8. $(45 \div 5) + (25 \div 5) = (a + b) \div c$

9. $(d \div 6) + (24 \div 6) = (18 + e) \div f$

10. $(54 \div g) + (h \div i) = (j + 27) \div 9$

11. $(21 \div 7) + (49 \div 7) = k + l = m$

We have learned several ways to set up our divisions. Method C here is like the one for multiplication on page 321.

A	B	C
4763 9)42869 36000 6869 6300 569 540 29 27 R 2	4763 9)42869 36 68 63 56 54 29 27 R 2	4763 R2 9)42869 652

EXERCISES

Use method A to name the quotients and remainders.

1. $8\overline{)56824}$
2. $7\overline{)69571}$
3. $9\overline{)32473}$
4. $6\overline{)45832}$

Use method B to name the quotients and remainders.

5. $5\overline{)48352}$
6. $4\overline{)34376}$
7. $3\overline{)31928}$
8. $6\overline{)52196}$

Use method C to name the quotients and remainders.

9. $7\overline{)32398}$
10. $8\overline{)44672}$
11. $5\overline{)28349}$
12. $9\overline{)49672}$

Use your regular method to name the quotients and remainders.

13. $9\overline{)38973}$
14. $6\overline{)45302}$
15. $8\overline{)59987}$
16. $5\overline{)39376}$
17. $4\overline{)32756}$
18. $7\overline{)69568}$
19. $3\overline{)26402}$
20. $2\overline{)19237}$

Division

Study these methods for dividing 577,962 by 23.

A

$$\begin{array}{r|l} 23\overline{)577962} & \\ 460000 & 20000 \\ \hline 117962 & \\ 115000 & 5000 \\ \hline 2962 & \\ 2300 & 100 \\ \hline 662 & \\ 460 & 20 \\ \hline 202 & \\ 184 & 8 \\ \hline 18 & 25128 \end{array}$$

B

$$\begin{array}{r} 25128 \\ 23\overline{)577962} \\ 460000 \\ \hline 117962 \\ 115000 \\ \hline 2962 \\ 2300 \\ \hline 662 \\ 460 \\ \hline 202 \\ 184 \\ \hline 18 \end{array}$$

$$577{,}962 - (23 \times 25{,}128) = 18$$

EXERCISES

Use method A to name the quotients and remainders.

1. $27\overline{)395426}$ **3.** $12\overline{)543981}$ **5.** $17\overline{)898215}$ **7.** $43\overline{)975032}$

2. $36\overline{)751978}$ **4.** $65\overline{)734206}$ **6.** $28\overline{)672573}$ **8.** $93\overline{)642975}$

Use method B to name the quotients and remainders.

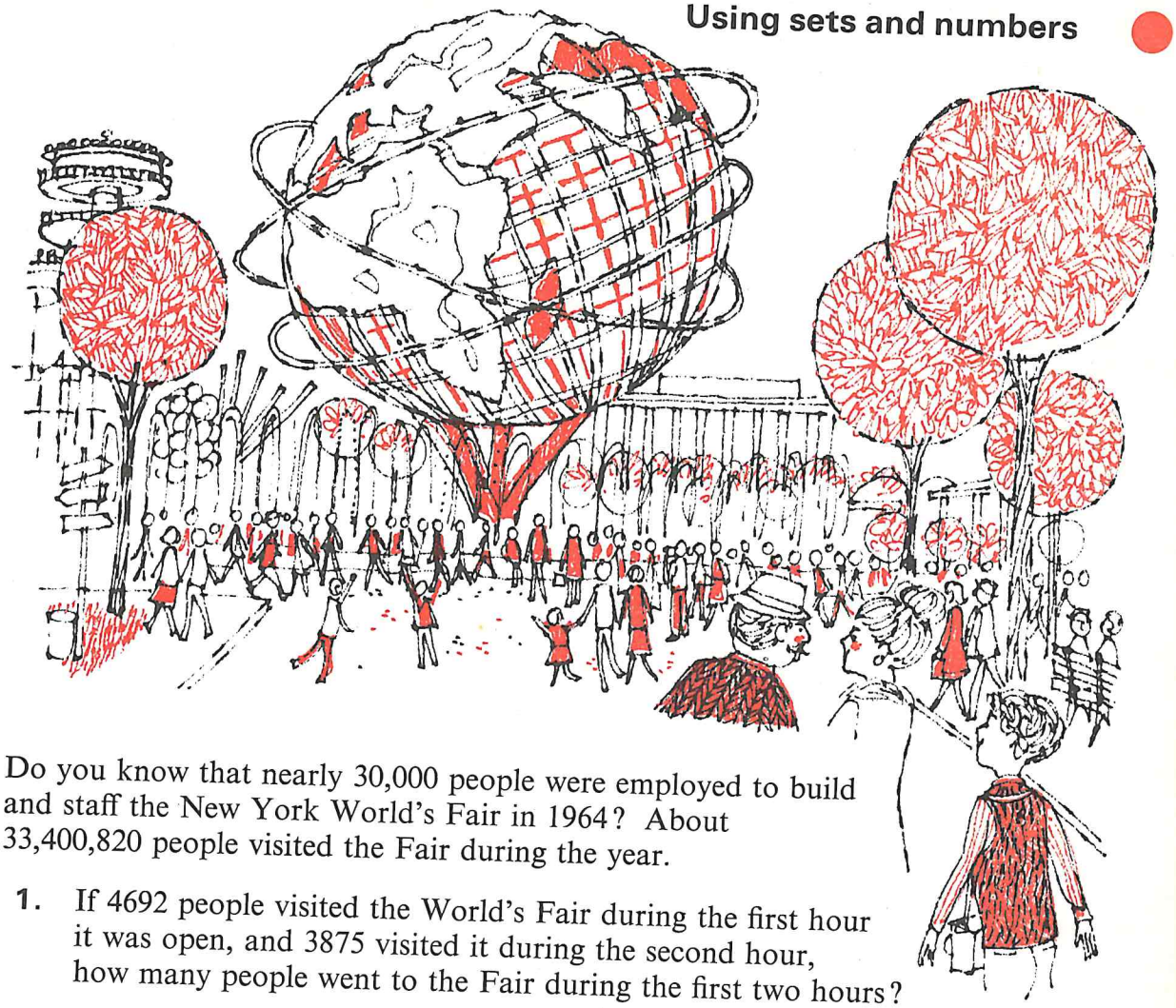
9. $32\overline{)985672}$ **11.** $25\overline{)893476}$ **13.** $14\overline{)389527}$ **15.** $36\overline{)957283}$

10. $19\overline{)628398}$ **12.** $48\overline{)423875}$ **14.** $21\overline{)673249}$ **16.** $68\overline{)578924}$

Solve the problems.

17. If 338,580 people attended the play during its 90 performances, what was the average number of people at each performance?

18. If 578,160 fans attended the 25 football games held in the stadium, what was the average attendance per game?



Do you know that nearly 30,000 people were employed to build and staff the New York World's Fair in 1964? About 33,400,820 people visited the Fair during the year.

1. If 4692 people visited the World's Fair during the first hour it was open, and 3875 visited it during the second hour, how many people went to the Fair during the first two hours?

2. If 9376 persons visited the Fair one morning and 5689 were adults, how many were children?

3. If there were 27,468 people at the Fair and they each spent $4.00, how much did they spend all together?

4. If 23,184 children under twelve, 16,172 children over twelve, and 27,386 adults visited the Fair one day, how many Fair visitors were there that day?

5. Suppose 45,762 people visited the Fair during a 6 hour period. What was the average number per hour?

6. Suppose 574,776 people went to the Fair in 24 days. What was the average number per day?

Fractional numbers

Study this number pattern.

$$24 \div 6 = 4$$
$$12 \div 6 = 2$$
$$6 \div 6 = 1$$
$$3 \div 6 = \frac{3}{6} = \frac{1}{2}$$

Compare these quotients.

$$24 \div 6 = 4 \qquad 25 \div 6 = 4\frac{1}{6}$$
$$4 \times 6 = 24 \qquad 4\frac{1}{6} \times 6 = 25$$

EXERCISES

Complete the number patterns.

1.
$$81 \div 3 = 27$$
$$27 \div 3 = 9$$
$$9 \div 3 = a$$
$$3 \div 3 = b$$
$$1 \div 3 = c$$

2.
$$16 \div 2 = 8$$
$$8 \div 2 = 4$$
$$4 \div 2 = d$$
$$2 \div 2 = e$$
$$1 \div 2 = f$$

3.
$$250 \div 5 = 50$$
$$50 \div 5 = 10$$
$$10 \div 5 = j$$
$$5 \div 5 = k$$
$$1 \div 5 = l$$

4.
$$32 \div 4 = 8$$
$$16 \div 4 = 4$$
$$8 \div 4 = m$$
$$4 \div 4 = n$$
$$2 \div 4 = a$$

5.
$$108 \div 9 = 12$$
$$54 \div 9 = 6$$
$$27 \div 9 = s$$
$$9 \div 9 = t$$
$$3 \div 9 = u$$

6.
$$162 \div 6 = 27$$
$$54 \div 6 = 9$$
$$18 \div 6 = v$$
$$6 \div 6 = w$$
$$2 \div 6 = x$$

Solve the equations.

7.
$$21 \div 3 = a$$
$$7 \times 3 = b$$
$$22 \div 3 = c$$
$$7\frac{1}{3} \times 3 = d$$

8.
$$4 \times 7 = a$$
$$4 \times 7\frac{1}{4} = b$$
$$5 \times 6 = c$$
$$5 \times 6\frac{1}{5} = d$$

9.
$$25 \div 5 = e$$
$$5 \times 5 = f$$
$$27 \div 5 = g$$
$$5\frac{2}{5} \times h = 27$$

10.
$$3 \times 8 = e$$
$$3 \times 8\frac{1}{3} = f$$
$$4 \times 8 = g$$
$$4 \times 8\frac{1}{4} = h$$

11.
$$54 \div 9 = i$$
$$6 \times 9 = j$$
$$57 \div 9 = k$$
$$6\frac{3}{9} \times l = 57$$

12.
$$6 \times 4 = i$$
$$6 \times 4\frac{1}{6} = j$$
$$8 \times 3 = k$$
$$8 \times 3\frac{1}{8} = l$$

Study these examples.

$$4\frac{2}{3} = 4\frac{4}{6}$$
$$+\ 5\frac{5}{6} = 5\frac{5}{6}$$
$$9\frac{9}{6} = 10\frac{3}{6} = 10\frac{1}{2}$$

$$6\frac{1}{4} = 5\frac{10}{8}\ 6\frac{2}{8}$$
$$-\ 4\frac{3}{8} = -\ 4\frac{3}{8}$$
$$1\frac{7}{8}$$

EXERCISES

Solve the equations.

1. $\frac{5}{4} = \frac{4}{4} + \frac{1}{4} = a$
2. $\frac{5}{3} = \frac{3}{3} + \frac{2}{3} = b$
3. $\frac{7}{5} = \frac{c}{5} + \frac{2}{5} = d$
4. $\frac{11}{8} = \frac{8}{8} + \frac{e}{8} = f$
5. $\frac{10}{6} = \frac{a}{6} + \frac{x}{6} = i$
6. $\frac{10}{7} = \frac{u}{7} + \frac{v}{7} = l$
7. $\frac{15}{9} = \frac{z}{9} + \frac{n}{9} = t$
8. $\frac{17}{10} = \frac{a}{10} + \frac{e}{10} = r$
9. $\frac{14}{9} = \frac{s}{9} + \frac{v}{9} = u$

Name the sums.

10. $3\frac{1}{4} = 3\frac{1}{4}$ / $+\ 2\frac{1}{2} = 2\frac{2}{4}$
11. $6\frac{1}{6} = 6\frac{1}{6}$ / $+\ 3\frac{2}{3} = 3\frac{4}{6}$
12. $8\frac{3}{5}$ / $+\ 1\frac{7}{10}$
13. $9\frac{3}{4}$ / $+\ 3\frac{5}{8}$
14. $7\frac{2}{3}$ / $+\ 8\frac{7}{9}$

Name the differences.

15. $7\frac{1}{2}$ / $-\ 2\frac{1}{4}$
16. $9\frac{1}{2}$ / $-\ 5\frac{3}{8}$
17. $10\frac{1}{3}$ / $-\ 4\frac{1}{6}$
18. $11\frac{7}{10}$ / $-\ 6\frac{1}{5}$
19. $13\frac{3}{4}$ / $-\ 7\frac{5}{8}$
20. $14\frac{1}{6}$ / $-\ 8\frac{4}{6}$
21. $15\frac{2}{5}$ / $-\ 9\frac{4}{5}$
22. $7\frac{3}{7}$ / $-\ 1\frac{6}{7}$
23. $9\frac{4}{9}$ / $-\ 4\frac{8}{9}$
24. $12\frac{3}{8}$ / $-\ 3\frac{5}{8}$

Fractional numbers

Study these examples.

$$\frac{3}{8} \times \frac{2}{5} = \frac{3 \times 2}{8 \times 5} = \frac{6}{40} = \frac{3}{20}$$

$$2\frac{1}{3} \times 3\frac{3}{4} = \frac{7}{3} \times \frac{15}{4} = \frac{105}{12} = 8\frac{9}{12} = 8\frac{3}{4}$$

EXERCISES

Name the products.

1. $\frac{2}{3} \times \frac{4}{5} = a$
2. $\frac{3}{7} \times \frac{4}{5} = b$
3. $\frac{2}{9} \times \frac{3}{8} = c$
4. $\frac{1}{2} \times \frac{3}{10} = d$
5. $\frac{3}{8} \times \frac{3}{4} = e$
6. $\frac{6}{7} \times \frac{3}{5} = f$
7. $\frac{4}{5} \times \frac{1}{3} = g$
8. $\frac{5}{8} \times \frac{2}{5} = h$
9. $\frac{4}{7} \times \frac{3}{7} = i$
10. $\frac{5}{6} \times \frac{4}{7} = j$
11. $\frac{5}{9} \times \frac{7}{9} = k$
12. $\frac{2}{9} \times \frac{2}{3} = l$
13. $\frac{7}{8} \times \frac{5}{6} = m$
14. $\frac{1}{3} \times \frac{7}{10} = n$
15. $\frac{3}{4} \times \frac{5}{11} = p$
16. $\frac{2}{5} \times \frac{4}{9} = q$

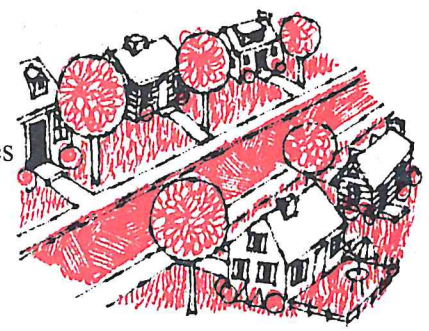
Name the products.

17. $4\frac{1}{2} \times 2\frac{2}{3} = a$
18. $2\frac{1}{5} \times 2\frac{3}{4} = b$
19. $6\frac{1}{4} \times 3\frac{1}{5} = c$
20. $6\frac{2}{3} \times 5\frac{2}{3} = d$
21. $1\frac{5}{8} \times 4\frac{2}{3} = e$
22. $2\frac{5}{6} \times 3\frac{1}{3} = f$
23. $5\frac{1}{5} \times 4\frac{1}{4} = g$
24. $2\frac{4}{9} \times 3\frac{2}{7} = h$
25. $3\frac{2}{3} \times 1\frac{7}{9} = i$

*Solve the word problems.

26. If $\frac{1}{2}$ of the houses are white and $\frac{2}{3}$ of the white houses have black doors, what part of the houses are white with black doors?

27. If $\frac{1}{4}$ of the houses are gray and $\frac{3}{5}$ of the gray houses have white shutters, what part of the houses are gray with white shutters?



Study these equations.

A $30 \div 5 = 6$

B $30 \div 10 = 3$

C $3 \div \frac{1}{5} = 3 \times 5 = 15$

D $3 \div \frac{2}{5} = (3 \times 5) \div 2$

$= \frac{15}{2} = 7\frac{1}{2}$

In B and D we **doubled** the **divisor** and **halved** the **quotient.**

EXERCISES

Solve the equations.

1. $3 \div \frac{1}{4} = a$ so $3 \div \frac{3}{4} = b$
2. $5 \div 3 = c$ so $5 \div \frac{2}{3} = d$
3. $7 \div \frac{1}{2} = e$ so $7 \div \frac{2}{2} = f$
4. $8 \div \frac{1}{5} = g$ so $8 \div \frac{3}{5} = h$
5. $2 \div \frac{1}{6} = i$ so $2 \div \frac{5}{6} = j$
6. $3 \div \frac{1}{7} = k$ so $3 \div \frac{3}{7} = l$
7. $6 \div \frac{1}{8} = m$ so $6 \div \frac{7}{8} = n$
8. $1 \div \frac{1}{9} = p$ so $1 \div \frac{5}{9} = q$

Study these equations.

A $30 \div 5 = 6$

B $15 \div 5 = 3$

C $3 \div \frac{1}{5} = 3 \times 5 = 15$

D $\frac{3}{2} \div \frac{1}{5} = (3 \times 5) \div 2$

$= \frac{15}{2} = 7\frac{1}{2}$

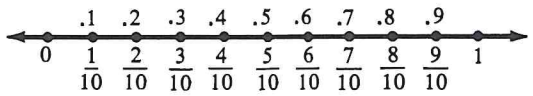
In B and D we **halved** the **dividend** and **halved** the **quotient.**

Solve the equations.

9. $5 \div \frac{1}{3} = a$ so $\frac{5}{2} \div \frac{1}{3} = b$
10. $4 \div \frac{1}{2} = c$ so $\frac{4}{3} \div \frac{1}{2} = d$
11. $7 \div \frac{1}{4} = e$ so $\frac{7}{2} \div \frac{1}{4} = f$
12. $6 \div \frac{1}{3} = g$ so $\frac{6}{5} \div \frac{1}{3} = h$
13. $9 \div \frac{1}{2} = i$ so $\frac{9}{5} \div \frac{1}{2} = j$
14. $5 \div \frac{1}{5} = k$ so $\frac{5}{2} \div \frac{1}{5} = l$
15. $3 \div \frac{1}{7} = m$ so $\frac{3}{5} \div \frac{1}{7} = n$
16. $8 \div \frac{1}{4} = p$ so $\frac{8}{3} \div \frac{1}{4} = q$

Another kind of fraction

The numerals above the number line use place value and a **decimal point.** They are **decimal fractions.** We shall call them **decimals.**



EXERCISES

Write decimals for the fractional numbers. The first one has been done for you.

1. $\frac{5}{10} = .5 =$ 5 tenths
2. $\frac{3}{10} = a = b$ tenths
3. $\frac{1}{10} = c = d$ tenths
4. $\frac{4}{10} = e = f$ tenths
5. $\frac{8}{10} = g = h$ tenths
6. $\frac{9}{10} = i = j$ tenths

Write common fractions for the decimals. The first one has been done for you.

7. $.4 = \frac{4}{10}$
8. $.7 = \frac{x}{10}$
9. $.2 = \frac{n}{10}$
10. $.9 = f$
11. $.1 = h$

Name the sums as decimals. The first one has been done for you.

12. $\frac{3}{10} + \frac{4}{10} = \frac{7}{10} = .7$
13. $\frac{2}{10} + \frac{6}{10} = \frac{a}{10} = b$
14. $\frac{5}{10} + \frac{2}{10} = \frac{c}{10} = d$
15. $\frac{1}{10} + \frac{7}{10} = \frac{e}{10} = f$
16. $\frac{4}{10} + \frac{4}{10} = \frac{s}{10} = h$
17. $\frac{3}{10} + \frac{6}{10} = \frac{i}{10} = j$

Name the differences as decimals. The first one has been done for you.

18. $\frac{7}{10} - \frac{4}{10} = \frac{3}{10} = .3$
19. $\frac{8}{10} - \frac{3}{10} = \frac{a}{10} = b$
20. $\frac{7}{10} - \frac{2}{10} = \frac{c}{10} = d$
21. $\frac{9}{10} - \frac{5}{10} = \frac{e}{10} = f$
22. $\frac{3}{10} - \frac{2}{10} = \frac{x}{10} = h$
23. $\frac{6}{10} - \frac{4}{10} = \frac{i}{10} = j$

Study these examples.

$\frac{3}{10}$ $+\ \frac{4}{10}$ $\frac{7}{10} = .7$	$.3$ $+\ .4$ $.7$	$\frac{6}{10}$ $+\ \frac{7}{10}$ $\frac{13}{10} = 1\frac{3}{10} = 1.3$	$.6$ $+\ .7$ 1.3

EXERCISES

Name the sums or differences as common and as decimal fractions.

1. $\frac{3}{10} + \frac{5}{10}$ **2.** $\frac{2}{10} + \frac{3}{10}$ **3.** $\frac{7}{10} - \frac{3}{10}$ **4.** $\frac{1}{10} + \frac{5}{10}$ **5.** $\frac{5}{10} + \frac{2}{10}$ **6.** $\frac{9}{10} - \frac{4}{10}$ **7.** $\frac{4}{10} + \frac{3}{10}$

Name the sums or differences as common and as decimal fractions. The first one has been done for you.

8. $\frac{7}{10} - \frac{2}{10}$
$\frac{5}{10} = .5$

9. $\frac{3}{10} + \frac{2}{10}$
$\frac{a}{10} = b$

10. $\frac{11}{10} - \frac{6}{10}$
$\frac{c}{10} = d$

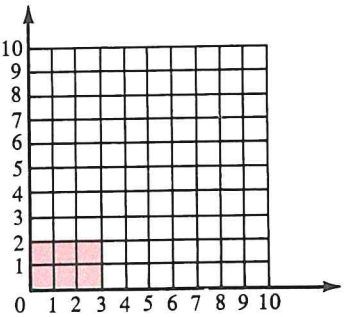
11. $\frac{5}{10} + \frac{7}{10}$
$\frac{e}{10} = f$

12. $\frac{6}{10} + \frac{8}{10}$
$\frac{i}{10} = j$

Name the sums or differences as decimals.

13. $.2 + .5$	**17.** $.9 - .3$	**21.** $.2 + .7$	**25.** $.8 - .4$	**29.** $.7 + .4$	**33.** $.9 + .3$
14. $.3 + .5$	**18.** $.6 + .2$	**22.** $.5 - .1$	**26.** $.8 + .3$	**30.** $.8 - .2$	**34.** $.8 - .5$
15. $.7 - .2$	**19.** $.4 + .5$	**23.** $.7 - .6$	**27.** $.4 - .2$	**31.** $.8 + .4$	**35.** $.6 + .5$
16. $.1 + .8$	**20.** $.6 - .4$	**24.** $.4 + .8$	**28.** $.6 + .3$	**32.** $.9 - .6$	**36.** $.6 - .4$

Hundredths



Each small square is **one hundredth** of the large square. We may write $\frac{6}{100}$ as .06 and read it "six hundredths."

The chart shows place values. The decimal point marks off the fractional numbers from the whole numbers.

hundreds	tens	ones	tenths	hundredths
$100 = 10 \times 10$	$10 = 10 \times 1$	$1 = \frac{1}{10} \times 10$ or $10 \times \frac{1}{10}$	$.1 = \frac{1}{10} \times 1$	$.01 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$
10^2	10	1	$\frac{1}{10}$	$\frac{1}{10^2}$

EXERCISES

Complete the sentences.

1. The first place to the left of the ones' place is the ___?___ place.
2. The first place to the right of the ones' place is the ___?___ place.
3. The second place to the left of the ones' place is the ___?___ place.
4. The second place to the right of the ones' place is the ___?___ place.

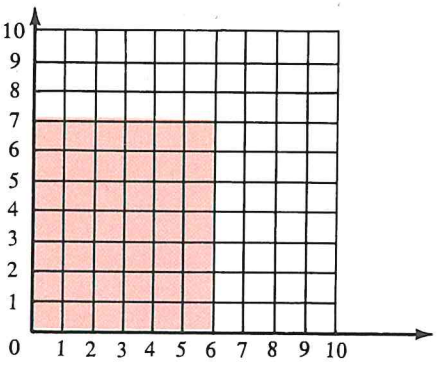
Rename the common fractions as decimals.

5. $\frac{8}{100} = a$
6. $\frac{7}{100} = b$
7. $\frac{3}{100} = c$
8. $\frac{8}{100} = d$
9. $\frac{5}{100} = e$
10. $\frac{1}{100} = f$
11. $\frac{4}{100} = g$
12. $\frac{2}{100} = h$

Rename these decimals as common fractions.

13. $.09 = \frac{a}{100}$
14. $.03 = \frac{s}{100}$
15. $.07 = \frac{c}{100}$
16. $.04 = \frac{x}{100}$
17. $.05 = \frac{e}{100}$
18. $.02 = \frac{n}{100}$
19. $.06 = \frac{v}{100}$
20. $.08 = \frac{i}{100}$

Each small square is $\frac{1}{100}$ of the large square; 42 small squares are $\frac{42}{100}$ of the large square. We may write $\frac{42}{100}$ as .42 and read it "forty-two hundredths."



.42 means 4 tenths and 2 hundredths and can be written as an expanded fraction: $.42 = .4 + .02 = \frac{4}{10} + \frac{2}{100}$.

EXERCISES

Rename the fractions in decimal form.

1. $\frac{16}{100} = a$
2. $\frac{47}{100} = b$
3. $\frac{52}{100} = c$
4. $\frac{6}{100} = d$
5. $\frac{21}{100} = e$
6. $\frac{39}{100} = f$
7. $\frac{61}{100} = g$
8. $\frac{75}{100} = h$
9. $\frac{83}{100} = i$
10. $\frac{14}{100} = j$
11. $\frac{94}{100} = k$
12. $\frac{67}{100} = l$

Rename the decimals as fractions.

13. $.43 = \frac{x}{100}$
14. $.28 = \frac{u}{100}$
15. $.56 = \frac{c}{100}$
16. $.11 = \frac{s}{100}$
17. $.32 = \frac{e}{100}$
18. $.08 = \frac{n}{100}$
19. $.84 = \frac{z}{100}$
20. $.91 = \frac{i}{100}$

Complete the equations.

21. $.76 = .7 + a$
22. $.81 = .8 + b$
23. $.17 = d + .07$
24. $.24 = .2 + e$
25. $.80 = .8 + g$
26. $.08 = h + .08$

Rename the decimals as expanded fractions. The first one has been done for you.

27. $.54 = \frac{5}{10} + \frac{4}{100}$
28. $.67 = \frac{a}{10} + \frac{x}{100}$
29. $.82 = \frac{e}{10} + \frac{v}{100}$
30. $.45 = \frac{r}{10} + \frac{u}{100}$
31. $.16 = \frac{e}{10} + \frac{x}{100}$
32. $.71 = \frac{s}{10} + \frac{n}{100}$

Decimal fractions in addition and subtraction

Study these methods for naming the sum 2.57 + 3.84.

Column

O	Tths	Hths
2	5	7
3	8	4
	1	1
1	3	
5		
6	4	1

Four step

2.57
+ 3.84
.11
1.3
5.
6.41

.07 + .04 = .11
.5 + .8 = 1.3
2. + 3. = 5.
.11 + 1.3 + 5. = 6.41

Short form

2.57
+ 3.84
6.41

EXERCISES

Use the column method to name the sums.

1. 4.24 + 6.32 = *a* **2.** 5.17 + 2.46 = *b* **3.** 3.01 + 7.84 = *c*

Name the sums, first using the 4-step method, then the short form.

4. 6.23 + 7.81 **5.** 2.72 + 9.53 **6.** 6.05 + 7.42 **7.** 5.93 + 2.87 **8.** 8.15 + 3.79 **9.** 4.14 + 8.53

Study these methods for naming the difference 6.42 − 3.67.

Column

O	Tths	Hths
5	13	12
~~6~~	~~4~~	~~2~~
−3	6	7
2	7	5

Four step

5 13 12
~~6.4 2~~
− 3.6 7
.0 5
.7 0
2.0 0
2.7 5

.12 − .07 = .05
1.3 − .6 = .7
5. − 3. = 2.
.05 + .7 + 2. = 2.75

Short form

6.42
− 3.67
2.75

Name the differences using the column method.

10. 4.82 − 3.61 = *a* **11.** 5.16 − 3.78 = *b* **12.** 7.93 − 4.07 = *c*

Name the differences, first using the 4-step method, then the short form.

13. 3.92 − 1.84 **14.** 7.13 − 5.64 **15.** 4.54 − 2.78 **16.** 6.75 − 3.98 **17.** 8.17 − 3.53 **18.** 9.23 − 6.54

Decimal fractions in multiplication

Study these examples.

$3 \times \frac{5}{10} = \frac{15}{10} = 1\frac{5}{10}$

$3 \times .5 = 1.5$

$8 \times \frac{4}{100} = \frac{32}{100}$

$8 \times .04 = .32$

$\frac{7}{10} \times \frac{6}{10} = \frac{42}{100}$

$.7 \times .6 = .42$

EXERCISES

Name the products as common and decimal fractions.
The first one has been done for you.

1. $7 \times \frac{3}{10} = \frac{21}{10} = 2.1$

2. $5 \times \frac{7}{10} = \frac{a}{10} = b$

3. $4 \times \frac{2}{10} = \frac{c}{10} = d$

4. $9 \times \frac{7}{100} = \frac{e}{100} = f$

5. $8 \times \frac{3}{100} = \frac{i}{100} = h$

6. $6 \times \frac{6}{100} = \frac{n}{100} = p$

7. $\frac{4}{10} \times \frac{7}{10} = \frac{r}{100} = q$

8. $\frac{9}{10} \times \frac{3}{10} = \frac{s}{100} = t$

9. $\frac{6}{10} \times \frac{9}{10} = \frac{u}{100} = g$

10. $13 \times \frac{3}{10} = \frac{v}{10} = w$

11. $11 \times \frac{7}{100} = \frac{a}{100} = m$

12. $\frac{3}{10} \times \frac{6}{10} = \frac{z}{100} = y$

Name the products.

13. $6 \times .4 = a$ **14.** $.08 \times 3 = b$ **15.** $.7 \times .9 = c$ **16.** $.4 \times 3 = d$ **17.** $.3 \times .2 = e$ **18.** $5 \times .07 = f$ **19.** $.2 \times 1 = h$ **20.** $.05 \times 12 = i$

Name the products.

21. .07 × 8	**25.** 6 × .8	**29.** .9 × .5	**33.** .08 × 9	**37.** 9 × .05	**41.** .06 × 11
22. .5 × 7	**26.** .4 × .8	**30.** .03 × 5	**34.** .6 × 7	**38.** 13 × .2	**42.** 14 × .3
23. .6 × .3	**27.** .05 × 3	**31.** 7 × .8	**35.** .8 × .3	**39.** 12 × .07	**43.** 17 × .04
24. .09 × 5	**28.** 9 × .9	**32.** .4 × .7	**36.** .02 × 7	**40.** .4 × .3	**44.** .8 × .8

Multiplication

Study these methods for naming the product 3×7.45.

Four-step	Expanded numerals	Short form
$\begin{array}{r} 7.45 \\ \times\ 3 \\ \hline .15 \\ 1.2 \\ 21. \\ \hline 22.35 \end{array}$ $\begin{array}{l} 3 \times .05 = .15 \\ 3 \times .4 = 1.2 \\ 3 \times 7. = 21. \\ \hline 3 \times 7.45 = 22.35 \end{array}$	$\begin{array}{r} 7.45 \\ \times\ 3 \\ \hline \end{array} = \begin{array}{r} 7. + .4 + .05 \\ \times\ 3 \\ \hline 21. + 1.2 + .15 \end{array} = 22.35$	$\begin{array}{r} 7.45 \\ \times\ 3 \\ \hline 22.35 \end{array}$

EXERCISES

Name the products using expanded numerals.

1. $5 \times 1.23 = a$

2. $4 \times 2.76 = b$

3. $2 \times 4.13 = c$

4. $7 \times 3.25 = d$

5. $3 \times 5.26 = e$

6. $8 \times 6.51 = f$

7. $9 \times 1.72 = g$

8. $6 \times 2.43 = h$

9. $7 \times 3.28 = i$

Name the products using the 4-step method.

10. $\begin{array}{r} 2.18 \\ \times\ 4 \\ \hline \end{array}$

11. $\begin{array}{r} 6.37 \\ \times\ 2 \\ \hline \end{array}$

12. $\begin{array}{r} 5.16 \\ \times\ 3 \\ \hline \end{array}$

13. $\begin{array}{r} 7.25 \\ \times\ 7 \\ \hline \end{array}$

14. $\begin{array}{r} 3.86 \\ \times\ .2 \\ \hline \end{array}$

15. $\begin{array}{r} 4.52 \\ \times\ 3 \\ \hline \end{array}$

16. $\begin{array}{r} 9.43 \\ \times\ 8 \\ \hline \end{array}$

17. $\begin{array}{r} 5.24 \\ \times\ 3 \\ \hline \end{array}$

18. $\begin{array}{r} 4.36 \\ \times\ 2 \\ \hline \end{array}$

19. $\begin{array}{r} 6.81 \\ \times\ 4 \\ \hline \end{array}$

20. $\begin{array}{r} 1.31 \\ \times\ 5 \\ \hline \end{array}$

21. $\begin{array}{r} 2.73 \\ \times\ 4 \\ \hline \end{array}$

22. $\begin{array}{r} 4.22 \\ \times\ 3 \\ \hline \end{array}$

23. $\begin{array}{r} 2.34 \\ \times\ 6 \\ \hline \end{array}$

24. $\begin{array}{r} 9.11 \\ \times\ 8 \\ \hline \end{array}$

Name the products using the short form.

25. $\begin{array}{r} 1.37 \\ \times\ 3 \\ \hline \end{array}$

26. $\begin{array}{r} 2.19 \\ \times\ 9 \\ \hline \end{array}$

27. $\begin{array}{r} 6.54 \\ \times\ 7 \\ \hline \end{array}$

28. $\begin{array}{r} 3.18 \\ \times\ 5 \\ \hline \end{array}$

29. $\begin{array}{r} 4.37 \\ \times\ 6 \\ \hline \end{array}$

30. $\begin{array}{r} 9.72 \\ \times\ 2 \\ \hline \end{array}$

31. $\begin{array}{r} 6.83 \\ \times\ 8 \\ \hline \end{array}$

32. $\begin{array}{r} 5.43 \\ \times\ 4 \\ \hline \end{array}$

33. $\begin{array}{r} 8.75 \\ \times\ 3 \\ \hline \end{array}$

34. $\begin{array}{r} 1.13 \\ \times\ 9 \\ \hline \end{array}$

35. $\begin{array}{r} 4.92 \\ \times\ 5 \\ \hline \end{array}$

36. $\begin{array}{r} 7.84 \\ \times\ 2 \\ \hline \end{array}$

37. $\begin{array}{r} 3.72 \\ \times\ 4 \\ \hline \end{array}$

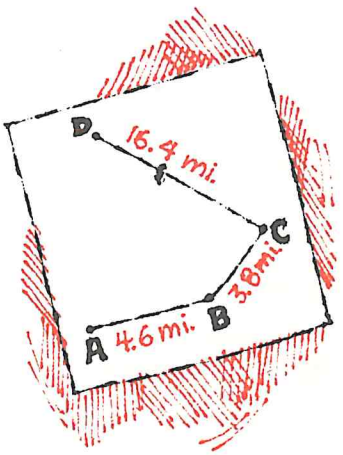
38. $\begin{array}{r} 6.23 \\ \times\ 6 \\ \hline \end{array}$

39. $\begin{array}{r} 9.35 \\ \times\ 8 \\ \hline \end{array}$

1. Before their summer trip the Bevans recorded the mileage on their car. It was 36,288.4 mi. After the trip it was 39,926.8 mi. How far had they traveled?

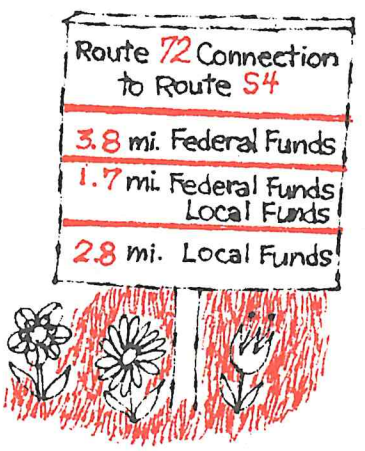
2. The temperature rose one hot summer day until it was 2.6 degrees above the record. If the record had been 97.8 degrees, what was the new record temperature?

3. The distance from A to B is 4.6 mi., from B to C is 3.8 mi., from C to D is 16.4 mi. What is the total distance from A to D? How much farther is it from C to D than from A to B?



4. Where Sean lives the wettest month in the year has an average rainfall of 6.8 inches and the driest has an average rainfall of .4 inches. What is the difference in average rainfall for these months?

5. From 1950 to 1960 the population of Los Angeles grew from 1.97 million to 2.18 million. Express this growth as a decimal fraction of a million.



6. How many miles of road are under construction in the project with this sign?

CHECKING UP

Name the sums and differences.

1. $\begin{array}{r} 43795 \\ +\ 69846 \\ \hline \end{array}$ **2.** $\begin{array}{r} 42962 \\ -\ 28584 \\ \hline \end{array}$ **3.** $\begin{array}{r} 2\frac{1}{6} \\ +\ 3\frac{2}{3} \\ \hline \end{array}$ **4.** $\begin{array}{r} 4\frac{1}{8} \\ +\ 2\frac{1}{6} \\ \hline \end{array}$ **5.** $\begin{array}{r} 6\frac{3}{5} \\ -\ 4\frac{1}{3} \\ \hline \end{array}$

Name the products.

6. $\begin{array}{r} 4376 \\ \times\ 29 \\ \hline \end{array}$ **7.** $\begin{array}{r} 58792 \\ \times\ 46 \\ \hline \end{array}$ **8.** $\frac{2}{3} \times \frac{4}{5} = a$ **9.** $\frac{3}{8} \times \frac{4}{9} = b$ **10.** $\frac{6}{5} \times \frac{3}{4} = c$ **11.** $\frac{9}{10} \times \frac{5}{3} = d$

Name the quotients and remainders.

12. $58\overline{)4002}$ **13.** $76\overline{)11628}$ **14.** $23\overline{)67526}$ **15.** $86\overline{)65582}$

Name the sums and differences.

16. $\begin{array}{r} 4.7 \\ +\ 3.2 \\ \hline \end{array}$ **17.** $\begin{array}{r} 5.8 \\ +\ 5.6 \\ \hline \end{array}$ **18.** $\begin{array}{r} 12.92 \\ +\ 7.64 \\ \hline \end{array}$ **19.** $\begin{array}{r} 3.64 \\ +\ 8.47 \\ \hline \end{array}$ **20.** $\begin{array}{r} 9.87 \\ +\ 6.76 \\ \hline \end{array}$

Name the products.

21. $\begin{array}{r} 4.5 \\ \times\ 6 \\ \hline \end{array}$ **22.** $\begin{array}{r} 6.7 \\ \times\ 8 \\ \hline \end{array}$ **23.** $\begin{array}{r} 8.72 \\ \times\ 4 \\ \hline \end{array}$ **24.** $\begin{array}{r} 10.56 \\ \times\ 8 \\ \hline \end{array}$ **25.** $\begin{array}{r} 22.39 \\ \times\ 9 \\ \hline \end{array}$

Name the total cost of

26. 7 records at \$.69 each

27. 9 albums at \$3.98 each

28. 13 phonographs at \$49.25 each

Name the cost per dress, on the average

29. if 6 dresses cost \$74.82

30. if 7 dresses cost \$92.54

31. if 8 dresses cost \$153.44

32. if 9 dresses cost \$218.31

We use the commutative, associative, and distributive properties in addition and multiplication. We also use the properties of zero as an addend and one as a factor. We use some of these properties and the ideas of opposites in subtraction and division.

1. Show how the associative property is used to rename $47 + 2$ as 49.

2. Show how the commutative property is used to lessen the number of addition or multiplication facts we have to learn.

3. Show how the distributive property is used to name the sum $(4 \times 8) + (7 \times 8)$.

4. Show how the distributive property is used to name the quotient $84 \div 4$.

A decimal fraction uses a decimal point to mark the ones' place. The first place to the right of the ones' place is for tenths, the second place is for hundredths.

5. Name the total value of the 6 in .64.

6. Name the total value of the 6 in .36.

7. Rename **a.** $\frac{4}{10}$ and **b.** $\frac{32}{100}$ as decimal fractions.

8. Rename **a.** .47 and **b.** .03 as common fractions.

Name the sums or differences.

9. $\begin{array}{r} 9.4 \\ +\ 6.8 \\ \hline \end{array}$ **10.** $\begin{array}{r} 8.54 \\ +\ 2.28 \\ \hline \end{array}$ **11.** $\begin{array}{r} 6.57 \\ +\ 8.95 \\ \hline \end{array}$ **12.** $\begin{array}{r} 9.84 \\ -\ 4.37 \\ \hline \end{array}$ **13.** $\begin{array}{r} 6.52 \\ -\ 2.71 \\ \hline \end{array}$ **14.** $\begin{array}{r} 8.32 \\ -\ 1.46 \\ \hline \end{array}$

Name the products.

15. $\begin{array}{r} 7.6 \\ \times\ 8 \\ \hline \end{array}$ **16.** $\begin{array}{r} 4.36 \\ \times\ 7 \\ \hline \end{array}$ **17.** $\begin{array}{r} 18.3 \\ \times\ 4 \\ \hline \end{array}$ **18.** $\begin{array}{r} 5.69 \\ \times\ 9 \\ \hline \end{array}$ **19.** $\begin{array}{r} 6.07 \\ \times\ 5 \\ \hline \end{array}$

LOOKING WAY BACK

Write *T* or *F* to show whether these statements are true or false.

1. It is always possible to divide one whole number by another whole number without having a remainder.
2. If you are working with counting numbers, it is possible to complete any subtraction.
3. A mixed fraction is a name for a fractional number greater than 1.
4. A fraction is in its simplest form if its numerator and denominator have a common factor of 2.
5. In division the order of the product and given factor does not affect the quotient.
6. We cannot use the distributive property to name the sum $(3 \times 4) + (5 \times 7)$ because the addends do not have a common factor.

Name the sums or differences.

7. $4\frac{1}{4} + 2\frac{1}{2}$
8. $3\frac{5}{6} + 2\frac{3}{8}$
9. $5\frac{3}{4} + 2\frac{5}{8}$
10. $5\frac{1}{6} + 6\frac{8}{9}$
11. $6\frac{1}{2} + 4\frac{3}{4}$
12. $1\frac{5}{7} + 3\frac{5}{6}$
13. $9\frac{1}{2} - 5\frac{3}{4}$
14. $8\frac{2}{3} - 4\frac{4}{5}$
15. $2\frac{1}{2} - 1\frac{1}{3}$
16. $3 - 1\frac{2}{7}$
17. $6\frac{1}{4} - 2\frac{1}{3}$
18. $5\frac{1}{4} - 4\frac{5}{6}$

Complete the examples.

19. $94076 + 273854$
20. 46853×27
21. $43\overline{)2463}$
22. $171693 + 484007$
23. 45629×43
24. $54\overline{)8496}$
25. $432891 - 26906$
26. 85209×57
27. $76\overline{)835941}$
28. $90462 - 78388$
29. 13691×73
30. $48\overline{)69351}$

Study these equations.

$$\frac{5}{2} \times \frac{2}{5} = \frac{10}{10} = 1, \text{ so } \frac{5}{2} = 1 \div \frac{2}{5}$$

$$\frac{4}{3} \div \frac{2}{5} = \left(\frac{4}{3} \times 1\right) \div \frac{2}{5}$$

$$= \frac{4}{3} \times \left(1 \div \frac{2}{5}\right)$$

$$= \frac{4}{3} \times \frac{5}{2} = \frac{20}{6} = \frac{10}{3} = 3\frac{1}{3}$$

Solve the equations.

1. $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{a}{3}$

$= \frac{c}{9}$

2. $\frac{3}{5} \div \frac{4}{7} = \frac{3}{5} \times \frac{e}{4}$

$= \frac{i}{20}$

3. $\frac{1}{3} \div \frac{5}{9} = \frac{1}{3} \times \frac{n}{5}$

$= \frac{r}{15}$

4. $\frac{4}{9} \div \frac{2}{3} = \frac{4}{9} \times \frac{3}{v}$

$= \frac{12}{x}$

5. $\frac{5}{8} \div \frac{1}{2} = \frac{5}{8} \times y$

$= \frac{10}{z}$

6. $\frac{5}{6} \div \frac{2}{5} = \frac{5}{6} \times \frac{5}{a}$

$= \frac{25}{c}$

7. $\frac{3}{11} \div \frac{2}{3} = \frac{3}{11} \times e$

$= f$

8. $\frac{3}{5} \div \frac{3}{4} = \frac{3}{5} \times g$

$= h$

9. $\frac{3}{10} \div \frac{2}{9} = i \times j$

$= k$

Study these equations and the division example.

$3 = 3\frac{0}{10}$ or $3\frac{0}{100} = 3.0$ or 3.00

$\frac{3}{4} = .75$

$$\begin{array}{r|l} 4\overline{)3.00} & \\ 2.8 & .7 \\ \hline .20 & \\ .20 & .05 \\ \hline 0 & .75 \end{array}$$

Use division to name the fractional numbers as decimals.

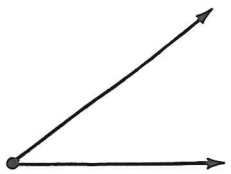
10. $\frac{2}{5}$ **11.** $\frac{3}{5}$ **12.** $\frac{1}{4}$ **13.** $\frac{1}{2}$ **14.** $\frac{3}{5}$ **15.** $\frac{7}{20}$ **16.** $\frac{3}{8}$ **17.** $\frac{3}{25}$ **18.** $\frac{5}{8}$ **19.** $\frac{3}{4}$ **20.** $\frac{7}{8}$ **21.** $\frac{1}{5}$

Abacus (p. 17)
Rods and beads used to show place values.

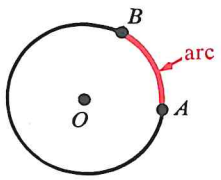
Addend (p. 32)
In an addition, the numbers used to name the sum.

Addition (p. 32)
The renaming of two addends as a sum.

Angle (p. 70)
The union of 2 rays with a common end point.



Arc (p. 262)
Any part of a circle. AB is an arc.



Area (pp. 86, 114)
The measure of a region together with the unit used for measurement.

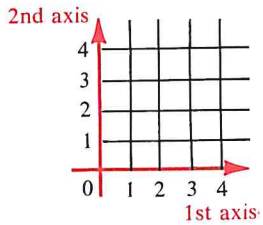
Associative Property (of Addition or Multiplication) (p. 14)
The way in which the addends (or factors) are grouped does not affect the sum (or product).

$$a + (b + c) = (a + b) + c$$

$$a \times (b \times c) = (a \times b) \times c$$

Average (pp. 157–158, 284)
The average of a set of numbers is the number all the numbers would be if they were all the same and their sum did not change.

Axis (p. 88)
One of the two number lines which form a number plane. The axes are usually called the x-axis and the y-axis or the first axis and the second axis.



Bar graph (pp. 92–93)
A graph that shows data by columns.

Base (p. 22)
The number on which a place value system is built. The base of our system is ten.

Base five numeration system (p. 22)
A numeration system using five as its base. The digits used are {0, 1, 2, 3, 4}.

Base ten numeration system (p. 24)
A numeration system using ten as its base. The digits used are {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

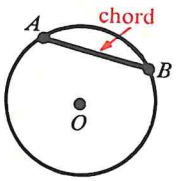
Basic fact (pp. 37, 103)
An addition or subtraction with addends less than 10, or a multiplication or division with factors less than 10.

Braces { } (p. 26)
Symbols used to enclose elements in a set.

Cardinal number (p. 8)
The number that tells how many objects are in a set.

Center (of a circle) (p. 76)
Point in a circular region such that the distance from it to points on the circle is always the same.

Chord (of a circle) (p. 262)
Any line segment that has both end points on the circle.



Circle (pp. 65, 76).
A circle is a closed curve with all its points a fixed distance from a given point.

Clock arithmetic (pp. 187–188)
A system of numbers which may be shown on a circular number line.

Closed curve (p. 64)
A curve that begins and ends at the same point.

Closed surface (p. 82)
A surface which separates space into 3 sets of points, those inside, on, and outside the surface.

Common factor (p. 165)
A number which is the factor of two or more other numbers.

Common multiple (p. 171)
A number which is a multiple of two or more numbers.

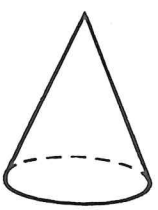
Commutative Property (of Addition or Multiplication) (p. 14)
The order of the addends (or factors) does not affect the sum (or product).

$$a + b = b + a$$

$$a \times b = b \times a$$

Compact numeral (p. 13)
A numeral which makes full use of digits and place value, as 67.

Cone (p. 84)
A closed surface formed by the union of a circular region and a curved surface and coming to a point at the top.

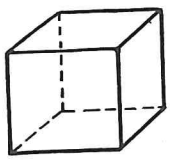


Congruent (pp. 68, 70, 266, 272)
Having the same measure using the same unit.

Counting numbers (p. 2)
The numbers in $\{1, 2, 3, 4, \ldots\}$.

Cross product (pp. 100–101, 135)
The cross product of sets A and B, written $A \otimes B$, is the set of pairs with first members from A, and second members from B.

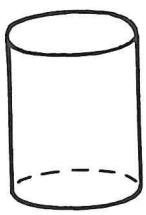
Cube (p. 82)
A cube is a closed surface formed by the union of 6 congruent square regions with common sides.



Cubic unit (p. 87)
A unit of volume used to measure the space inside a closed surface.

Curve (p. 64)
A set of connected points that form a path.

Cylinder (p. 84)
A closed surface formed by the union of two circular regions and a curved rectangular region.



Data (p. 91)
A set of facts using numbers to show something.

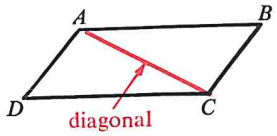
Decimal fraction (p. 330)
A numeral that uses place value and a decimal point to name a fractional number.

Decimal point (p. 330)
In the decimal fraction .32, the point is called the decimal point.

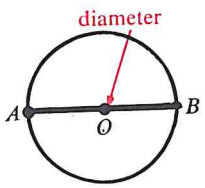
Degree (p. 71)
A standard unit for measuring angles. There are 360° (360 degrees) in a circle.

Denominator (p. 197)
9 is the denominator of the fraction $\frac{2}{9}$. It names the number in the total set.

Diagonal (p. 75)
A line segment in a polygon which joins 2 vertices but is not one of its sides. $\overline{AC}$ is a diagonal of $ABCD$.



Diameter (pp. 76, 85)
A diameter of a circle is any line segment that passes through the center, and has both end points on the circle. *The* diameter is the length of such a line segment.



Difference (p. 32)
The missing addend in a subtraction.

Digit (p. 13)
A symbol used to write numerals. In our system of numeration the digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Disjoint sets (p. 6)
Sets that have no common elements.

Distributive Property of Multiplication (p. 14)
To name the sum of two addends with a common factor, for example

$$(3 \times 5) + (4 \times 5),$$

we may multiply, then add

$$(15 + 20 = 35)$$

or add, then multiply

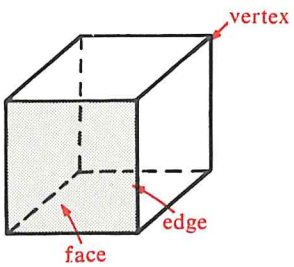
$$(3 + 4 = 7, 7 \times 5 = 35).$$

Divisible (p. 182)
Because $9 \times 3 = 27$, we say that 27 is divisible by 9 and by 3.

Division (pp. 114, 99, 118)
The renaming of a product and one factor as the other factor.

Divisor (p. 119)
The given factor in a division.

Edge (p. 82)
In certain space figures, the intersection of the flat surfaces (faces) of a simple closed surface.



Ellipse (p. 77)
The intersection of a cylinder and a plane not parallel or perpendicular to the base of the cylinder.

Empty set (p. 2)
The set that has no elements. Its cardinal number is zero.

Equal sets (p. 2)
Sets that have exactly the same elements.

Equation (p. 11)
A mathematical sentence stating that 2 numerals name the same number.

Equivalent fractions (p. 209)
Fractions that name the same fractional number.

Equivalent sets (p. 8)
Sets with the same cardinal number.

Estimate (p. 128)
To say what you expect the answer or measurement to be before you do the work.

Expanded numeral (pp. 13, 215)
A numeral such as

$$30 + 7 \quad \text{or} \quad 7 + \tfrac{3}{4}.$$

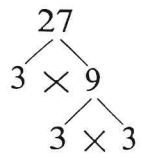
Exponent (pp. 20, 177)
A small numeral written above and to the right of another numeral. The exponent tells how many times the base number appears as a factor. For example, in 5^3, 3 is the exponent; it shows 5 appears as a factor 3 times. $5^3 = 5 \times 5 \times 5$.

Faces (p. 82)
(See "Edge.") The plane regions of a simple closed surface.

Face value (p. 13)
The value a digit always has. In 637 the face value of the 6 is 6.

Factor (pp. 99, 164)
A number that is multiplied by another number to name a product. In the equation $4 \times 3 = 12$, 4 and 3 are factors.

Factor tree (p. 175)
A tree-like arrangement of factors to show a prime factorization as



Finite set (p. 9)
A set whose members can be listed and counted. {All the people in the world}, though very large, is finite.

Fraction (p. 195)
A numeral naming a fractional number.

Fractional number (p. 194)
A number used to compare a subset and a set.

Function (pp. 38, 104, 138)
A set of ordered pairs like

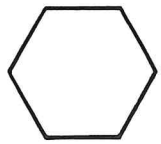
$\{(0, 2), (1, 3), (2, 4), (3, 5), \ldots\}$.

In this function we have the rule "add 2."

Greatest Common Factor (p. 166)
The greatest number in a set of common factors of two numbers.

Graph (p. 91)
A picture used to show data. Different types of graphs are bar, line, and circle.

Hexagon (p. 65)
A six-sided polygon.



Inequality (p. 11)
A number sentence which states that one number is greater (or less) than another.

Infinite set (p. 9)
A non-empty set which is not finite. All of its members cannot be listed and counted.

Intersection (of sets) (p. 6)
The intersection of A and B, written $A \cap B$, is the set of all elements that are common to both A and B.

Kilogram (p. 26)
A basic unit of measurement of weight in the metric system.

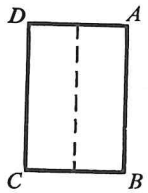
Lattice Method (p. 133)
A way of multiplying.

Least Common Multiple (p. 171)
The least nonzero member of the set of common multiples of two numbers.

Line (p. 64)
A straight curve that goes on indefinitely in two directions. A line is named by any 2 points on it. $\overleftrightarrow{AB}$ is read "line AB."

Line segment (p. 64)
Part of a line with 2 end points. $\overline{AB}$ is read "line segment AB."

Line of symmetry (p. 282)
Separates a curve into matching parts.



Liter (p. 26)
A basic unit of measurement of capacity in the metric system.

Measure (pp. 58, 68, 260)
A number that tells how many units match an object for the property being measured: length, area, or volume.

Median (p. 157)
The number which comes in the middle of a set of numbers when they are arranged in order.

Meter (p. 26)
A basic unit of measurement of length in the metric system.

Metric system (pp. 26–27)
A system of measurement which uses ten as its base.

Mixed numeral (p. 215)
A numeral with part naming a whole number and part naming a fractional number less than 1.

Multiple (p. 168)
The multiple of a number is a product of that number and a whole number. The multiples of 3 may be found by multiplying 3 by the members of the set $\{0, 1, 2, 3, 4, \ldots\}$.

Multiplication (pp. 14, 99)
The renaming of two or more factors as a product.

Number line (p. 12)
A line with points labeled by numbers.

Number pattern (p. 44)
An arrangement of numbers according to a rule.

Number plane (pp. 89, 280)
A plane with points labeled by number pairs.

Numeral (p. 10)
A name for a number.

Numerator (p. 107)
The numerator of the fraction $\frac{2}{9}$ is 2. It names the cardinal number of a subset.

One Property (p. 14)
When one is a factor, the product and the other factor are the same.

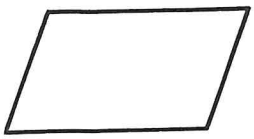
Opposites Property (pp. 14, 36, 308–309)
Subtraction is the opposite of addition, and division is the opposite of multiplication.

Ordered pair (p. 89)
A pair of numbers in which the order is important. For example (3, 6) is not the same as (6, 3).

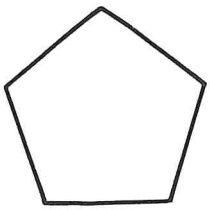
Parallel lines (p. 67)
Two (or more) lines in the same plane that never intersect. The distance between 2 parallel lines is everywhere the same.

Parallel planes (p. 67)
Planes which do not intersect.

Parallelogram (pp. 74, 265)
A quadrilateral with opposite sides parallel and congruent.



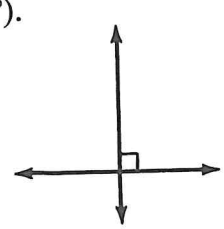
Pentagon (p. 65)
A five-sided polygon.



Perimeter (p. 69)
The distance around a plane figure.

Periods (p. 18)
Grouping place values in hundreds, tens, and ones for ones, thousands, and millions periods.

Perpendicular lines (p. 72)
Lines that intersect to form right angles (90°).



Placeholder (pp. 34–35)
A symbol holding a place for a numeral in a number sentence. In the number sentence $3 + 4 = n$, n is a placeholder.

Place value (p. 13)
The value given to the place in which a digit appears. In 437, 4 is in the hundred's place, 3 is in the ten's place, and 7 is in the one's place.

Plane (pp. 64, 66)
A flat surface that extends without end in all directions.

Point (p. 64)
Undefined term in geometry. It may be thought of as the intersection of 2 lines, and represented by a dot.

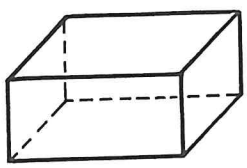
Polygon (p. 65)
A polygon is a simple closed curve formed as the union of line segments.

Power (p. 20)
Any product shown by a base and an exponent. $3 \times 3 = 3^2$ is the second power of 3.

Prime factorization (p. 176)
A numeral which names a number as a product of its prime factors.

Prime number (p. 172)
A number with only 2 factors, itself and 1.

Prism (p. 82)
A prism is a simple closed surface formed as the union of 6 rectangular regions.

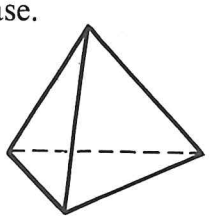


Probability (p. 252)
Comparing the chance of a particular thing happening with all the possibilities.

Product (p. 99)
A number that results from renaming factors. In the equation $2 \times 5 = 10$, 10 is the product.

Protractor (p. 71)
An instrument used for measuring and drawing angles. It is usually circular, and uses degrees as units.

Pyramid (p. 83)
A pyramid is a simple closed surface made up of 3 or more triangular regions and a base.



Quadrilateral (p. 65)
A four-sided polygon.

Quotient (p. 119)
The number resulting from the division of one number by another.

Radius (pp. 76, 85)
A radius of a circle is any line segment with one end point on the circle, the other the center of the circle. *The* radius of a circle is the length of such a line segment.

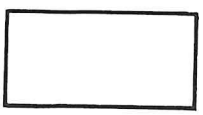
Range (pp. 157, 284)
The difference between the greatest and least members of a set of data.

Rate (pp. 116, 220)
A many-to-one matching of two sets with different members. Gary reads his book at the *rate* of 10 pages in 1 day.

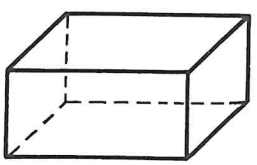
Ratio (pp. 218, 276)
A fractional number used to compare two sets of like elements, one of which may be a subset of the other.

Ray (p. 64)
Part of a line that has one end point and goes endlessly in one direction. $\overrightarrow{AB}$ is read "ray AB" and A is the endpoint.

Rectangle (p. 75)
A quadrilateral with 4 right angles.



Rectangular prism (p. 82)
Closed surface formed by the union of three pairs of rectangular regions.



Region (p. 65)
Points inside a closed curve.

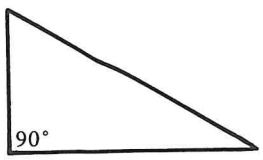
Regular polygon (p. 83)
A polygon with all its sides and all its angles congruent.

Remainder (pp. 118–119)
The number remaining in a division when the greatest multiple of the divisor has been subtracted from the dividend. 5 is the remainder in

$$\begin{array}{r} 4 \\ 6\overline{)29} \\ 24 \\ \hline 5 \end{array}$$

Right angle (p. 72)
The angle formed by the intersection of two perpendicular rays. A right angle has a measurement of 90°

Right triangle (p. 268)
A triangle which has one angle of 90°.



Scale drawing (p. 279)
A similar but smaller or larger figure drawn using a ratio.

Set (p. 2)
A collection of objects or ideas.

Side (pp. 65, 70, 82)
The line segment of a polygon; one of the two rays forming an angle; a plane region of a simple closed surface.

Sieve of Eratosthenes (p. 172)
A method of finding prime numbers discovered by the Greek mathematician, Eratosthenes.

Similar polygons (pp. 277–278)
Two polygons which have the same shape.

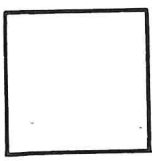
Simple closed curve (p. 64)
A closed curve in a plane such that if you draw a picture of the curve, your pencil will return to the starting point, it will never leave the paper, and it will not go through any point twice.

Size (p. 70)
The measure together with the unit of measure.

Solution (of a number sentence) (p. 35)
A number whose name makes the sentence true.

Sphere (p. 85)
A round closed surface, the points of which are a fixed distance from a fixed point.

Square (p. 75)
A square is a quadrilateral with 4 right angles and 4 congruent sides.



Square unit (p. 86)
A unit of area used to measure the area contained in a region.

Subset (p. 3)
A is a subset of B if the members of A are all members of B.

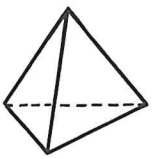
Subtraction (p. 14, 32–33)
The renaming of a sum and an addend; the opposite of addition.

Superset (pp. 3–4)
B is a superset of A if A is a subset of B.

Surface Area (p. 272)
The combined area of all the faces of a closed surface.

Symmetrical curve (p. 282)
A curve that can be separated into two matching parts.

Tetrahedron (p. 82)
A closed surface formed by the union of four triangular regions.

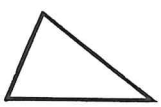


Theorem of Pythagoras (p. 269)
The square of the measure of the longest side of a right triangle is equal to the sum of the squares of the measures of the other two sides. If c is the longest side, then

$$a^2 + b^2 = c^2.$$

Total value (p. 13)
The product of the face and place values of a digit.

Triangle (p. 65)
A simple closed curve formed as the union of three line segments. A polygon with 3 sides.



Triangular prism (p. 82)
Closed surface formed by the union of 3 rectangular regions and a pair of triangular regions.

Union (p. 7)
The union of A and B, written $A \cup B$, is the set of all the elements that are in A or in B.

Universal set (p. 4)
The superset for sets we are talking about.

Vertex (pp. 65, 70, 82)
(See "Edge.") The common end point of the 2 rays that form an angle or the point in which two sides of a polygon intersect.

Volume (p. 87, 270)
The measure of a closed surface together with the units used for measurement.

Whole numbers (p. 2)
The numbers in $\{0, 1, 2, 3, \ldots\}$.

Zero Property (p. 14)
If zero is one addend, the sum and the other addend are the same.

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